Scaling Properties at the Crumpling Transition

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ABSTRACT

We investigate numerically a dynamically triangulated random surface with extrinsic curvature embedded in a three-dimensional space. Toroidal topology is used, to allow for string-tension measurements by a new method. A high-statistics test of the finite-size scaling of the specific heat is performed.

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I report here on a study done in collaboration with J. Ambjørn, J. Jurkiewicz, B. Petersson and S. Varsted. We have investigated the dynamically triangulated random surface model defined by the partition function [1]

$$Z = \sum_T \int \left[\prod_{i=1}^N d^d x_i \right] \delta^d(\Sigma \mathbf{x}_i) e^{-(S_g + \lambda S_e)} . \quad (1)$$

The action consists of the Gaussian term

$$S_g = \frac{1}{2} \sum_{\langle ij \rangle} (\mathbf{x}_i - \mathbf{x}_j)^2 \quad (2)$$

and the extrinsic curvature term

$$S_e = \sum_{\langle ij \rangle} (1 - \cos \theta_{ij}) , \quad (3)$$

where θ_{ij} is the angle between the two triangles sharing the link ij and $\mathbf{x}_i$ denotes vertex coordinates in the $d = 3$ target space. Previous numerical results have indicated that this model undergoes a second-order phase transition at a finite value of the coupling, $\lambda \approx 1.5$, from a crumpled phase at low λ to a flat phase at high λ [2–4]. We have studied various properties of the model at this transition [5,6] and present here results for the string tension and the specific heat.

Our method to measure the string tension [5] differs in important aspects from the one that was used in an earlier investigation [7]. We have chosen to consider closed systems with the topology of a torus. We can then introduce a non-zero external dimension without having to define a boundary. This is done by imposing twisted boundary conditions: the sum of link vectors along a closed path is $n_1 \mathbf{E}_1 + n_2 \mathbf{E}_2$, where n_1 and n_2 count, with sign, the number of windings around the two axes of the torus. In the string-tension calculations we use $\mathbf{E}_1 = (y, 0, 0)$ and $\mathbf{E}_2 = (0, y, 0)$ and obtain in this way a translationally invariant frame of projected area $A_p = y^2$. In the canonical ensemble with fixed

N considered in the numerical simulations, we define the string tension by (cf. [8])

$$\begin{aligned} \sigma(\lambda, A_p, N) &= - \frac{\partial \ln Z(\lambda, A_p, N)}{\partial A_p} \\ &= \frac{2 \langle S_g \rangle - d(N-1)}{2A_p} . \end{aligned} \quad (4)$$

The last expression, obtained by a rescaling of the coordinates, permits a direct measurement of the string tension in the numerical experiment.

The measurements of the canonical string tension are given in Fig. 1. We expect the results to depend on A_p and N only through the ratio $r = A_p/N$ and interpret the deviations from this behaviour seen at small r and high λ as finite-size effects. At large values of r the results are nearly independent of λ , which is expected from the dominance of the Gaussian term. At small values of r the string tension shows a strong λ dependence and it appears to approach zero near $\lambda = 1.5$.

How to relate the canonical results to the grand canonical continuum limit of the theory was discussed in [5]. The relevant region is that of small r , where we expect

$$\sigma(r, \lambda) = \sigma_0(\lambda) + \sigma_1(\lambda) r^{2\nu/(1-2\nu)} + \dots , \quad (5)$$

corresponding to the grand canonical behaviour $\bar{\sigma}(\mu, \lambda) = \sigma_0(\lambda) + d(\lambda)(\mu - \mu_c(\lambda))^{2\nu}$. Here the first term $\sigma_0(\lambda)$ must vanish for the physical string tension to be finite. The results in Fig. 1 suggest that this is the case at some λ close to 1.5. An additional requirement for an interesting scaling limit is that different physical observables scale in a consistent way. Using (5) the critical exponent ν can be determined, which then has to be compared with the corresponding result for other quantities. To this end we have performed calculations also of the mass gap. Preliminary results, obtained by a method similar to that for the string tension, are given in [5].

Fig. 1. String tension $\sigma(\lambda, r)$ vs r for $N = 64, 144$ and 256 .

The numerical simulations have been performed by using a program in which vectorisation is obtained by simulating in parallel a number of systems [9]. On the CRAY-YMP we simulated 64 parallel systems and obtained a CPU time per sweep per bond of $5.7 \mu\text{s}$. The integrated autocorrelation time τ_{int}^R for the most non-local quantity measured, the radius of gyration, is shown in Fig. 2. Data at fixed coupling are consistent with a behaviour $\tau_{int}^R \sim N^{z'}$ with $1.5 < z' < 2$.

We have analysed the specific heat

$$C = \frac{\lambda^2}{N} (\langle S_e^2 \rangle - \langle S_e \rangle^2) \quad (6)$$

in the transition region by using the spectral density method. The results presented are for system sizes $N = 36, 64, 144, 256$ and 576 and periodic boundary conditions, $E_1 = E_2 = 0$. The multiple-histogram technique [10] was used to continue, in λ , data for fixed N taken at a few different couplings. For $N = 576$ only one cou-

Fig. 2. Integrated autocorrelation time τ_{int}^R in units of sweeps vs N for $\lambda = 1.25$ (circles), 1.4 (squares), 1.47 (triangles) and 1.5 (diamonds).

pling was simulated. For the other values of N we checked that the coupling range considered could be covered by consistent single-histogram continuations. Around $20\tau_{int}^R$ sweeps were used for equilibration, and measurements were taken over at least twice that number of sweeps.

As in earlier investigations using spherical topology, we find that the specific heat has a peak near $\lambda = 1.5$ and that the maximum C_{max} increases with increasing N . The scaling of C_{max} with N was studied quantitatively in [4,11], where it was fitted to a power law, $C_{max} \sim AN^\omega + B$. As can be seen from Fig. 3, our results, which cover a similar range of system sizes but have smaller statistical errors, are not consistent with such a scaling. They increase slower than logarithmically with N and suggest that C_{max} may stay finite in the limit $N \rightarrow \infty$. This raises doubts about the second-order nature of the transition and presumably more detailed information on other observables

Fig. 3. Maximum of the specific heat vs N .

will be required to settle this question.

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