**NUMERICAL RESULTS ON GRIBOV COPIES
IN LATTICE $SU(3)$ THEORY**

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ABSTRACT

Recent results obtained in collaboration with G. Parrinello, A. Vladikas, and with M.L. Paciello and B. Taglienti on the subject of Gribov copies on the lattice are presented.

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NUMERICAL RESULTS ON GRIBOV COPIES IN LATTICE $SU(3)$ THEORY

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Recent results obtained in collaboration with C. Parrinello, A. Vladikas, and with M.L. Paciello and B. Taglienti on the subject of Gribov copies on the lattice are presented.

In 1978 V.N. Gribov showed [1] that Coulomb and Landau gauge-fixing in the continuum theory could be ambiguous when the potential becomes strong; later on it was realized that this pathology affects all the realistic gauges [2]. A possible solution has been investigated [1,3] along the line of adding to $\partial A = 0$ the more stringent condition that the Faddeev-Popov operator be: $F.P. \geq 0$. When the generic gauge field $g(x)$ varies over the gauge transformations, the rotated potential A^g moves along the gauge orbit. The intersection points of the gauge orbit with the plane $\partial A = 0$ are stationary points of the functional $F_A[g] = -\sum_\mu \text{Tr} \int d^4x (A_\mu^g)^2$. Enforcing the positivity condition one selects the minima of F_A ; the boundary of this convex region is called (first) Gribov horizon. On the lattice the gauge-fixing is done minimizing $F \equiv -\frac{1}{V} \sum_{\mu,x} \text{ReTr} (U_\mu^g(x) + U_\mu^{g\dagger}(x - \hat{\mu}))$; in such a way the stronger condition is taken into account automatically and the copies are located inside the Gribov horizon. The precision in reaching the gauge-fixing is measured [4] by θ that is the quantity corresponding on the lattice to $\text{Tr} \int d^4x (\partial_\mu A_\mu)^2$.

The standard method adopted to generate Gribov copies consists in applying (random) gauge transformations to a link configuration

and minimizing F starting from such rotated configurations. In this way an ensemble of gauge-fixed configurations is produced [5], with some probability to obtain different final F values, i.e. Gribov copies.

It is worth while to notice that there is a complete equivalence, on a formal ground, between continuum and lattice formulation [3].

Here I would like to report on some results that we have recently obtained on this subject.

First of all, working with low statistics in Coulomb gauge $SU(3)$ theory [6] at $\beta = 6$ and $V = 10^3 \cdot 20$ on a Cray Y-MP, we have faced the *a priori* curious phenomenon of finding, for each rotated configuration, Gribov copies. We ended up with Gribov copies even when using as starting gauge rotation either a random gauge transformation close to the identity or a global random gauge transformation. In particular in the last case we did not expect, before attempting the test, to obtain a Gribov copy. Actually *a posteriori* we realized that there is no reason for the equivalence among configurations related by global transformations to hold along the gauge fixing process.

In Fig. 1 θ is shown as a function of the iteration sweeps; after a certain number of sweeps, in this case $\simeq 2000$, the smooth behaviour reached

Fig. 1. Plot of θ as a function of the Coulomb gauge-fixing sweeps for $SU(3)$, $V = 10^3 \cdot 20$ and $\beta = 6$.

seems to be a common property of all the copies.

A nice explanation of the high probability to find Gribov copies in Coulomb gauge with periodic boundary condition, comes from the fact that F is invariant under gauge transformations with the property to be constant within each time slice: $g(\mathbf{x}, t) = g(t)$. Therefore one is still free to rotate each time slice with respect to the others so that the sub-space of each time slice can have its own Gribov copies. Let me show the case of 4 gauge-fixed configurations obtained from the same thermalized configuration. We find that for the first time slice each configuration has the same F value: $F_{1,2,3,4}(t = 1) = 0.792044$ but at $t=4$ the first configuration has $F_1(t = 4) = 0.791700$ and the others have: $F_{2,3,4}(t = 4) = 0.792002$. At $t=12$ $F_1(t = 12) = 0.785009$, $F_{2,4}(t = 12) = 0.786018$ and $F_3(t = 12) = 0.784967$.

In order to gain computer time during the minimization, we have studied the over-relaxation algorithm as described in [7]; we refer to that paper for the description of the method and for further references. The following results have been obtained, in double precision, on an IBM RISC System/6000 mod 520 with 32 Mbyte of memory. Typically for $V = 8^4$ a single gauge-fixing

Fig. 2. Plot of θ versus Landau gauge-fixing sweeps. Different curves correspond to different ω values.

sweep takes roughly 2 seconds.

Besides confirming the strong reduction in the iteration sweeps required to converge at a prefixed accuracy, we have found [8] that different values of the over-relaxation parameter ω may generate Gribov copies. This unconventional way of producing Gribov copies strengthens the correspondence between the gauge-fixing algorithm and the evolution of a non-linear dynamic system with a large number of degrees of freedom. From this point of view the evolution may depend dramatically both on the initial condition and on the relevant parameters of the evolution equation.

In Fig. 2 the behaviour of θ versus the gauge-fixing sweeps is shown for a lattice with $\beta = 6$ and $V = 8^4$. Different curves corresponding to different copies have been labelled with the letters ($\alpha, \beta, \gamma, \delta$) and the same letter indicates the same final F value.

Fig. 2 shows that different behaviours are not directly related to the final F value and the same final F value is sometimes reached with different ω values without having a clear relationship between them. In Fig. 3 the complete θ behaviour is shown up to the required accuracy $\theta \leq 10^{-18}$.

Fig. 3. The same plot as in Fig. 2 up to $\theta \leq 10^{-18}$.

β	$La(GeV^{-1})$	N_{conf}	$V = 6^4$
5.55	9.7	3	4, 1, 9
5.65	8.7	3	2, 4, 6
5.70	8.2	3	3, 2, 4
6.00(*)	5.9	6	1, 1, 1, 1, 1, 4
β	$La(GeV^{-1})$	N_{conf}	$V = 8^4$
5.65	11.6	3	9, 10, 4
5.80	9.8	6	1, 3, 1, 7, 3, 1
6.00(C)	7.8	3	5, 6, 11
6.10(*)	7.0	3	2, 2, 2
6.20(*)	6.3	6	1, 1, 1, 2, 1, 1
β	$La(GeV^{-1})$	N_{conf}	$V = 10^4$
5.65	14.5	3	12, 21, 3

Table 1

The number of Gribov copies in SU(3) lattices with varying β and V . La is the linear lattice size in GeV^{-1} according to the two-loop perturbative asymptotic relation. N_{conf} is the number of thermalized configurations generated in each case. The number of gauge-equivalent configurations fixed to the Landau gauge is always 21. Each entry stands for the number of copies found in each case; 1 stands for no copies found. The symbol (C) indicates the critical value of β ; (*) stands for β values characteristic of deconfinement.

Moreover the final value of F and the pattern of F values is analogous to what is found with the conventional method of changing the starting random transformation.

Finally, using the over-relaxation algorithm for

its high efficiency, we have studied the behaviour of the number of copies with varying β and V in the Landau gauge [8]. The gauge has been fixed at $\theta \leq 10^{-18}$ and the value of ω has been optimized. Our data are reported in Table 1.

They show the decrease of the number of copies with the shrinking of the physical lattice volume. This is a clear trend, even if it is not confirmed at all points, probably as a consequence of our low statistics. A few points of Table 1 refer to the deconfined phase; here we found copies with low probability.

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