Light pair corrections to the Z lineshape parameters¹

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Abstract

We study the effect of the light fermion pair correction in the Z lineshape parameters. We observe corrections which are modest with respect to the present experimental accuracy but will become relevant in the near future: M_Z changes by about 1.3 MeV, Γ_Z by more than 2 MeV and the peak cross section by around 0.25 %. This estimate has been obtained for a realistic experimental strategy.

A new third order formula for total cross section with multiple photon and lepton pair contributions (featuring exponentiation) is presented. It provides the total cross section near the Z resonance with precision $\Delta\sigma/\sigma \leq 3.7 \cdot 10^{-4}$ for $|\sqrt{s} - M_Z| \leq 7.5$ GeV. This precision includes $\Delta\sigma/\sigma \leq 3.5 \cdot 10^{-4}$ from pure bremsstrahlung and $\Delta\sigma/\sigma \leq 0.2 \cdot 10^{-4}$ from lepton pairs. We also discuss light quark pairs contribution. We estimate (conservatively) its uncertainty to be $\Delta\sigma/\sigma \leq 1.8 \cdot 10^{-4}$.

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In this letter we discuss the effect of light fermion pair emission on the s dependence of the total cross section at LEP. The most relevant effects are the change of the Z width Γ_Z , and the peak cross section σ_{peak}^0 .

In the case of the total width, the reason is the following: the change of $\sigma(s)$ on the resonance wings corresponds, with the coefficient one, to the change of the width (see below) if a Breit-Wigner shape is assumed. Therefore since, as we will show, the light pair emission distorts the lineshape with a correction of the order of few per mil, we can expect also a few per mil (i.e. a few MeV) change of Γ_Z due to pair emission. To emphasize the importance of this effect, let us recall that if the total Z width measured at LEP is used to estimate the top mass, a shift of 1 MeV in Γ_Z corresponds to about a 5 GeV shift in m_{top} . In addition, this effect is comparable to the statistical error that one can expect to have at the end of LEP-I (around 2 MeV) and is of the same order as the present systematic error in Γ_Z coming from machine energy point-to-point uncertainty³ (about 4 MeV).

Concerning the hadronic peak cross section, the few per mil change in $\sigma(s)$ produced by the light pair emission correction will directly translate into a few per mil change in the peak cross section σ_{peak}^0 . This correction is therefore much larger than the statistical error that one can expect to have in the near future and is comparable to the systematic error in the peak cross section measurement which comes from the theoretical systematic uncertainty in the luminosity determination, which at present is of about 0.25 % [1]⁴.

In this letter we want to give both a qualitative and a quantitative analysis of this effect. We start with the presentation of a new formula for the QED pair radiation in the form of a correction to the total cross section. Afterwards, we show its relation to the formulae available in the literature and we discuss its actual accuracy (physical precision). Next we present a simple, semi-quantitative explanation for the width distortion. Finally we give the exact numbers coming from fitting M_Z, Γ_Z and σ_{peak}^0 from $\sigma(s)$ measured in several s -points as in the real experiment.

Close to the Z resonance the dominant contribution of pair emission comes from the bremsstrahlung-like mechanism, in which pairs are emitted in the same manner as additional photons (Fig. 1 (a)). In the case of photon emission, the most important soft radiation can be resummed to all orders of perturbation theory in a systematic way by means of the Yennie-Frautschi-Suura (YFS) prescription [2]. This is a useful technique which substantially improves perturbative convergence, see eg. [3, 4]. We have made an attempt to extend the YFS prescription to simultaneous resummation of both soft photons and soft lepton pairs. The resulting formula includes also the exponentiation of soft lepton pairs [5]:

$$\sigma(s) = \int_{z_{min}}^1 dz \rho(z, s) \sigma_{Born}(zs), \quad (1)$$

³This number is expected to improve sizably in the near future thanks to the energy measurement using the beam polarization at LEP.

⁴The experimental systematic in the determination of the hadronic peak cross section, if the results of the four LEP experiments are combined, is already of the same order as the luminosity theoretical uncertainty and it is expected to improve in the near future for some LEP experiments thanks to the installation of more accurate luminosity monitors.

where

$$\begin{aligned} \rho(z, s) = & (1-z)^{\beta_S-1} \exp\left(\frac{3}{4}\beta_S + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)\right) \Phi(\bar{\beta}, l_f) \exp\left[\left(\frac{\alpha}{\pi}\right)^2 \sum_f \left(\frac{4}{9}l_f^3\right.\right. \\ & \left.\left. + \frac{2}{3}\left(\frac{31}{18} - \frac{\pi^2}{3}\right)l_f - \frac{1}{18}L_f^3 + \frac{19}{36}L_f^2 + \left(\frac{\pi^2}{9} - \frac{265}{108}\right)L_f\right)\right] \left(\Delta_{JW}^{(3)LL}(z) + \Delta_{JW}^{(3)NLL}(z)\right), \end{aligned} \quad (2)$$

being

$$\begin{aligned} \Phi(\bar{\beta}, l_f) = & e^{\frac{4}{9}(\frac{\alpha}{\pi})^2 \frac{\pi^2}{2} \sum_f l_f} F(\bar{\beta}) \left[\bar{\beta} - \frac{4}{9}\left(\frac{\alpha}{\pi}\right)^2 \sum_f \left(\frac{\pi^2}{2} + \left(\frac{3}{2}\pi^2 l_f - 8\zeta(3)\right)\bar{\beta}\right) + \mathcal{O}(\alpha^4) \right], \\ \Delta_{JW}^{(3)LL}(z) = & \left[\frac{1}{2}(1+z^2) + \frac{1}{4}\beta_R^{LL} \left(-(1-z)^2 - \frac{1}{2}(1+3z^2)\ln z \right) \right. \\ & \left. + \frac{1}{8}(\beta_R^{LL})^2 \left(\frac{3z^2-4z+1}{2}\ln z + \frac{1+7z^2}{12}\ln^2 z + (1-z)^2 + (1-z^2)\text{Li}_2(1-z) \right) \right], \\ \Delta_{JW}^{(3)NLL}(z) = & \frac{\alpha}{\pi}\beta_S \left(\frac{3}{32} - \frac{\pi^2}{8} + \frac{3}{2}\zeta(3) \right) + \frac{1}{8}\frac{\alpha}{\pi} \left(4(1+z^2)(\text{Li}_2(1-z) + \ln z \ln(1-z)) \right. \\ & \left. - (1+3z^2)\ln^2 z + 4(2+z+2z^2)\ln z + 2(1-z)(5-4z) \right), \end{aligned} \quad (3)$$

where

$$\begin{aligned} F(\beta) &= \exp(-\beta C)/\Gamma(1+\beta), \\ \bar{\beta} &= \beta_S + (4/3)(\alpha/\pi)^2 \sum_f (l_f^2 + 31/36), \\ l_f &= \ln(\sqrt{s}(1-z)/m_f) - (5/6), \\ \beta_S &= 2(\alpha/\pi)(\ln(s/m_e^2) - 1), \\ L_f &= \ln(s/m_f^2), \end{aligned}$$

$C = 0.57721566\dots$ is the Euler constant and $f = e, \mu, \tau$.⁵ The treatment for quark pair production is discussed later. The dominant contribution is due to electron pairs, i.e. $f = e$. On the other hand (ignoring the relation to YFS exponentiation), this formula may be treated as a phenomenological ansatz which reproduces all other results existing in the literature as special cases and includes new dominant higher order contributions. In the following, we shall give a short overview of existing formulae and we will emphasize their connection to eq. (2).

⁵The function $\Phi(\bar{\beta}, l_f)$ is defined by

$$\Phi(\bar{\beta}, l_f) = \frac{\sqrt{s}}{2}(1-z) \frac{1}{2\pi} \int_{-\infty}^{\infty} dy \exp\left[iy \frac{\sqrt{s}}{2}(1-z) - \beta_S \mathcal{L} - \frac{4}{9} \frac{\alpha^2}{\pi^2} \sum_f \left(\mathcal{L}^3 - 3\mathcal{L}^2 l_f + 3\mathcal{L} l_f^2 + \frac{31}{12} \mathcal{L} - \frac{\pi^2}{2} l_f \right) \right]$$

where $\mathcal{L} = C + \ln(y\sqrt{s}(1-z)/2) + i\pi/2$. In eq. (2) we expand this integral, keeping the minimum number of terms necessary for our numerical precision.

(i) Restricted to the second order, formula (2) reproduces, up to *noninfrared, nonleading* terms, the exact $\mathcal{O}(\alpha^2)$ result of Refs. [6, 7]:

$$\begin{aligned} \rho_f^{(2)}(z, s) = & \frac{1}{3} \left(\frac{\alpha}{\pi} \right)^2 \left[\frac{1+z^2}{1-z} \left(\frac{1}{2} (2l_f - \ln z)^2 + \frac{31}{18} - \frac{\pi^2}{3} \right) + \mathcal{O}(1-z) \right] \theta \left(1 - \frac{4m_f}{\sqrt{s}} - z \right) \\ & + 2 \left(\frac{\alpha}{\pi} \right)^2 \left[-\frac{1}{36} L_f^3 + \frac{19}{72} L_f^2 + \left(\frac{1}{3} \zeta(2) - \frac{265}{216} \right) L_f + \mathcal{O}(1) \right] \delta(1-z). \end{aligned} \quad (4)$$

(ii) Restricted to leading-logarithmic (LL) approximation the formula (2) reproduces the LL exponentiated $\mathcal{O}(\beta^3)$ formula of [3]:

$$\rho^{(LL)}(z, s) = \frac{\exp(\beta_R^{LL}(\frac{3}{4} - C))}{\Gamma(1 + \beta_R^{LL})} \beta_R^{LL} (1-z)^{\beta_R^{LL}-1} \Delta_{JW}^{(3)LL}(z) \quad (5)$$

with running coupling constant⁶

$$\beta_R^{LL} = (2\alpha/\pi) \ln(s/m_e^2) + (1/3)(\alpha/\pi)^2 \sum_f \ln^2(s/m_f^2) + \mathcal{O}(\alpha^3).$$

(iii) In the case of only one additional e^+e^- pair emitted, formula (2) reproduces the ansatz proposed by Kuraev and Fadin [8]. It includes numerically dominant $\mathcal{O}(\alpha^2)$ contributions from single e^+e^- pair ($f = e$) emission combined with the soft multiphotonic radiation:

$$\begin{aligned} \rho^{(KF)}(z, s) = & \beta_S (1-z)^{\beta_S-1} \beta_S^2 \left(-\frac{L_e}{72} + \frac{10}{96} \right) + \left(\frac{\alpha}{\pi} \right)^2 \theta \left(1 - z - \frac{4m_e}{\sqrt{s}} \right) \\ & \frac{1}{6(1-z)} \left(1 - z - \frac{4m_e}{\sqrt{s}} \right)^{\beta_S} 4l_e^2 \left(1 + z^2 + \frac{1}{3} \beta_S 2l_e \right), \end{aligned} \quad (6)$$

Formula (2) as compared to eq. (6) includes also nonleading IR divergent ($\mathcal{O}(\alpha^2 L^0)$) contributions from soft pair, LL pair with additional bremsstrahlung photons corrections ($\mathcal{O}(\alpha^3 L^3)$) and contributions from two and more (also of different type) pairs ($\mathcal{O}(\alpha^4 L^4)$).

(iv) Formula (2) is a natural extension of the “pragmatic” formula for total cross section of Ref. [3] (which includes exponentiated photonic corrections up to the $\mathcal{O}(\alpha^3 L^3)$) and inherits its very high precision of $\Delta\sigma/\sigma \leq 3.5 \cdot 10^{-4}$ for *pure bremsstrahlung* corrections (see Ref. [4]). Finally, in eq. (2) we explicitly sum over all possible lepton pairs.

The striking feature of all of these formulae is that subleading corrections in pair contributions are extremely important. In Fig. 2 we compare cross sections for additional e^+e^- pairs resulting from the following formulae: $\mathcal{O}(\alpha^2)$ of eq. (4), eq. (4) restricted to LL and exponentiated $\mathcal{O}(\alpha^3)$ LL formula of eq. (5). All curves are normalized to the $\mathcal{O}(\alpha^3)$ exponentiated photonic cross section; the cut-off $z \geq z_{min} = 0.04$ is used⁷. The

⁶Actually, $\bar{\beta}|_{LL} = 2(\alpha/\pi) \ln(s/m_e^2) + (1/3)(\alpha/\pi)^2 \sum_f \ln^2(s/m_f^2)$ reproduces the first terms of the β_R^{LL} expansion in α . The $\mathcal{O}(\alpha^3)$ terms, neglected in this expansion, due to additional photon self energy corrections not included in eq. (2) are numerically negligible.

⁷This cut-off corresponds to an experimental selection of Z decays by requiring the invariant mass of the Z products being at least of 20% of the nominal one. In fact, so far the LEP experiments [12] do not apply an invariant mass cut but rather a visible energy cut to select hadronic decays and there is no simple relationship among both. Anyway, for a cut in visible energy of about $0.1 \cdot E_{cm}$ (the one applied for instance by ALEPH [10]) the invariant mass value for which the efficiency has around a 50% drop is at about 20%. At any rate, we shall discuss the final correction size for a wide range of z_{min} values.

difference between $\mathcal{O}(\alpha^2)$ LL+NLL and LL results, i.e. subleading correction, is about 100% of the leading correction at the peak. The physical reason for this is that the soft pair divergence is not properly reproduced in the LL formula, whereas any kind of soft divergent corrections are important close to narrow resonance. On the other hand the $\mathcal{O}(\alpha^2)$ LL result differs from the exponentiated (i.e. infinite order) LL one by 50%. Here the source of discrepancies is the additional soft photon contribution which is not included in $\mathcal{O}(\alpha^2)$ formula⁸. The above comparison shows clearly that both nonleading terms in multiple pair production and multiple soft photons are equally important.

What is the actual size of the lepton pair contribution and its theoretical uncertainty?

Let us calculate the actual size of the pair contribution to the total cross section and discuss its QED uncertainty. In Fig. 3 we compare the corrections resulting from YFS-like formula (2) and the Kuraev-Fadin ansatz (6) for additional e^+e^- pairs. Both curves are normalized to the total cross section corrected for photon emission $\sigma^{photonic}$. The whole contribution within the range $|\sqrt{s} - M_Z| \leq \Gamma$ is about $\sigma^{pairs} \leq 2 \cdot 10^{-3} \cdot \sigma^{photonic}$ for the cut $z_{min} = 0.04$. As expected, higher order contributions reduce the second order result by about 30%. This is due to the soft photons which carry away the beam energy and shift the process away from the resonance. The third curve of Fig. 3 shows the difference between second and third order hard corrections of eq. (5) to the YFS formula (2). One can see that the difference is below the scale of the picture, $(\sigma^{pairs(3)} - \sigma^{pairs(2)}) \leq 10^{-5} \cdot \sigma^{photonic} \simeq 10^{-2} \cdot \sigma^{pairs}$. We can therefore reasonably assume that fourth order corrections to e^+e^- emission (not included) are of order $10^{-5} \cdot \sigma^{photonic} \simeq 10^{-2} \cdot \sigma^{pairs(2)}$, and use this number as the physical accuracy of the lepton pairs production part of formula (2)⁹. Combining this with the accuracy of photonic correction $3.5 \cdot 10^{-4}$, see Ref. [4], we get the total accuracy of formula (2) to be $3.7 \cdot 10^{-4}$ for $|\sqrt{s} - M_Z| \leq 7.5\text{GeV}$, see later for the discussion of quark pairs contribution.

Which QED corrections are still neglected? Apart from the bremsstrahlung-like process shown in Fig. 1 (a) there are two other LL contributions to initial state e^+e^- pair production at LEP: the Singlet Structure Function mechanism (Fig. 1 (b)) and the $\gamma\gamma$ fusion mechanism (Fig. 1 (c)). As far as the total cross section is concerned, processes (b) and (c) were analyzed in [6, 8, 9, 7]. In Ref. [6] the initial state $\mathcal{O}(\alpha^2)$ processes (b) is presented. In Refs. [8] and [9] the Structure Functions formalism is used. The process (b) in the $\mathcal{O}(\alpha^2)$ Leading Logarithmic (LL) approximation is calculated. The process (c) is discussed in [9]. The conclusion of [9] is that around the resonance the most important contributions to total cross section come from (a) and (b) processes. In the language of Structure Functions (SF), the process (b) corresponds to the Singlet Electron SF. In addition to the $\mathcal{O}(\alpha^2)$ LL result of [8], we have also computed the $\mathcal{O}(\alpha^3)$ correction and calculated the corresponding cross sections [4]. We come to the conclusion that within the range $|\sqrt{s} - M_Z| \leq \Gamma_Z$ the initial state LL singlet contribution to the total cross section obeys $\sigma^{pair(b)} \leq 2 \cdot 10^{-5} \cdot \sigma^{photonic}$ for strong cuts ($z_{min} \geq 0.6$) and increases to $7 \cdot 10^{-4} \cdot \sigma^{photonic}$ for loose cuts ($z_{min} = 0.04$). Similarly, the LL calculation for process (c) gives $\sigma^{pair(c)} \leq 2.5 \cdot 10^{-4} \cdot \sigma^{photonic}$ for $z_{min} \geq 0.6$ and $10^{-1} \cdot \sigma^{photonic}$ for $z_{min} = 0.04$,

⁸The same picture, scaled down by the factor of five, holds for muon pairs.

⁹We also traced back the difference between YFS and KF curves to be due to the nonleading pair and photonic terms, surviving also in the soft limit, present in eq. (2) but not included in eq. (6).

$|\sqrt{s} - M_Z| \leq \Gamma_Z$. Therefore, we conclude that the processes (b)+(c) contribute less than 10% with respect to the dominant process (a), in the range $|\sqrt{s} - M_Z| \leq \Gamma_Z$ only for cuts $z_{min} \geq 0.6$.

How relevant are the neglected contributions (b) and (c) experimentally? To understand the practical relevance of each one of these contributions, it is important to think about how this effect is treated experimentally. If just the resonant cross section is selected in the region of small z [10], then process (a) is selected as signal (and therefore should be included in the fitting procedure) whereas processes (b) and (c) are subtracted as background¹⁰. The subtraction of the non-resonant part can be computed by using the data [10] and therefore we can forget about these contributions in our discussion. We should remark here that this separation is the most natural one since, in fact, the aim of the lineshape study is the determination of the resonance parameters.

Quark pair correction: In order to get a complete numerical estimate of the pair production effect on the measured total cross section, besides the lepton pair production discussed in detail in the previous sections, the quark pair contribution has to be taken into account. For that we have followed the approach of Ref. [7]. In this work, only $O(\alpha^2)$ corrections are taken into account and, as we have shown above, the effect of considering initial state photon radiation in addition, changes the correction significantly. We have corrected numerically the $O(\alpha^2)$ results of ref. [7] to account for this effect, which basically reduces the correction in the resonance region by about 30%. We also take the same 30% as the quark pair contribution uncertainty.

Table 1 and Fig. 4 show the final estimated correction for the hadron and the muon total cross sections and the different contributions coming from pairs of e , μ , τ and hadrons. The size of the correction for other choices of the z_{min} cut in a broad range is also shown. It is clear that the actual correction is almost independent of this choice. Combined uncertainty of lepton and quark pairs correction δ_{tot} in Table 1, we obtain $\Delta\delta_{tot} \leq 2 \cdot 10^{-4}$ for $|\sqrt{s} - M_Z| \leq 10\text{GeV}$ and cut $z_{min} \geq 0.04$.

A realistic and rigorous estimate of the effect of the inclusion of pair production in the lineshape parameters requires the use of an accurate procedure to fit the total cross section similar to the one actually used by the LEP experiments [11].

However, we are able to explain the main result (the size of the Γ_Z change) in a simple semi-quantitative way without fitting. Let us take the Breit-Wigner expression for the total cross section:

$$\sigma(s) = \sigma_{peak}^0 \frac{M_Z^2 \Gamma_Z^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}. \quad (7)$$

The geometrical meaning of Γ_Z is the following: the cross section is exactly half of the peak value at $s = M_Z^2 \pm M_Z \Gamma_Z$, that is $E_{cm} \cong M_Z \pm \Gamma_Z/2$. We can compute the expected variation in Γ_Z due to the inclusion of pair production simply by calculating the derivative

¹⁰For the potentially dangerous low z region, process (b) can be seen as a radiation of a virtual Z producing a pair in Bhabha scattering and therefore, it is not resonant with respect to the center-of-mass energy. Process (c) is never resonant.

of the cross section with respect to s at $s = M_Z^2 \pm M_Z \Gamma_Z$. By doing that, we obtain

$$\frac{\Delta \Gamma_Z}{\Gamma_Z} = \frac{\Delta \sigma}{\sigma} \Big|_{s=M_Z^2 \mp M_Z \Gamma_Z} , \quad (8)$$

so that we get the simple prediction¹¹

$$\begin{aligned} \Delta \Gamma_Z &= \Gamma_Z \frac{\Delta \sigma|_{M_Z^2 - M_Z \Gamma_Z} + \Delta \sigma|_{M_Z^2 + M_Z \Gamma_Z} - 2\Delta \sigma|_{M_Z^2}}{\sigma} \\ &= \Gamma_Z \times (-0.27 - 0.17 + 2 \times 0.26)\% \simeq -2 \text{ MeV}. \end{aligned} \quad (9)$$

A fitting formula is used to simulate a χ^2 fit of fake data. This enables us, not only to have a more accurate theoretical estimate of the changes in the parameters, but also to evaluate the effect of the actual scan strategy of the LEP machine. This strategy specifies which energy points are to be considered and how much statistical significance each of them will have. In fact it determines how the total cross section distortion is reflected in the fit parameters. In practice we proceed in the following way: by correcting the predictions of the fitting formula for the pair production effect, we produce fake data which we refit without including the pair production correction.

We simulate the LEP scan by generating hadron cross section points at the center-of-mass energies of $M_Z, M_Z \pm 1, M_Z \pm 2$ and $M_Z \pm 3$ GeV. The obtained cross sections are then corrected to include the pair production effect. We assign statistical errors to these points, corresponding to a scanning strategy with 2/3 of the total luminosity at the peak and 1/3 off the peak. We have checked other scanning strategies (for instance 1/2 at the peak) and we obtained similar effects. We have also checked that if the true LEP 1989+1990 scanning points and luminosity [12] are used, the observed distortion increases slightly (we get about 10% larger distortion for Γ_Z).

The results for a 3 parameter fit (M_Z, Γ_Z and σ_{peak}^0) to the hadron cross section are summarized in Table 2. Given the experimental limitations on the measurement of these three parameters [12], the main effects of the pair production correction are, as estimated qualitatively, an increase of Γ_Z by about 2 MeV and a decrease on the peak cross section by about 0.25%.

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FIGURE CAPTIONS

1. Different e^+e^- pair contributions to the Z production cross section. (a) the bremsstrahlung like process, (b) the Singlet Structure function and (c) the $\gamma\gamma$ fusion. The dashed line denotes a Z propagator.

¹¹This formula takes into account only the symmetric component of the σ variation across the Z peak. The asymmetric one, which affects M_Z is in general also present.

$\sqrt{s}$ [GeV]	σ_{had}							σ_μ
	$z_{min} = 0.04$					$z_{min} = 0.01$	$z_{min} = 0.25$	$z_{min} = 0.04$
	δ_e	δ_μ	δ_τ	δ_{had}	δ_{tot}	δ_{tot}	δ_{tot}	δ_{tot}
88.175	-1.56	-0.26	-0.03	-0.55	-2.40	-2.38	-2.43	-2.45
89.175	-1.69	-0.27	-0.04	-0.59	-2.60	-2.58	-2.61	-2.62
90.175	-1.80	-0.29	-0.04	-0.63	-2.76	-2.75	-2.76	-2.77
91.175	-1.70	-0.29	-0.03	-0.61	-2.63	-2.62	-2.63	-2.63
92.175	-1.14	-0.25	-0.01	-0.45	-1.85	-1.85	-1.86	-1.86
93.175	-0.39	-0.16	-0.02	-0.22	-0.79	-0.78	-0.80	-0.79
94.175	0.30	-0.06	-0.03	-0.03	0.24	0.25	0.23	0.24

Table 1: *Pair production correction $\delta = \Delta\sigma/\sigma^{photonic}$ in per mil. The numbers have been obtained for $M_Z=91.175$ GeV, $\Gamma_Z=2.487$ GeV and the main result is for $z > 0.04$. The last three columns show how little the total numbers change if the cut on z is modified or if the muon lineshape is considered instead of the hadronic one.*

	M_Z	Γ_Z	σ_{peak}^0
	[MeV]	[MeV]	[nb]
Input value (true)	91175.0	2487.0	41.23
Fitting result (no pairs)	91176.3	2489.1	41.12
Shift	-1.3	-2.1	+0.11

Table 2: *Three parameter fit for hadron production. The last line of numbers shows the shift that should be applied to the present fit numbers (if pair production has not been taken into account) to reproduce the true numbers.*

2. Ratio of various e^+e^- pair contributions to the total cross section. The contributions are the exact $\mathcal{O}(\alpha^2)$ eq. (4) (dots), LL $\mathcal{O}(\alpha^2)$ eq. (4) (filled circles) and LL $\mathcal{O}(\alpha^\infty)$ eq. (5) (open circles).
3. Ratio of various e^+e^- pair contributions to the total cross section. The contributions are the $\mathcal{O}(\alpha^2)$ KF formula eq. (6) (open circles), $\mathcal{O}(\alpha^3)$ YFS formula eq. (2) (dots) and the difference between $\mathcal{O}(\alpha^3)$ YFS and $\mathcal{O}(\alpha^2)$ YFS formulae eq. (2) (filled circles).
4. The contributions from e , μ , τ and hadron pairs separately and the whole correction to the total hadron lineshape as a function of the center-of-mass energy.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4