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SOME SIGNALS OF COMPOSITENESS OF QUARKS AND LEPTONS

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ABSTRACT. If quarks and leptons are composed of some subconstituents, ("preons"), the matrix elements of the various currents in the Standard Model contain induced terms ("Pauli terms"). The presence of Pauli terms leads to well-defined deviations from the predictions of the Standard Model both in some decays of the Z^0 Boson and in the ratio of inclusive pair annihilation to $e^+ e^- \rightarrow \mu^+ \mu^-$. These deviations will be measurable at LEP and LEP2. The effect of the induced terms on "oblique corrections" is outlined.

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The Standard Model (SM) of electroweak and strong interactions, based on a gauge group $SU(3)_c \times SU(2)_L \times U(1)$ is in an excellent agreement with the experimental data available at present. Nevertheless, due to several unsatisfactory features of the model, it is rather clear that the SM in its present form is not the ultimate theory. The existence of a substructure (“preons”) has been proposed as one possible remedy for the diseases of the SM, see, *e.g.* Ibáñez for a recent review [1] and references quoted there. Although, for the time being, nobody was able to propose a viable theory of composite quarks and leptons, the search for a substructure remains an important topic, both from the theoretical and experimental points of view.

In analyzing experimental data, limits on a substructure are conventionally parametrized in terms of the energy scale of extra four Fermion interactions such a substructure, presumably, gives rise to [2]. The Eichten–Lane–Peskin parametrization has been criticized by Kroha [3], who pointed out that from the point of view of meaningfully combining experimental data, the square of the corresponding *length* scale should be used.

It is perhaps more important to realize that the Eichten–Lane–Peskin parametrization implicitly contains certain assumptions about the physics beyond the SM which may or may not be correct. For instance, if the dynamics binding the preons inside quarks is a gauge theory in its confining phase, there will be no extra four Fermion terms present in quark–quark interactions at energies below the characteristic scale of the theory. Rather, one expects non–renormalizable terms to appear in the form of a σ –model action. It is, therefore, important to look for alternative parametrizations of the signatures of the “new physics” in analyzing data. In doing so, it is advantageous to make use of information presently available at energies low compared to the scale of the “new physics”. In particular, one should use the fact that the SM is a *gauge theory*.

In all known composite systems the existence of a structure at some higher energy scale manifests itself in the presence of induced terms (“anomalous magnetic moments”) in matrix elements of currents; for a review, see [4] and references quoted there. The presence of such terms, in turn, can be used as a low energy parametrization of physics beyond the currently known theory. It takes into account the known low energy structure of the theory and it gives rise to well defined and measurable deviations from the predictions of the SM. Previously, we studied the effect of anomalous moments on the forward–backward asymmetry in the annihilation of electron pairs into heavy quarks and leptons. Whereas, from the theoretical point of view, the measurement of the forward–backward asymmetry is the most desirable observable, see [4], it is clear that at present the asymmetries can be measured only with a rather limited accuracy, and hence, one cannot get a meaningful lower limit on an eventual compositeness scale.

The presence of anomalous moments in the matrix elements of currents influences, however, other observables as well: in particular, both the real and imaginary parts of the vacuum polarization tensor are modified; hence, various annihilation rates and the hadron to muon pair ratio (R) is also affected. (This is in contrast to the deviations from the SM considered by Altarelli and Barbieri [5]; see also [6]. In the case studied in those works only the real part of the vacuum polarization is affected at presently available energies.) Some of the quantities just mentioned are being measured to a high accuracy at currently available energies and will be remeasured at future machines, such as LEP2.

In this letter we show how the presence of anomalous moments influences the *rates* of the

processes $Z^0 \rightarrow l\bar{l}$ and the similar process, $Z^0 \rightarrow q\bar{q}$, where l and q stand for some lepton and quark. (The calculation is the same for the decays of the W and it is not repeated here.) In addition, we derive the contribution of the anomalous moments to the ratio R . Finally, we indicate how the presence of anomalous moments influences the vacuum polarization off-shell (the *oblique corrections*, [7]).

The matrix element of a conserved current between states of spin-1/2 particles can be parametrized in terms of the standard Dirac and Pauli form factors:

$$\langle p, \bar{p} | V_\mu | 0 \rangle = g_V \bar{u}(p) [\gamma_\mu F_1 + i\sigma_{\mu\nu} q^\nu / 2M F_2] v(\bar{p}). \quad (1)$$

In this equation, p and $\bar{p}$ stand for the four-momenta of the particle and antiparticle respectively and $q_\mu = p_\mu + \bar{p}_\mu$. The quantities $F_{1,2}$ are the usual Dirac and Pauli form factors. The mass M serves conventional normalization purposes. In the case of a neutral current, it is chosen to be *the* mass ($p^2 = \bar{p}^2 = M^2$), whereas in the case of a charged current it is conveniently chosen as the geometric mean of the masses in the final state.² All internal symmetry and spin indices are suppressed. The constant g_V in front of the expression on the *rhs* is chosen to be the one given by the SM, *evaluated at the appropriate q^2* . Thus, the structural contributions to the form factors are expected to depend on q^2/Λ^2 where Λ stands for the characteristic energy scale of the “new physics”, in the case investigated here, the one characterizing the substructure. (It is to be noted that without extracting a coupling constant, the form factors would receive contributions both from effects within the SM and from compositeness effects. We thus accomplish an approximate separation of the energy scales.) As a consequence, at $q^2 \ll \Lambda^2$ one has $F_1 \approx 1$ and $F_2 \approx \kappa$, where κ is the “anomalous magnetic moment”. In a composite model, one expects the coefficient of $\sigma_{\mu\nu} q^\nu$ to be of the order of $1/\Lambda$. (In fact, this estimate works quite well in various QCD-inspired bag-like models: the magnetic moments of baryons are obtained reasonably accurately in this way.) Hence, the order of magnitude of the conventionally normalized anomalous moments is³ M/Λ . Hence, heavy quarks and leptons are preferred in searching for compositeness effects. In what follows, we occasionally use an expression of the form,

$$\kappa = f \frac{M}{\Lambda}, \quad (2)$$

where f is a model-dependent factor, presumably of order unity.

The matrix element of the axial current does not contain a Pauli term. However, in the case of composite models with spontaneously broken chiral symmetries [4] the matrix element contains an irrotational part proportional to q_μ which is related to the standard $\gamma_5 \gamma_\mu$ term by means of a Goldberger–Treiman relation. With the possible exception of a few oblique corrections (e.g. W , Z coupled to heavy quarks) the contribution of this term is of the order of m^2/q^2 , where m stands for the mass of the Fermion the gauge Boson is coupled to. In what follows, we neglect these terms: compositeness effects can be efficiently searched for if $q^2 \gg m^2$. In the same way as we described in the case of the vector current, a running coupling constant, g_A , can be extracted. The conclusion is, however, that in the regime of

²In the case of quarks in the final state, one would have to further worry about properly defining the running masses due to QCD effects. This will not be considered here.

³One recalls that the conventional normalization amounts to properly normalizing the amplitudes of helicity flip 0 and 1 relative to each other.

current interest, viz. $m^2 \ll q^2 \ll \Lambda^2$, where the form factors are, to a good approximation, constant, the matrix elements of axial currents do not differ from those given the SM; hence, they will not be listed here.

Next, one should determine the internal symmetry structure of the Pauli terms in the vector currents. Without a knowledge of the symmetry breaking mechanism in the theory, this is ambiguous. Relying upon the known low energy structure, we therefore choose to associate the anomalous moments with the currents to which the physical gauge Bosons are coupled. When necessary, appropriate Boson is indicated by means of a subscript on the parameter κ .

The calculation of the physical observables is a straightforward exercise; its details are skipped. We give the results without QCD corrections; the $O(\alpha_s)$ correction – which should be adequate for the values of q^2 of interest – is given by the usual multiplicative factor.

We define the vacuum polarization tensor in the usual way,

$$\Pi_{\mu,\nu}^{a,b} = (g_{\mu,\nu}q^2 - q_\mu q_\nu)\Pi^{a,b}, \quad (3)$$

where a, b stand for γ , Z or W as appropriate. Decay rates are related to the imaginary part of Π in the usual way, $\Gamma^a = M_a \Sigma \text{Im} \Pi^{a,a}$, where the summation extends over the vector and axial contributions to the width.

The end result of the calculation of the imaginary part of Π is very simple. One has:

$$\text{Im} \Pi^{a,b} = \text{Im} \Pi_0^{a,b} \left(1 + \frac{\kappa^a \kappa^b}{8m_\alpha m_\beta}\right) \quad (4)$$

where it is understood that only the vector parts of the currents contribute and $\text{Im} \Pi_0^{a,b}$ stands for the expression of the vacuum polarization computed within the framework of the SM, (i.e. $\kappa = 0$), see, e.g. [8]. The symbols m_α, m_β stand for the Fermion masses in the loop.

What are the prospects of observing the deviations exhibited in the previous equation at currently available devices, such as LEP or by devices of the near future? It is immediately obvious that one has to look at the channels with heavy quarks or leptons. At current energies only c, b and τ are available. Because of the structure of $\text{Im} \Pi$ found above, the quantities of interest are the *ratios* of the various partial decay widths of the Z^0 computed for a nonvanishing κ to that of the expressions obtained in the SM. (As remarked above, these ratios are independent of QCD corrections to one loop accuracy.)

Define the ratio of partial widths of the Z^0 :

$$G(\alpha) = \frac{\Gamma^Z(\alpha)}{\Gamma_0^Z(\alpha)}, \quad (5)$$

where α stands for c, b, and τ , respectively. The expression for $G(\alpha)$ is:

$$G(\alpha) = \frac{g_V^2(m_{Z^0}^2 + 2m_\alpha^2)(1 + \frac{m_{Z^0}^2 \kappa_Z^2}{8m_\alpha^2}) + g_A^2(m_{Z^0}^2 - 4m_\alpha^2)}{g_V^2(m_{Z^0}^2 + 2m_\alpha^2) + g_A^2(m_{Z^0}^2 - 4m_\alpha^2)} \quad (6)$$

Here g_V and g_A stand for the appropriate vector and axial couplings as extracted from the SM, respectively. Note that $G(\alpha) \geq 1$. Because of the structure of the SM (and because

of the value of θ_W around the mass of the Z^0), the coupling to τ is almost a purely axial one: the ratio g_V^2/g_A^2 is, approximately, 0.14, 0.47 and 0.0041 for c, b and τ , respectively. Neglecting $m_\alpha^2/m_{Z^0}^2$ and using eq.(1), one gets the estimate for $Q = -1/3$ quarks (the most favorable case):

$$G(\alpha) \approx 0.04 f^2 \frac{m_{Z^0}^2}{\Lambda^2}. \quad (7)$$

Clearly, the effect is small (as expected), however, not entirely out of range for future, precision measurements of partial widths of the Z^0 .

The ratio, $S(q^2)$, of the annihilation cross section to that computed within the framework of the SM can be calculated in the same way. Due to the fact that we expect any anomalous moment of the muon arising as a consequence of its composite nature to be a very small quantity, we can also write to a very good approximation,

$$S(q^2) = \frac{R(e\bar{e} \rightarrow F\bar{F})}{[R(e\bar{e} \rightarrow F\bar{F})]_0} \quad (8)$$

where, as before, the subscript 0 denotes an expression obtained within the framework of the SM. As usual, $R = \sigma(e\bar{e} \rightarrow F\bar{F})/\sigma(e\bar{e} \rightarrow \mu\bar{\mu})$ and F stands for any Fermion of a given flavor produced in the final state. (Although eq.(8) is an – almost – trivial one, it is of some practical importance, since it shows that in an attempt to discover a composite structure, one does not have to worry about the absolute normalization of the cross sections.)

In view of the fact that, at least initially, one just wants to establish the existence of an anomalous moment, we exhibit the corresponding formula in the limit, $q^2 \gg m_{Z^0}^2, q^2 \gg m_\alpha^2$. We have:

$$S(q^2) \sim \frac{1 + \frac{(Q+g_V)^2}{g_A^2} (1 + \frac{q^2 \kappa^2}{8m_\alpha^2})}{1 + \frac{(Q+g_V)^2}{g_A^2}} \quad (9)$$

(Everywhere, Q stands for the electric charge of the Fermion.) In order to simplify matters, we also put the anomalous moments of Z^0 and of γ equal to each other. Barring accidental cancellations, such as $g_V \kappa_Z + Q \kappa_\gamma \approx 0$, this should be acceptable at a first attempt. The ratios, $(Q + g_V)^2/g_A^2$ are, roughly, 2.25, 1.57 and 2.89 for c, b and τ , respectively. Using again eq.(1) and assuming that the model dependent factors, f , are of the same order of magnitude, the masses drop out. Hence one can take the ratio of *total* cross sections and conclude that at, say, LEP2, the ratio S exceeds G by about a factor of 30. (*It pays to look for a substructure at higher energies.*) In practice, one should measure the hadron to muon ratio at several energies, divide it by the corresponding expression in the SM – including QCD radiative corrections to one loop order – and search for a linear dependence on q^2 with a *positive* slope.

Finally, we discuss oblique corrections, using the low- q^2 expansion introduced in ref.[5]. Given the imaginary part of the vacuum polarization, the real part can be calculated by means of a dispersion relation. It is well known that within the framework of a renormalizable theory, the dispersion relation for the vacuum polarization requires exactly one subtraction: by definition, the self energy has to vanish at the observable mass at which the propagator has a pole. (In the case of unstable particles, this is a somewhat subtle question, since the “mass” lies in the continuum created by open decay channels, see[9]. The problem together with its

solution has been recently rediscovered [10] in connection with the precision measurements of m_{Z^0} .) In our case, since we want to compute the deviation from the SM up to $O(q^2)$ in the presence of anomalous moments, it is more reasonable to cut off the dispersion integrals at a q^2 where “new physics” is expected to begin. In what follows, we choose the cutoff at $q^2 = \Lambda^2$. We compute $Re\Pi(q^2)$ in this approximation. As before, the calculations are elementary and we need not give their details.

We illustrate the result by exhibiting the self energy corrections arising from a $\bar{t}, b$ (or $\bar{b}, t$, respectively) intermediate state in the self energy of the W . (Due to the large mass of the t -quark, *a priori* this is expected to be the dominant contribution.) Furthermore, we neglect the mass of the b -quark with respect to that of the top and Λ . We write:

$$\Pi = a_0 + x a_1 + \dots, \quad (10)$$

where $x = q^2/m_t^2$. We find:

$$a_0 = A\left\{\xi - 2\ln\xi - \frac{7}{12} - \frac{1}{4\xi} - \frac{1}{6\xi^2}\right\}, a_1 = A\left\{\ln\xi - \frac{19}{12} + \frac{2}{\xi} + \frac{1}{4\xi^2} + \frac{1}{6\xi^3}\right\}. \quad (11)$$

In this equation, $\xi = \Lambda^2/m_t^2$ and the constant A is given in terms of the charged current coupling constant, $\hat{g}$:

$$A = \kappa^2 \frac{m_t}{m_b} \frac{g^2}{384\pi^2} |V_{tb}|^2. \quad (12)$$

The quantity V_{tb} is an element of the Kobayashi–Maskawa matrix; to a reasonable approximation, $V_{tb} = 1$.

We notice that a_0 is a relative mass shift of the W due to the composite structure of the intermediate state, whereas a_1 is a change in the normalization of the propagator. The mass shift is quadratically divergent in the limit when $\Lambda/m_t \rightarrow \infty$. Retaining only the quadratically divergent term and using eq.(1), the mass shift becomes independent of the masses *and of the characteristic scale of compositeness*. For a single flavor, its expression becomes:

$$\frac{\delta m_W^2}{m_W^2} \approx f^2 \frac{g^2}{384\pi^2} |V_{tb}|^2. \quad (13)$$

Numerically, the *rhs* of the last equation is about 8×10^{-5} , or summing over all flavors, one gets a relative mass shift of about 5×10^{-4} . Due to the fact that radiative corrections in the SM either are already known to a high accuracy or will be known in the near future, the measurement of such excess mass shifts appears to be a realistic possibility.

We merely remark that in the same approximation of retaining quadratically divergent parts only, the mass shift of the Z^0 is given by a similar expression. However, due to the fact that the Kobayashi–Maskawa matrix elements have to be replaced by the appropriate combinations of the weak mixing angles, the result is considerably smaller (for all practical purposes, only d -type quarks contribute). One finds that the relative mass shift of Z^0 is only about 1.1×10^{-4} .

To conclude, a composite structure of quarks and leptons (if it exists) can be discovered by means of precision measurements performed at energies well below the characteristic energy scale of the structure. Such measurements are, of course, no substitutes for a detailed exploration of a “post-SM physics” at the appropriately high energies.

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