CERN-TH-6238/91

Determining the CP-Violating Phase γ

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ABSTRACT

The weak phase γ is conventionally probed by the $B_s \rightarrow \rho^0 K_S$ mode. The predicted rate is tiny. Even if a $B_s \rightarrow \rho^0 K_S$ rate difference could be established, it would not be clear that $\sin 2\gamma$ had been measured, because amplitudes with other weak phases may contribute significantly. Non-CP eigenstates, such as $B_s \rightarrow D_s^\pm K^\mp$, have a two-fold advantage over $B_s \rightarrow \rho^0 K_S$. Their rates are orders of magnitude above that for $B_s \rightarrow \rho^0 K_S$, and they probe the CP-violating phase γ , without any contamination from other weak phases. Detailed time-dependent studies of non-CP eigenstates remove possible final-state phases and extract the weak phase γ .

CERN-TH-6238/91
December 1991

1. Introduction

The Standard Model, where CP non-invariance arises due to complex elements of the 3×3 CKM (Cabibbo-Kobayashi-Maskawa) matrix,¹ does not explain but merely accommodates CP violation. It predicts large effects in the B system, which tests the CKM model.² Rate differences of B^0 decays into CP eigenstates measure $\sin 2\phi$, where ϕ is one of the angles of the unitarity triangle (see Figure 1).³

Although the elements of the CKM matrix depend on only one independent complex phase, they also depend on three mixing angles. Thus, while the constraint $\alpha + \beta + \gamma = \pi$ can be used to eliminate one of the three angles of the unitarity triangle, the remaining two angles, which are the phases of certain combinations of CKM elements, are independent quantities. To test the Standard Model, one would like to overconstrain the triangle through measurements of all three angles. The angles α and β can be determined accurately with many B_d modes.^{4, 5, 6} In contrast, the conventional way of extracting the angle γ uses only a few processes, such as $B_s \rightarrow \rho^0 K_S, \omega K_S$, with predicted tiny rates. Even if a $B_s \rightarrow \rho^0 K_S$ rate difference could be established, it would not be clear that $\sin 2\gamma$ was measured, because amplitudes with other weak phases may contribute significantly. Another way to determine γ involves exclusive B modes with a D^0 .^{7, 8} It suffers, however, from small expected rates, and will not be discussed here.

Time-dependent studies of non-CP eigenstates offer a different way to extract γ . Recently a general method was developed to determine the underlying weak phases from non-CP eigenstates.⁵ A specific application extracts γ from processes such as $B_s \rightarrow D_s^\pm K^\mp$, $B_s \rightarrow D_s^\pm K^\mp$, and $B_s \rightarrow D_s^\pm K^\mp$. That those processes might show large CP violating effects was discussed

earlier in the literature.⁹ Additional modes (for example, $B_s \rightarrow D_s^\pm K^\mp$) can be used to determine γ , once the various helicity amplitudes have been obtained from the angular distribution.¹⁰ The predicted rates are orders of magnitude above that for $B_s \rightarrow \rho^0 K_S$.

Section 2 reviews the extraction of the weak phase from time-dependent studies. Section 3 estimates the branching ratios for various processes. Factorization alone suffices to predict branching ratios above 10^{-4} for $B_s \rightarrow D_s^\pm K^\mp$, $D_s^\pm K^\mp$, $D_s^\pm K^\mp$. In contrast, we find a tiny branching ratio for the CP eigenstate, $B_s \rightarrow \rho^0 K_S$. Equipped with the estimates, Section 4 compares the extraction of the weak phase γ from the different modes. Section 5 presents our conclusions.

2. Extraction of the Weak Phase

To review the extraction of the weak phase from non-CP eigenstates, a few preliminaries must be introduced first.² The eigenstates of the 2×2 B-mass matrix are related to the B^0 and $\bar{B}^0$ states by

$$|B_{L(H)}\rangle = p |B^0\rangle + (-)q |\bar{B}^0\rangle, \quad (2.1)$$

the light (heavy) B^0 being denoted by $B_{L(H)}$. The Standard Model predicts

$$|q/p| \approx 1, \quad (2.2)$$

which holds to a better order than 10^{-2} for the B_d system and to 10^{-4} for the B_s system. The quantity q/p is given by a ratio of CKM elements

$$(q/p)_{B_d} = \frac{V_{td}V_{tb}^*}{V_{ub}V_{ub}^*}, \quad (q/p)_{B_s} = \frac{V_{ts}V_{tb}^*}{V_{ub}V_{ub}^*}. \quad (2.3)$$

A final state of B^0 decay is denoted by f , and its CP conjugate by $\bar{f}$,

$$\bar{f} \equiv \text{CP } f. \quad (2.4)$$

The weak phase can be determined from the four measurables

$$\lambda(f) = \frac{q < f | \bar{B}^0 >}{p < f | B^0 >}, \quad \bar{\lambda}(f) = \frac{p < \bar{f} | B^0 >}{q < \bar{f} | \bar{B}^0 >}, \quad \lambda(\bar{f}), \quad \bar{\lambda}(\bar{f}). \quad (2.5)$$

They are rephasing invariants.¹¹ Not only their magnitudes, but even their phases are observables. Note that $\lambda(\bar{f})$ and $\bar{\lambda}(\bar{f})$ are not independent, because $\lambda(\bar{f}) = 1 / \bar{\lambda}(f)$ and $\bar{\lambda}(\bar{f}) = 1 / \lambda(f)$.

Theorem: If the ratio q/p is a weak phase, as in eq. (2.3), and the amplitudes $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$ each involve a **unique weak phase**, then

$$\begin{aligned} \lambda(f) &= p e^{i(\phi + \Delta)}, \\ \bar{\lambda}(f) &= p e^{i(-\phi + \Delta)}, \\ \lambda(\bar{f}) &= \frac{1}{p} e^{i(\phi - \Delta)}, \\ \bar{\lambda}(\bar{f}) &= \frac{1}{p} e^{-i(\phi + \Delta)}. \end{aligned} \quad (2.6)$$

Here the weak phase of λ is denoted by ϕ , and the strong interaction final-state phase difference between the $\bar{B}^0 \rightarrow f$ and $B^0 \rightarrow f$ amplitudes by Δ . The magnitude of λ is

$$\rho \equiv |\lambda(f)|. \quad (2.7)$$

Because the four λ 's are observables, the weak phase ϕ can, in principle, be determined without any ambiguities from

$$\lambda(f) \lambda(\bar{f}) = e^{2i\phi} \quad \text{or} \quad \bar{\lambda}(f) \bar{\lambda}(\bar{f}) = e^{-2i\phi}. \quad (2.8)$$

Similarly, the phase difference Δ can be unambiguously extracted.

Time-dependent studies of the four $\bar{B}^0(t) \rightarrow f, \bar{f}$ rates determine the magnitudes and phases of the λ 's. The complex mass difference between the

heavy and light B mass eigenstates governs the time-evolution of a B and $\bar{B}$ meson. It is predicted to be mainly real within the Standard Model,

$$\Delta\Gamma/\Delta m \sim 10^{-2}. \quad (2.9)$$

One usually neglects the lifetime difference between the heavy and light B 's,

$$\Delta\Gamma \approx 0. \quad (2.10)$$

When $\Delta\Gamma$ vanishes, a time-dependent study obtains $|\lambda|$ and $\text{Im } \lambda$. A sign ambiguity in $\text{Re } \lambda$ remains. If, however, a non-negligible lifetime difference exists ($\Delta\Gamma \neq 0$), then a careful time-dependent study reveals $|\lambda|$, $\text{Im } \lambda$ and $\text{Re } \lambda$. No ambiguities in the determination of λ remain. This comment holds for any f , for CP eigenstates as well as for non-CP ones.

The determination of $\text{Re } \lambda$ from time-dependent measurements is not realistic for B_d modes, because $\Delta\Gamma/\Gamma \sim 10^{-2}$. It might have more of a chance for B_s modes, where $\Delta\Gamma/\Gamma \lesssim 0.1$.

We now demonstrate explicitly how the λ 's can be determined from experiment. For that purpose, we assume that the conditions of the above theorem hold; the following is then true for the decays of unmixed B^0 and $\bar{B}^0$:

$$|< f | B^0 >| = |< \bar{f} | \bar{B}^0 >| \quad \text{and} \quad |< \bar{f} | B^0 >| = |< f | \bar{B}^0 >|. \quad (2.11)$$

We simplify and use $\Delta\Gamma = 0$. The time-dependent rates of the four processes become

$$\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f) = |< f | B^0 >|^2 e^{-\Gamma t} \left\{ \cos^2 \frac{\Delta m t}{2} + \rho^2 \sin^2 \frac{\Delta m t}{2} - \rho \sin(\phi + \Delta) \sin \Delta m t \right\}, \quad (2.12)$$

$$\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow \bar{f}) = |< \bar{f} | B^0 >|^2 e^{-\Gamma t} \left\{ \cos^2 \frac{\Delta m t}{2} + \rho^2 \sin^2 \frac{\Delta m t}{2} + \rho \sin(\phi - \Delta) \sin \Delta m t \right\}, \quad (2.13)$$

$$\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow \bar{f}) = |\langle f | B^0 \rangle|^2 e^{-\Gamma t} \left\{ \rho^2 \cos^2 \frac{\Delta m t}{2} + \sin^2 \frac{\Delta m t}{2} - \rho \sin(\phi - \Delta) \sin \Delta m t \right\}, \quad (2.14)$$

$$\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f) = |\langle f | B^0 \rangle|^2 e^{-\Gamma t} \left\{ \rho^2 \cos^2 \frac{\Delta m t}{2} + \sin^2 \frac{\Delta m t}{2} + \rho \sin(\phi + \Delta) \sin \Delta m t \right\}, \quad (2.15)$$

where the time-evolved B-meson, which was a B^0 at $t = 0$, is denoted by $\bar{B}_{\text{phys}}^0(t)$.

Fig. 2 shows the $\bar{B}_{s,\text{phys}}^0(t) \rightarrow f, \bar{f}$ rates--where $f = D_s^- K^+, D_s^{*-} K^+$, or $D_s^- K^{*+}$ --for $\Delta m/\Gamma = 10$, $\Delta = 0$, $\sin \phi = \sin(-\gamma) = -0.4$, and $\rho = 1/\sqrt{2}$. Here $\rho = |\langle f | \bar{B}_s \rangle| / \langle f | B_s \rangle|$. CP violation occurs when the time distribution of a process differs from that of its CP-conjugated partner. Note that at $t = 0$ no CP violation happens, because a unique weak phase contributes to the unmixed B^0 decay amplitude. In contrast, at later times large CP violating effects can occur.

From this time-dependent study, the quantities ρ , $\sin(\phi + \Delta)$ and $\sin(\phi - \Delta)$ can be extracted. Within a discrete ambiguity, trigonometry derives the fundamental weak phase ϕ from the latter two quantities.⁵ Explicitly the quantity $\sin \phi$ can be extracted up to a four-fold ambiguity,

$$\sin^2 \phi = \frac{1}{2} \left\{ 1 + s_+ s_- \pm \sqrt{(1 - s_+^2)(1 - s_-^2)} \right\} \quad (2.16)$$

where $s_{\pm} \equiv \sin(\phi \pm \Delta)$ and $s_{\pm} \equiv \sin(\phi - \Delta)$. Here one of the signs on the right hand side gives the true $\sin^2 \phi$, while the other gives $\cos^2 \Delta$. Only a two-fold ambiguity in $\sin \phi$ remains, for processes where $\phi = -\gamma$, such as, $B_s \rightarrow D_s^{\pm} K^{\mp}$, $B_s \rightarrow D_s^{*\pm} K^{\mp}$, and $B_s \rightarrow D_s^{\pm} K^{*\mp}$, because within the CKM model,¹²

$$0.1 \lesssim \sin \gamma \lesssim 1. \quad (2.17)$$

The quantity $\sin \gamma$ is never zero, always positive, and larger than about 0.1.

In fact, because those modes are dominated by colour-allowed amplitudes (see Fig. 3),¹³ it is likely that $\Delta \approx 0$.¹⁴ These modes also involve W-exchange amplitudes, defined by $\bar{b}s \rightarrow \bar{c}u$ and $\bar{b}s \rightarrow \bar{u}c$, but the CKM phases of these amplitudes are the same as those of the colour-allowed ones. Thus the weak phase remains $\phi = -\gamma$. If it can be established that Δ is negligible, then $\sin \gamma$ can be extracted without any ambiguity from the four time-dependent processes, eqs. (2.12)-(2.15).¹⁵

Finding modes where $\Delta \approx 0$ is important. For non-vanishing Δ , the weak phase ϕ must be disentangled from the unknown final-state phase difference Δ . This disentanglement could introduce large errors on $\sin^2 \phi$, see eq. (2.16).

Before comparing the extraction of the weak phase γ from various modes, their rates need to be estimated. This is done in the next section.

3. Branching ratios

This section estimates rates for a few B_s modes. Our estimates use experimental results combined with the factorization assumption.^{14,16-21} Recently factorization was derived from first principles in a particular limit.²² Factorization for two-body modes may be more justifiable for colour-allowed processes than for colour-suppressed ones (see Fig. 4).¹³ The reason is simple. Within the factorization approximation, rates of colour-suppressed modes are tiny with respect to colour-allowed ones. The latter may rescatter and yield a larger rate for the colour-suppressed processes.

Our rate estimates are summarized in Table 1. Reasonable ones can be made for $B_s \rightarrow D_s^- K^+$, $D_s^{*-} K^+$, $D_s^- K^{*+}$, from the observed $B_d \rightarrow D^{(*)-} \pi^+$. In contrast, the values for the other modes in Table 1 reflect order of magnitudes

only. The large uncertainties occur because either a colour-suppressed mode or a heavy $\rightarrow$ light mesonic transition are involved. The appendix explains the entries of Table 1 in detail.

It is worthwhile to point out the reason for the large uncertainty in the $B_s \rightarrow \rho^0 K_S$ rate. The Appendix shows that

$$\frac{B(B_s \rightarrow \rho^0 K_S)}{B(B_d \rightarrow \pi^- \pi^+)} \sim 2 \times 10^{-2}. \quad (3.1)$$

What remains is to estimate the $B_d \rightarrow \pi^- \pi^+$ rate, where large uncertainties exist.

For $|V_{ub}V_{cd}| \approx 0.11$,²³ the BSW model expects¹⁶

$$B(B_d \rightarrow \pi^- \pi^+) \sim 2 \times 10^{-5}, \quad (3.2)$$

while Ref. 21 predicts much smaller values,

$$B(B_d \rightarrow \pi^- \pi^+) \sim 2 \times 10^{-6} - 6 \times 10^{-8}. \quad (3.3)$$

In the table, the first estimate for the $B_s \rightarrow \rho^0 K_S$ rate uses the BSW calculation [eq. (3.2)], while the range within the parenthesis comes from eq. (3.3).

Because the $B_s \rightarrow \rho^0 K_S$ rate is tiny, amplitudes with different weak phases become important and may no longer be neglected. They consist of either rescattering contributions, such as $B_s \rightarrow D_s^- D^{*+} \rightarrow \rho^0 K_S$, or of penguin diagrams ($\bar{b} \rightarrow \bar{d}$). Now that the rate estimates have been discussed, we are able to compare the various processes for extracting γ .

4. Extraction of γ

The extraction of the weak phase γ is compared among various B_s modes. To succeed in the extraction of the weak phase for any of the processes discussed in this note, time-dependent measurements as well as the disinction between a B_s and a $\bar{B}_s$ are required.

The quantity $\sin 2\gamma$ could be extracted directly from a time-dependent study of the CP eigenstates $B_s \rightarrow \omega K_S, \rho^0 K_S$, were the tree-level amplitude to dominate—that is, were the contributions from other weak phases small. Contributions from other weak phases may well be important, however. That is, because they as well as the tree-level amplitude depend on similar CKM magnitudes, and the tree-diagram is colour-suppressed. No reliable determination of $\sin 2\gamma$ is possible from those modes.

To extract γ consider the following. The non-CP eigenstates, $B_s \rightarrow f$ (where f stands for $D_s^- K^+, D_s^- K^{*+}$, or $D_s^- K^{*+}$), have estimated rates which are orders of magnitude above that for the CP eigenstates under consideration. Sizeable CP violating effects can occur because the two interfering amplitudes, $B_s \rightarrow f$ and $B_s \rightarrow \bar{f}$, are comparable. Another nice feature of the non-CP eigenstates is that unique weak phases are involved, which arise from tree diagrams alone. No contaminations from other weak phases are possible. Neither penguin diagrams nor rescatterings with different weak phases exist. The weak phase is truly γ .

We have seen that the non-CP eigenstates have a two-fold advantage over the CP eigenstates in extracting the weak phase γ . Their rates are orders of magnitude larger, and they have no contaminations from other weak phases. On the other hand, an above-mentioned advantage of modes like $\rho^0 K_S, \omega K_S$, is that $\sin 2\gamma$ could be extracted directly from a time-dependent study, if it were possible to neglect the contamination from other weak phases.

In contrast, with the non-CP eigenstates, three observables, $\rho, \sin(\gamma + \Delta)$ and $\sin(\gamma - \Delta)$, are obtained from time-dependent fits. The weak phase is derived from the latter two observables. As a result, γ could suffer from large errors. Because the final state phase difference Δ might be small for the non-CP

eigenstates, as argued in Section 2, there may be no need to disentangle γ from Δ . The extraction of γ would then be more accurate.

We have placed the non-CP eigenstates in contrast to the CP eigenstates. However, the two approaches yield complementary information. Whereas the first measures $\sin \gamma$, the latter measures $\sin 2\gamma$.²⁴ For instance, when $\gamma \approx 90^\circ$, the non-CP eigenstates would show spectacular CP violating effects, while the CP eigenstates would show none. The CKM model predicts the ranges¹²

$$0.1 \lesssim \sin \gamma \lesssim 1, \quad (2.17)$$

$$-1 \lesssim \sin 2\gamma \lesssim 1. \quad (4.1)$$

The prediction, eq. (2.17), would appear to guarantee sizeable CP-violating effects for the non-CP eigenstates.

There exists one caveat connected to the final-state phase difference Δ . Although very unlikely, time-dependent studies of the non-CP eigenstates may show no CP violation—that is, the $\sin \Delta$ terms would have identical coefficients for a process and its CP-conjugated partner [see eqs. (2.12)-(2.15)],

$$\sin(\gamma + \Delta) = -\sin(\gamma - \Delta) = \pm \cos \gamma. \quad (4.2)$$

The final-state phase difference would have to be $\Delta = (1 + 2n) 90^\circ$, in order to satisfy simultaneously $\sin \gamma \neq 0$ [see eq. (2.17)] and the first equality of eq. (4.2). The second equality of eq. (4.2) follows immediately. Even for such a scenario, the angle γ (really $\cos \gamma$) would still be measured. Within a discrete ambiguity, the weak phase γ can be extracted whatever the final-state phase difference happens to be.

The modes $B_s^- \rightarrow D^{(\pm)} \phi$ have also been shown to have potentially large CP violating effects.⁹ Gronau and London further demonstrated that the $B_s^- \rightarrow D^{(\pm)} \phi$ modes in principle allow the unambiguous extraction of γ .⁷ Our rate estimates

indicate, however, that those modes are much suppressed relative to the non-CP eigenstates considered in this paper (see Table 1).

5. Conclusions

The conventional way to measure the angle γ of the unitarity triangle is time-dependent studies of the $B_s^- \rightarrow \rho^0 K_S^-$ mode. The rate of the $B_s^- \rightarrow \rho^0 K_S^-$ process is tiny. Thus, contaminations from other weak phases, in the form of rescattering amplitudes and penguin diagrams, make a clean extraction of γ impossible.

Another way to obtain γ is a time-dependent study of the non-CP eigenstates $B_s^- \rightarrow D_s^\pm K^\mp$, $D_s^\pm K^\mp$, and $D_s^\pm K^{*\mp}$. Not only are their rates orders of magnitude above that for the CP eigenstate, $B_s^- \rightarrow \rho^0 K_S^-$, but they also satisfy a necessary condition for the observation of large CP violation. The two interfering amplitudes are comparable in size. Another important advantage of those non-CP eigenstates is that they probe the weak phase γ , without any contamination from other weak phases. Neither penguin contributions nor rescatterings with different weak phases exist. Those modes truly test the weak phase γ , in contrast to the $B_s^- \rightarrow \rho^0 K_S^-$ mode.

Acknowledgements

We thank F. Le Diberder for a fruitful earlier collaboration, and for useful discussions on this note.

Appendix: Derivation of Rates

This appendix estimates the rates shown in Table 1. In Table 1 the rates for the first three modes can be predicted rather well, in contradistinction to the rest. The first three predictions use the observed²³

$$B(B_d \rightarrow D^{(*)-} \pi^+) = (0.32 \pm 0.07) \times 10^{-2}. \quad (A.1)$$

SU(3) breaking effects enter when the transition form factors of $B_s \rightarrow D_s^{*-}$ are related to the $B_d \rightarrow D^-$ ones. They are expected to be of the order of 10%-20%.²⁵ Neither the SU(3) breaking effects nor the errors they may induce will be considered in this paper. To the best of our knowledge those effects have not yet been studied. Using the well-known K and π decay constants, $f_K = 161$ MeV and $f_\pi = 132$ MeV, the rates for $B_s \rightarrow D_s^- K^+$ and $B_s \rightarrow D_s^{*-} K^+$ are obtained from eq. (A.1) and

$$\frac{B(B_s \rightarrow D^{(*)-} K^+)}{B(B_d \rightarrow D^{(*)-} \pi^+)} = \left| \frac{V_{us}}{V_{ud}} \right|^2 \left(\frac{f_K}{f_\pi} \right)^2 = 0.076. \quad (A.2)$$

They are

$$B(B_s \rightarrow D_s^{*-} K^+) \approx (2.4 \pm 0.5) \times 10^{-4}. \quad (A.3)$$

The $B(B_s \rightarrow D_s^- K^{*+})$ comes from eq. (A.3) and

$$\frac{B(B_s \rightarrow D_s^- K^{*+})}{B(B_s \rightarrow D_s^- K^+)} = 0.93 \left(\frac{f_{K^*}}{f_K} \right)^2 \approx \left(\frac{f_{K^*}}{f_K} \right)^2 = 1.84 \pm 15\%. \quad (A.4)$$

The K^* decay constant is extracted from tau-decays, $f_{K^*} = 218 \pm 15$ MeV.²⁰ We predict

$$B(B_s \rightarrow D_s^- K^{*+}) \approx (4.4 \pm 1.6) \times 10^{-4}. \quad (A.5)$$

Were the rate for $B_s \rightarrow D_s^- K^{*+}$ related to the measured²³

$$B(B_d \rightarrow D^- \rho^+) = (0.9 \pm 0.6) \times 10^{-2}, \quad (A.6)$$

larger errors would occur than given in eq. (A.5). Thus, we choose not to use this measured rate (eq. (A.6)).

Only colour-allowed factorizations have been used so far. The ensuing rate estimates involve either colour-suppressed factorizations or heavy $\rightarrow$ light mesonic transitions, and are less accurate. Because only order of magnitude estimates will be made, no errors will be quoted. The following discussion uses the colour-allowed and -suppressed parameters $|a_1| = 1.11$ and $|a_2| = 0.21$,²⁰ the CKM ratio $|V_{ub}V_{cb}| \approx 0.11$,²³ and the decay constants^{18, 20, 26} $f_{D_s^*} = f_{D_s} = 280$ MeV, $f_{D^*} = f_D = 220$ MeV, $f_\psi = 384$ MeV (from the leptonic width of the Ψ), and $f_\rho = 194$ MeV (from tau decay).

The $B_s \rightarrow K^- D_s^{*+}$ process factorizes into a $B_s \rightarrow K^-$ transition coupled to a D_s^{*+} creation from the vacuum. As a crude estimate for the $B_s \rightarrow K^-$ transition, we use the $B \rightarrow K$ one which is available from the measured rate²³

$$B(B \rightarrow K \Psi) \approx 8 \times 10^{-4}. \quad (A.7)$$

We guess that

$$\frac{B(B_s \rightarrow K^- D_s^{*+})}{B(B \rightarrow K \Psi)} \sim \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left(\frac{f_{D_s^*}}{f_\psi} \right)^2 \left| \frac{a_1}{a_2} \right|^2 = (0.11)^2 \left(\frac{280}{384} \right)^2 \left(\frac{1.11}{0.21} \right)^2 = 0.18. \quad (A.8)$$

Eqs. (A.7) and (A.8) yield

$$B(B_s \rightarrow K^- D_s^{*+}) \sim 1.4 \times 10^{-4}. \quad (A.9)$$

It is interesting to note that an application of a recent update of the BSW model arrives at essentially the same result,²⁰

$$B(B_s \rightarrow K^- D_s^{*+}) \sim 1 \times 10^{-4}. \quad (A.10)$$

Here the theoretical estimate for the colour-suppressed rate of $B^+ \rightarrow \pi^+ \bar{D}^{*0}$ was used in addition to a ratio,

$$B(B^+ \rightarrow \pi^+ \bar{D}^{*0})_{a_2\text{-piece}} = 0.255 (1.292 a_2)^2 10^{-2} = 1.88 \times 10^{-4}, \quad (A.11)$$

$$\frac{B(B_s \rightarrow K^- D_s^{*+})}{B(B^+ \rightarrow \pi^+ \bar{D}^{*0})_{a_2\text{-piece}}} \sim \left| \frac{V_{ub}}{V_{cb}} \right|^2 \left(\frac{f_{D_s^{*+}}}{f_{D^+}} \right)^2 \left| \frac{a_1}{a_2} \right|^2 \approx (0.11)^2 \left(\frac{280}{220} \right)^2 \left(\frac{1.11}{0.21} \right)^2 = 0.55. \quad (A.12)$$

Similarly, another straightforward application of Ref. 20 yields,

$$\frac{B(B_s \rightarrow K^- D_s^{*+})}{B(B_s \rightarrow K^- D_s^{*+})} = 0.94, \quad (A.13)$$

$$\frac{B(B_s \rightarrow K^{*-} D_s^{*+})}{B(B_s \rightarrow K^- D_s^{*+})} \approx 0.64. \quad (A.14)$$

Combining each of the above two equations with eq. (A.9) yields the two rates given in Table 1.

In order to observe large CP violating effects, it is necessary that the rate for the process $B_s \rightarrow f$ be comparable to that for $B_s \rightarrow \bar{f}$ --where $f = D_s^- K^+$, $D_s^{*-} K^+$, and $D_s^- K^{*+}$. Furthermore, the rates are predicted to be large at the 10^{-4} level.

In contrast, the rate for $B_s \rightarrow \rho^0 K_S$ is expected to be tiny. To estimate the tree-level $B_s \rightarrow \rho^0 K_S$ rate, we note that colour-allowed factorization alone reliably predicts⁵

$$\frac{B(B_d \rightarrow \pi^- \rho^+)}{B(B_d \rightarrow \pi^- \pi^+)} = \left(\frac{f_\rho}{f_\pi} \right)^2. \quad (A.15)$$

A less clean prediction is

$$\frac{B(B_s \rightarrow \rho^0 \bar{K}^0)}{B(B_d \rightarrow \pi^- \rho^+)} \sim \frac{1}{2} \left| \frac{a_2}{a_1} \right|^2. \quad (A.16)$$

The Clebsch-Gordan factor of 1/2 arises because the ρ^0 is made from a V-A up-quark current. Combining the last two equations, the ratio is

$$\frac{B(B_s \rightarrow \rho^0 K_S)}{B(B_d \rightarrow \pi^- \pi^+)} \sim \frac{1}{4} \left(\frac{f_\rho}{f_\pi} \right)^2 \left| \frac{a_2}{a_1} \right|^2 = 2 \times 10^{-2}. \quad (A.17)$$

There are wide uncertainties in the $B_d \rightarrow \pi^- \pi^+$ rate. Whereas the BSW model expects¹⁶

$$B(B_d \rightarrow \pi^- \pi^+) \sim 2 \times 10^{-5}, \quad (A.18)$$

Ref. 21 predicts much smaller values,

$$B(B_d \rightarrow \pi^- \pi^+) \sim 2 \times 10^{-6} - 6 \times 10^{-8}. \quad (A.19)$$

The expected rate is

$$B(B_s \rightarrow \rho^0 K_S) \sim 4 \times 10^{-7} (4 \times 10^{-8} - 1 \times 10^{-9}), \quad (A.20)$$

where the first estimate uses the BSW calculation, while the range within the parenthesis comes from eq. (A.19). Estimating $B(B_s \rightarrow \rho^0 K_S)$ by comparing it with $B(B_d \rightarrow \Psi K_S) \approx 3 \times 10^{-4}$ while using the measured $|V_{ub}/V_{cb}|$ ratio gives values consistent with eq. (A.20).

We now turn to the last two entries of Table 1. Various approaches were used to estimate the rates. They all obtain

$$B(B_s \rightarrow \bar{D}^0 \phi) \sim 2 \times 10^{-6}, \quad (A.21)$$

$$\text{and} \quad B(B_s \rightarrow D^0 \phi) \sim 5 \times 10^{-7}. \quad (A.22)$$

Here two approaches are reviewed. The first relates $B_s \rightarrow \bar{D}^0 \phi$ to the observed rate²³

$$B(B_d \rightarrow K^{*0} \Psi) \approx 1.2 \times 10^{-3}. \quad (A.23)$$

The relevant ratio is broken up into three more manageable pieces,

$$\frac{B(B_s \rightarrow \bar{D}^0 \phi)}{B(B_d \rightarrow K^{*0} \Psi)} = \frac{B(B_s \rightarrow \bar{D}^0 \phi)}{B(B_d \rightarrow \bar{D}^0 K^{*0})} \frac{B(B_d \rightarrow \bar{D}^0 K^{*0})}{B(B_d \rightarrow K^{*0} \Psi)}. \quad (A.24)$$

When we equate the $B_s \rightarrow \phi$ and $B_d \rightarrow K^{*0}$ formfactors, the first ratio on the right-hand side of eq. (A.24) is approximately 1,

$$\frac{B(B_s \rightarrow \bar{D}^0 \phi)}{B(B_d \rightarrow \bar{D}^0 K^0)} \approx 1. \quad (A.25)$$

The BSW model obtains for the second ratio¹⁶

$$\frac{B(B_d \rightarrow \bar{D}^0 K^0)}{B(B_d \rightarrow \bar{D}^0 K^0)} \sim \left(\frac{0.26}{0.85} \right)^2 = 0.094. \quad (A.26)$$

The last ratio was taken as

$$\frac{B(B_d \rightarrow \bar{D}^0 K^0)}{B(B_d \rightarrow \bar{D}^0 K^0)} \sim \left| \frac{V_{us}}{V_{cs}} \right|^2 \left(\frac{f_{D^*}}{f_{\psi}} \right)^2 \approx 1.7 \times 10^{-2}. \quad (A.27)$$

Thus,

$$\frac{B(B_s \rightarrow \bar{D}^0 \phi)}{B(B_d \rightarrow K^0 \psi)} \sim 1.6 \times 10^{-3}. \quad (A.28)$$

Combining all the above yields eq. (A.21).

The second approach dispenses with the $B_d \rightarrow K^0 \psi$ measurement altogether. Eq. (A.21) is obtained by using eq. (A.25) with the BSW prediction,

$$B(B_d \rightarrow \bar{D}^0 K^0) \sim 1.5 \times 10^{-6}. \quad (A.29)$$

The $B_s \rightarrow D^0 \phi$ rate is obtained by using eq. (A.21) and the estimated ratio

$$\frac{B(B_s \rightarrow D^0 \phi)}{B(B_s \rightarrow \bar{D}^0 \phi)} \approx \left| \frac{V_{ub} V_{cs}}{V_{cb} V_{us}} \right|^2 \approx \frac{1}{4}. \quad (A.30)$$

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Table I. Predicted Branching rates

<u>Process</u>	<u>Branching rate</u>
$B_S \rightarrow D_S^- K^+$	$(2.4 \pm 0.5) \cdot 10^{-4}$
$B_S \rightarrow D_S^{*-} K^+$	$(2.4 \pm 0.5) \cdot 10^{-4}$
$B_S \rightarrow D_S^- K^{*+}$	$(4.4 \pm 1.6) \cdot 10^{-4}$
$B_S \rightarrow K^- D_S^{*+}$	$\sim 1.4 \times 10^{-4}$
$B_S \rightarrow K^- D_S^+$	$\sim 1.4 \times 10^{-4}$
$B_S \rightarrow K^{*-} D_S^+$	$\sim 0.9 \times 10^{-4}$
$B_S \rightarrow \rho^0 K_S$	$\sim 4 \times 10^{-7} (\sim 4 \times 10^{-8} - 1 \times 10^{-9})$
$B_S \rightarrow \bar{D}^0 \phi$	$\sim 2 \times 10^{-6}$
$B_S \rightarrow D^0 \phi$	$\sim 5 \times 10^{-7}$

Figure Captions

Figure 1. The unitarity triangle shows graphically the unitarity between the first and third columns of the CKM matrix. The angles α , β and γ are defined.

Figure 2. Time distribution of the $B_{s,\text{phys}}(t) \rightarrow f, \bar{f}$ processes and their CP-conjugated partners, where $f = D_s^- K^+, D_s^{*-} K^+$, or $D_s^- K^{*+}$.

- (a) Solid line: $B_{s,\text{phys}}(t) \rightarrow f$. Dashed line: $\bar{B}_{s,\text{phys}}(t) \rightarrow \bar{f}$.
- (b) Solid line: $\bar{B}_{s,\text{phys}}(t) \rightarrow f$. Dashed line: $B_{s,\text{phys}}(t) \rightarrow \bar{f}$.

Figure 3. The colour-allowed processes:

- (a) $B_s \rightarrow D_s^{(*)-} K^{(*)+}$,
- (b) $B_s \rightarrow D_s^{(*)+} K^{(*)-}$.

Figure 4. A colour-suppressed process, $B \rightarrow K\psi$.

Fig. 2 (a)

Fig. 2 (b)

(a)

(b)

Fig. 3

Fig. 4