A Scaling Analysis of Heavy-Light-Meson Properties ¹

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Abstract

Within the quenched approximation, we study the lattice spacing dependence of various observables of mesons composed of one heavy and one light quark in the mass range 1 GeV to m_B and in the static approximation. Large corrections to the leading order of the heavy quark effective theory (HQET) are found. The distance of string breaking in the static quark potential is estimated.

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1. INTRODUCTION

The properties of B-mesons are very interesting for a further understanding of the Standard Model of particle physics. In particular, the calculation of the B-meson decay constant has been the subject of various investigations [1] over the last three years. In this talk I will focus on the lattice spacing dependence of a number of quantities, which we have calculated in the quenched approximation with the Wilson fermion action [2].

We have chosen three sets of parameters: $(\beta, L/a) = (5.74, 8), (6.0, 12), (6.26, 18)$, with the time extent L_t of each lattice three times as large as the space extent L . We have analysed about 100 configurations each for $\beta = 5.74, 6.0$ and 55 configurations at $\beta = 6.26$ (the latter is roughly a doubling of statistics compared with [3]). A plot of $m_V(m_l, m_l)L$ vs $m_P^2(m_l, m_l)L^2$ shows a universal curve well within the statistical errors. Together with the well determined scale at $\beta = 6.0$ [5], this universality gives $a_{m_\rho}^{-1} = m_\rho/(m_\rho a) = 1.533, 2.30, 3.45$ GeV at $\beta = 5.74, 6.0, 6.26$ respectively.

Motivated by perturbation theory, we shall fit

$$r(a) = r^{(cont)}(1 + c_r a) \quad (1)$$

if this is consistent with our data for dimensionless ratios $r(a)$ (when quantities are given in GeV, we have used $a_{m_\rho}^{-1}$, which is equivalent to dividing by the appropriate power of the ρ mass). We will then quote $r^{(cont)}$ as our estimate for the continuum result and use $\frac{1}{2}[r^{(cont)} - r(a_{m_\rho} = (3.45 \text{ GeV})^{-1})]$ as a systematic error for this procedure. One should bear in mind that Eq. (1) is only an educated guess in the case of the non-perturbative quantities discussed here.

2. PSEUDOSCALAR DECAY CONSTANT f_P

We have determined f_P with propagating quarks as well as in the static approximation [1]. For a mass of the light quark m_l corresponding to $m_P(m_l, m_l) = 1.17$ GeV, we are able to analyse finite a effects in some detail. In the following, the subscript l will denote this value of the light quark mass.

In the **static approximation**, we calculate

$$\hat{f}_P^{(stat)} = \lim_{m_P \rightarrow \infty} \hat{f}_P, \text{ with} \quad \hat{f}_P = f_P \sqrt{m_P} \left(\frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{6/33}. \quad (2)$$

Details have been described in ref. [4]. Here we only note the following technical points: at $\beta = 6.26$ we have used the "Gaussian" wave function denoted F^G in ref. [4], with parameters $(\alpha, n) = (4, 70), (4, 100), (5, 160)$. They give r.m.s. radii of $4.6a, 5.1a, 6.0a$, respectively. The overlap was best with the second choice. Here clear plateaux are visible in the "smeared-smeared" and the "local-smeared" local masses, and exponential fits to the correlation functions are performed in the range $t = 4a - 16a$. For the largest wavefunction, the plateaux do not extend to such small values of t and the fits have to be performed in different ranges (e.g. $t = 7a - 16a$). In contrast to the results of ref. [6], however, we find no significant dependence of $\hat{f}_P^{(stat)}$ and the "mass" in the static approximation $M^{(stat)}$ on the wave function. In particular we have $M^{(stat)} = 0.566(4), 0.562(4), 0.560(4)$ in the order of parameters given above. The tiny variation may not be considered significant. The proper choice of the fitting interval is subtle – even with our statistics – and we do not have the space to discuss it here.

With $Z_A^{(stat)} = 0.8$, we get $\hat{f}_P^{(stat)} / \text{GeV}^{3/2} = 1.34(4), 1.13(5), 1.06(4)$ at $\beta = 5.74, 6.0,$

6.26. Eq. (1) fits well ($\chi^2/\text{DoF}=0.53$) yielding $\hat{f}_P^{(\text{stat,cont})} = 0.82(8)(12) \text{ GeV}^{3/2}$ with the second (systematic) error as described above. A further uncertainty is due to the value of $Z_A^{(\text{stat})}$, the perturbative renormalisation of the current.

A different possibility to extrapolate to the continuum is shown in Fig. 3. For a canonical value of $\sqrt{\sigma} = 420 \text{ MeV}$, the extrapolated values agree nicely.

Fig. 1. $\hat{f}_P^{(\text{stat,cont})}$ (o) and $\hat{f}_P$ for $\beta = 5.74$ (o), $\beta = 6.0$ (o), $\beta = 6.26$ (Δ) plotted against $m_P^{-1} \cdot \text{GeV}$.

In Fig. 1, we show this result, together with the data from propagating heavy quarks, at the three different values of β (the standard 1-loop renormalisation of the axial current is used). The general trend of the latter is in agreement with the result in the static approximation after extrapolation to $a = 0$. However, the data with propagating quarks shows some scaling violations. Unexpectedly, these are seen mainly for $m_P^{-1} > 0.4 \text{ GeV}$.

We performed a global fit to the data (without the static point), describing the scaling violations to order a through two parameters, b (scaling vi-

olations independent of the heavy quark mass) and d (parametrizing the heavy quark mass dependence of the a -effects)

$$f_P = f_P^{(\text{cont})}(m_P)(1 + b a)(1 + d a m_P), \quad (3)$$

and the mass dependence through an asymptotic expansion of $\hat{f}$ in powers of $1/m_P$:

$$f_P^{(\text{cont})} \sqrt{m_P} \left(\frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{6/33} = \sum_{n=0}^N c_n m_P^{-n}. \quad (4)$$

The overall function $f_P^{(\text{cont})}(m_P)$ and the coefficient c_0 are stable under varying N from 2 to 4. The χ^2/DoF is smaller than one and the results for the fit parameters with $N = 3$ are $b = 0.94 \text{ GeV}$, $d = -0.17$, $c_0 = 0.72(5) \text{ GeV}^{3/2}$, $c_1 = -1.16 \text{ GeV}^{5/2}$, $c_2 = 0.96 \text{ GeV}^{7/2}$, $c_3 = 0.17 \text{ GeV}^{9/2}$. The value of c_0 is in good agreement with $\hat{f}_P^{(\text{stat,cont})} = 0.82(8)(12) \text{ GeV}^{3/2}$. The extrapolation to the continuum $\hat{f}_P^{(\text{cont})}(m_P)$ (curve in Fig. 1) is not far from the $\beta = 6.26$ data points. Further details are given in ref. [3].

Finally, $f_{bl}^{(\text{cont})} = 232(7)(5) \text{ MeV}$, $f_{cl}^{(\text{cont})} = 238(7)(10) \text{ MeV}$ can be extrapolated to light-quark mass zero by observing that $f_{hl}/f_{hd} = 0.88(4)$ independent of the heavy-quark mass at $\beta = 6.0$, and a consistent ratio is found at $\beta = 6.4$ [7] and in the static approximation ($\beta = 6.0$). This extrapolation gives

$$185 \text{ MeV} < f_{bd} < 225 \text{ MeV}, \quad (5)$$

$$186 \text{ MeV} < f_{cu} < 235 \text{ MeV}. \quad (6)$$

Eqs. (5) and (6) include the systematic error of the extrapolation $a \rightarrow 0$ (cf. introduction), but not the uncertainty in the perturbative renormalisation (may be 5 to 10%).

3. SPIN SYMMETRY?

The results for $\hat{f}_P^{(\text{cont})}(m_P)$ show large ($\simeq -25\%$ for B, $\simeq -40\%$ for D) violations of the

Fig. 2. U in the top part and $(m_V^2 - m_P^2)/2$ plotted against $2/(m_P + m_V)$ in units of GeV, for $\beta = 6.0$ (o), $\beta = 6.26$ (Δ) [3] and $\beta = 6.4$ ($\square$) [7]

flavor symmetry [1] present in the HQET. However, the spin symmetry could be realised with smaller corrections. Forming the ratio of the correlator of the zeroth component of the axial vector current to the correlator of the vector current one obtains

$$U e^{(m_V - m_P)t}, \quad U = (f_P f_V / m_V)^2, \quad (7)$$

for sufficiently large t . The HQET predicts $U = 1 + O((m_P + m_V)^{-1})$, $m_V^2 - m_P^2 = \text{const} + O((m_P + m_V)^{-1})$. The data [3,7], which shows no dependence on the mass of the light quark, is plotted in Fig. 2. The HQET behavior is seen for U – again with large $1/m$ corrections. The data for $m_V^2 - m_P^2$ shows a decrease with growing $m_P + m_V$, whereas the experimental values are the same for B - and D -mesons. Since the whole quantity is a small mass splitting, the discrepancy is not really unexpected for finite values of a . It is worrying, however, that we see no sign of improvement with decreasing lattice spacing.

4. S-WAVE - P-WAVE SPLITTING

We have given results for this splitting in ref. [4] and Fig. 4 of ref. [8]. The new point to add is $\Delta^{(\text{stat})} = 466(46)$ MeV, obtained in the static approximation at $\beta = 6.26$. It shows no scaling violation compared with the $\beta = 6.0$ result. Consequently, a splitting of about 450 MeV seems to emerge (cf. Fig. 4 of ref. [8]).

Fig. 3. Extrapolation $a \rightarrow 0$ of $R_b \sqrt{\sigma}$ (for light-quark mass m_l (o) and light-quark mass zero (Δ)) and of $\frac{1}{2} f_P^{(\text{stat})} \sigma^{-3/4}$ (o). σ is the string tension calculated from potential fits (with $-\pi/(12R)$ term) [10,11].

5. STRING BREAKING

For some time, simulations including dynamical fermions have searched for the breaking of the QCD string, i.e. the flattening of the heavy quark potential at large distances. No effect was found in the most serious effort [9]. We can easily – and I believe as reliably as anything that can be done in the quenched approximation – estimate the distance R_b where the string will break. This comes about because, we have a good approximation of the potential at intermediate distances, namely the quenched $Q\bar{Q}$

potential, and at large distances, the full QCD potential should be approximated well by the (static) meson-meson potential that can be calculated from the correlation function of two static heavy-light mesons separated by a distance R . This correlation function will factorise (up to weak non-leading terms that give the Van der Waals forces) giving an asymptotic value $\exp(-2 M^{(stat)} t)$.

R_b can therefore be calculated from

$$V(R_b) = 2M^{(stat)}, \quad (8)$$

$V(R)$ being the quenched potential including the self-energy term that cancels in Eq. (8).

In ref. [8] R_b was calculated from the $M^{(stat)}$ values Table 2 of [4]. Unfortunately, due to a copying mistake, the values given there for $\beta = 5.74$ are the ones for the $L = 1$ state: $M^{(stat)} + \Delta$. Therefore, the true scaling behavior is quite different from the one given in ref. [8]. Interpolating (in β) the potential from the literature [10,11], we obtain the a -dependence shown in Fig. 3. The lines represent fits to Eq. (1). The more reliable estimate is $R_b^{(cont)}(m_l)\sqrt{\sigma} = 4.24(15)(33)$. Fig. 3 indicates that $R_b^{(cont)}$ is only a little smaller in the chiral limit. Therefore, in order to observe the string breaking in a full QCD simulation, it is cheaper to work at largish values of the dynamical quark mass, since the dependence of R_b on m_l will be overcompensated by the speed of the Hybrid Monte Carlo algorithm.

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