CM-P00061086

CERN SL 91-44 (AP)
LHC 172

Towards a Better Understanding of Slow Particle Losses in Large Hadron Colliders

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Abstract

It is well known that the high-order multipole field errors, which cannot be avoided in modern superconducting proton accelerators, lead to particle losses, which may occur after many turns around the machine. The easiest, but most expensive approach, for the detection of such particle losses is brute-force tracking of many particles over long periods of time. Unfortunately, one is thereby always limited to some maximum number of turns owing to computational speed and the large number of cases to be studied. Predictive tools for an early detection of the phenomena leading to particle loss are therefore welcome. The detection of local chaos through the evaluation of Lyapunov exponents allows such a prediction in an easy and reliable manner. However, it is still an open question how the particles are actually driven to large amplitudes and why this may take long periods of time. Our aim here is to give a phenomenological picture for this slow instability, and to show how resonances, even of very high orders, enter into the loss mechanisms.

*Paper presented at the
“Fifth Advanced ICFA Beam Dynamics Workshop”
Corpus Christi, TX, USA, 2-8 October, 1991*

Geneva, Switzerland

October 1991

TOWARDS A BETTER UNDERSTANDING OF SLOW PARTICLE LOSSES IN LARGE HADRON COLLIDERS

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ABSTRACT

It is well known that the high-order multipole field errors, which cannot be avoided in modern superconducting proton accelerators, lead to particle losses, which may occur after many turns around the machine. The easiest, but most expensive approach, for the detection of such particle losses is brute-force tracking of many particles over long periods of time. Unfortunately, one is thereby always limited to some maximum number of turns owing to computational speed and the large number of cases to be studied. Predictive tools for an early detection of the phenomena leading to particle loss are therefore welcome. The detection of local chaos through the evaluation of Lyapunov exponents allows such a prediction in an easy and reliable manner. However, it is still an open question how the particles are actually driven to large amplitudes and why this may take long periods of time. Our aim here is to give a phenomenological picture for this slow instability, and to show how resonances, even of very high orders, enter into the loss mechanisms.

1 Introduction

The prediction and understanding of the long-term behaviour of conservative systems has always been a major concern in the field of non-linear dynamics. In particle accelerator physics this becomes more and more important in view of the large hadron colliders, which require a long lifetime of the beam in the presence of strong non-linear fields.

One of the well-known techniques to predict an unstable behaviour is the evaluation of the Lyapunov exponent,¹⁻⁵ which has also been applied to particle stability in accelerators.^{6,7} More recently, refined methods have been investigated^{8,9} to further improve the predictive power of chaos detection. In the first part of this paper we will discuss some of our post-processing procedures, which we have applied in the tracking studies done to evaluate the LHC dynamic aperture.¹⁰⁻¹² We will show how chaotic motion can be detected and we will demonstrate that the Lyapunov analysis is much more sensitive than other methods, especially when slightly chaotic particles are studied. We will also discuss how a small but sufficient set of parameters can be found to minimize the effort for a complete tracking study.

In general the results from tracking simulations are summarized in survival plots, which depict the particle-loss turn number as a function of the amplitude. In the second part of this paper we will show the results of a study that, while giving a phenomenological view of the chaotic motion in a strongly non-linear accelerator, allow us to explain the typical features of the survival plots.

2 Post-processing

2.1 Detection of chaotic motion

There are many ways to determine whether particles are chaotic and will eventually be lost.

If the particles are violently chaotic one can easily see their behaviour directly in phase space. More subtle methods allow the detection of what we call medium-strong chaos. One can, for instance, make a Fourier analysis of the motion¹³: chaos manifests itself by the appearance of a large number of frequency lines in the spectrum. Another way is to study the evolution of the particle amplitude, in particular if one is able to subtract the large phase-space fluctuations by means of methods based on the use of normal forms: small, irregular fluctuations, which indicate chaotic motion, can be more easily detected.¹⁴ At the border of the dynamic aperture, however, we enter the regime of weak chaos, and only the most sensitive tools work reliably there.

To determine a chaotic behaviour we use the Lyapunov method by following the distance in phase space of two initially close-by particles: if the motion is chaotic, this distance should grow exponentially even on a microscopic scale.

This last feature makes the Lyapunov method much more sensitive than the other approaches, that can only detect chaotic motion after the instability has become apparent on a much larger scale. Nevertheless, it must be stated that for a reliable chaos detection a certain minimum number of turns is always required. This number of turns is, however, orders of magnitude smaller than the loss turn number at which the particle escapes to very large amplitudes, so that it allows us to define a safe lower bound for the unstable regime. A serious limit of this method is that, whereas it tells us that the particle will eventually be lost, it gives no precise information on when this loss will occur. A different concept would clearly be needed to make real progress on this point.

Now we would like to illustrate the differences between regular motion, weak chaos, and medium-strong chaos.

In Fig. 1 the distance in phase space of two initially close-by particles is depicted together with their amplitude in the combined horizontal and vertical phase space ($X-X'-Y-Y'$) in the case of a regular motion. The non-linearity of the motion leads to a linear increase of the distance in phase space: slightly different initial amplitudes lead to a difference in tune, which causes the particles to depart linearly on average. The regular variations in both pictures are due to the phase-space deformations which are sampled at regular intervals.

In the medium-strong chaotic case (Fig. 2), it takes a considerable amount of time (40,000 turns) until the maximum distance in phase space, which is normalized not to exceed unity, is reached, and it needs 240,000 turns before the actual particle loss takes place. The amplitude shows only slight irregularities in the first period of time and then reveals rather large variations, though remaining confined; during the last few thousands of turns, however, a small but steady increase of the amplitude is noticeable.

In the case of the weakly chaotic particles (Fig. 3) the amplitude does not reveal any sign of irregularity over the first 100,000 turns, whilst the evolution of the distance in phase space does already show a clear deviation from the linear increase. Extending the analysis to 400,000 turns (Fig. 4), one sees that this distance has almost reached its maximum value, whilst the amplitude does not yet show any irregular variation. Note that also in this case we find the periodic variation both in the amplitude and in the distance in phase space.

2.2 Minimizing the number of parameters

The study of the dynamic performance of a big superconducting accelerator always implies the investigation of a considerable number of different parameters. We are forced, therefore, to try to reduce the task to the study of a small, but sufficient, set of cases; especially as we should always be prepared for some major design changes that will require a repetition of the whole study.

Now we would like to say a few words about issues that always come up when long-term particle tracking is considered.

The first question is whether it is sufficient, in order to determine the region of stability, to vary only the starting amplitude, or if it is necessary to vary also the initial phase. As our tracking results are recorded over a large number of turns for each starting amplitude, the information about the phase dependence of the amplitude can be extracted from the data corresponding to one single value of the starting phase, by simple post-processing. In the specific case of the LHC, which is a machine dominated by high-order resonances, we find that the amplitude depends very weakly on the phase, so that, for the definition of the dynamic aperture, it is enough to rely on the results obtained by considering only one value for the starting phase in each plane. As an example, in Figs. 5a and 5b we show the amplitude variation of a regular particle as a function of phase for the horizontal and vertical phase spaces respectively. Although this particle is at large amplitude, close to the dynamic aperture, the average amplitude varies by less than 5%: this is of the order of our uncertainty in the determination of the dynamic aperture itself, in which case the dependence of the amplitude on phase can be neglected.

Another problem that arises quite often is how to adjust the initial amplitude ratio between the horizontal and vertical planes. In a separate study we have checked that the ratio between the dynamic apertures found for flat and round beams is roughly $\sqrt{2}$, as expected, so we can restrict ourselves to the study of the

most realistic case which is that of a round beam. We must make sure, however, that our non-linear fields do not distort the initial amplitude ratio. Figure 5c shows this ratio as a function of turn number for the same regular case as before. Besides the large short-term fluctuations already observed in the plots of the amplitude versus turn number, the average ratio stays constant in time, differing from unity only by some 5%: we conclude that we are indeed considering round beams, and that the amplitude ratio is not changed over time.

3 Slow particle loss

3.1 *Survival plots and local instability*

Post-processing analyses like the ones presented above give a lot of insight into non-linear aspects of accelerators and are, therefore, an essential tool in our dynamic aperture studies for the LHC.

Another tool we have been using repeatedly to determine the dynamic aperture is the survival plot, from which one can guarantee particle stability over a number of turns as a function of the amplitude. We add to these plots the onset of chaotic motion, derived as described in the previous section. As we are mainly interested in the overall increase of stability towards small amplitudes, we generally restrict ourselves to a few amplitudes, spaced roughly by 5% of the expected dynamic aperture. Figure 6 is a typical example of a survival plot from the LHC aperture studies: the particle-loss turn number is recorded as a function of the amplitude for two different random distributions of magnet errors, and the border between regular and chaotic motion is indicated.

Using a much finer scan of the amplitude, the typical spread in the loss turn number for particles with very close amplitudes becomes apparent (Fig. 7). It is very important to understand the cause of this spread because its lower bound will be directly observed in the actual accelerator.

From the theory of the Lyapunov exponents we expect the chaotic behaviour to be a local phenomenon; to test these local properties of the instability, for each of the amplitudes considered in Fig. 6 we started 15 companion particles (Fig. 8) located on a four-dimensional sphere around the original particle, at a distance less than 10^{-4} mm (Fig. 8 shows only the companion particles that are lost before 10^5 turns).

The spread in loss turn numbers of particles scanning a microscopic portion of the phase space agrees very well with the spread in Fig. 7 where the particles are not close by, but equally spaced in amplitude. The local strength of the instability, rather than macroscopic differences in phase space, is therefore the cause of this spread. Moreover, as seen in Fig. 8, this local instability tends to become smaller when the dynamic aperture is approached, resulting in an increase of the spread in loss turn numbers for decreasing amplitudes. It has to be noted that in both cases (Figs. 7 and 8) some of the particles close to the dynamic aperture are not yet lost, so that the spread is even wider there, as indicated by the open dashed

lines.

The dense survival plots present another very interesting feature: in Fig. 8, for example, we can identify different amplitude ranges for which the average loss turn number and its spread, as a function of the amplitude, stay approximately the same. This can be related to different high-order resonances, or even to different nests of resonances. In the tune diagrams of Fig. 9 we see that at the beginning particles exhibit tune-shifts that vary smoothly with amplitude (Fig. 9a). However, shortly thereafter, chaotic particles belonging to different ranges of amplitudes are attracted towards different nests of resonances (Fig. 9b), each of them determining a different average lifetime.

3.2 *Following a chaotic particle*

To get a phenomenological understanding of the slow particle losses in large superconducting accelerators, we will now choose a chaotic particle at large amplitude and will follow in detail its tune evolution and that of one of its companions during their whole lifetimes.

In Figs. 10a and 10b we first show the averaged horizontal and vertical amplitudes for one of these particles. This particle has a rather strong stochastic behaviour, which results in wild and irregular amplitude variations. After about 12,000 turns (point C in Fig. 10) a coupling resonance becomes active, leading to an increase of the horizontal amplitude and a decrease of the vertical amplitude. The effect of this coupling resonance can be interpreted as an exchange of energy between the two transverse planes, in which case the four-dimensional amplitude remains unchanged at this specific point (C in Fig. 10c). However, the motion being six-dimensional, there is also in general the possibility of an exchange between transverse and longitudinal motion. Such a coupling can be invoked to explain the variation of the four-dimensional amplitude seen in Fig. 10c. At the end of the life of the particle, at some 60,000 turns (point F in Fig. 10), another exchange of energy takes place between the horizontal and the vertical planes, until both amplitudes increase rapidly, indicating the effect of a sum resonance.

The variations of the amplitude can be nicely described by means of the evolution in the tune diagram of Fig. 11, where we show the phase advance averaged over 760 turns, which correspond to 5 synchrotron oscillations. Our chaotic particle of Fig. 10 moves wildly in the tune diagram in a region spanned by resonances of at least 12^{th} order: soon after it has been started (point A in Figs. 10 and 11), it is attracted by a resonance of order 23 and spends some time there (point B); then it is quickly displaced towards a difference resonance of order between 12 and 29 (point C) which leads to the energy exchange mentioned above; after this very short event (less than 4000 turns), the particle is captured again on two main nests of resonances (points D and E), and bounces back and forth between them, until it is brought back to the same region of difference resonances (point F), and is finally driven out by a sum resonance. The difference resonance at point C/F, as well as the lethal sum resonance at the end, cannot be determined

precisely, owing to the short time for which they are active.

The role of difference resonances turns out to be very important: though not being capable of causing a particle loss themselves, they seem to have a catalytic effect, moving the particle to places where it finally reaches a sum resonance strong enough to extract it.

It is very instructive to follow the tunes of the other companions corresponding to the same amplitude; the transverse tunes of one of them are shown in Fig. 12 (and all other particles show a similar behaviour): after following the same path as the first particle in the tune diagram for a while (until point B in Figs. 11 and 12), suddenly the particle ‘decides’ to follow a different track, which leads to its loss due to a completely different resonance and after a different period of time.

We can explain now why at a lower amplitude the spread of the particle-loss turn number becomes wider: as the motion becomes less chaotic, particles move more slowly in the tune diagram; they thereby have more time to ‘choose’ a path different from the paths ‘chosen’ by the other companions; consequently they will have a much wider spread of lifetime.

4 Conclusions

We have shown that the Lyapunov exponent method works reliably and is more sensitive than other methods. Some of our post-processing techniques were presented, which allow a detailed analysis and understanding of the intricate motion in phase space.

We then demonstrated that the spread of the particle-loss turn number in the survival plots is due to the local nature of the instability. We also found that this spread is widened when the dynamic aperture is approached and that the step-wise variation of the average lifetime as a function of amplitude can be related to the influence of different nests of resonances. Moreover, analysing the evolution of the tunes of a chaotic particle, we have seen that very-high-order resonances, and a large number of them, may determine the loss mechanism, and that difference resonances can play an important role in it. Finally it became clear that the spread of the particle lifetime is due to the different paths in the tune diagram.

5 Acknowledgement

We would like to thank J. Gareyte for constant encouragement and for having time for many thorough discussions.

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Figure 1: Regular motion

The upper graph shows the amplitude of a regular particle as a function of turn number. The corresponding distance in phase space is depicted in the lower graph.

Figure 2: Medium strong chaotic motion

The upper graph shows the amplitude of a medium strong chaotic particle as a function of turn number; the corresponding distance in phase space is depicted in the lower graph. A particle loss takes place after a relatively long time (240,000 turns) and the maximum phase-space difference is reached after 40,000 turns. The chaotic behaviour leads to strongly irregular amplitude variations.

Figure 3: Weakly chaotic motion (first 100,000 turns)

The upper graph shows the amplitude of a weakly chaotic particle as a function of turn number and the corresponding distance in phase space is seen in the lower graph. There is no apparent irregularity in the evolution of the amplitude, whilst the distance in phase space shows a deviation from a linear increase which indicates that the motion is of stochastic nature.

Figure 4: Weakly chaotic motion (400,000 turns)

The upper graph shows the amplitude of the same weakly chaotic particle of *Fig. 3* as a function of turn number, whilst the corresponding distance in phase space is shown in the lower graph. There is still no apparent irregularity in the evolution of the amplitude, whilst the distance of phase space now increases almost to its maximum value.

Figure 5: Amplitude variation versus phase, and amplitude-ratio versus turn number.
a), b) The amplitude variations as a function of phase for the horizontal and vertical phase-space plane respectively. A typical regular case is shown near the dynamic aperture of the LHC. The variations of the average amplitude stay below 5%: as they are of the same order of magnitude as our uncertainty for the dynamic aperture, we can neglect them.
c) The ratio of the horizontal and vertical amplitudes for the same case. The deviation is less than 5% so that the beam can be considered as being round.

Figure 6: A typical survival plot from the LHC aperture studies

For two different random distributions of magnet errors the particle loss numbers are shown as a function of the amplitude. In addition, the onset of chaos is shown, which is determined by means of the Lyapunov analysis.

Figure 7: Spread of the loss turn number

For the same two cases as in *Fig. 6* the survival plot is shown with a finer amplitude scan, which reveals a spread of the loss turn number.

Figure 8: Survival plot of companion particles

The analysis of the particles considered in *Fig. 6* is repeated for 15 companions, separated by less than 10^{-4} mm. One finds a spread of the loss turn number which is comparable to the one seen in *Fig. 7*. Distinct domains of amplitudes can be seen where the loss turn number is roughly constant (compare with *Fig. 9*). Moreover one finds an increase of the spread in loss turn number when the dynamic aperture is approached.

Figure 9: Tune diagram for chaotic particles with different starting amplitudes

The average position in the tune diagram is shown for particles of 10 different amplitudes, tracked in the LHC with the error distribution No. 2 of Fig. 8: the first (a) and the third (b) synchrotron oscillations are considered.

The amplitude regimes with different average lifetimes, seen in Fig. 8, are due to different nests of resonances dominating the particle behaviour in different amplitude ranges.

The $\star$ indicates the nominal tune.

Figure 10: Energy exchange between horizontal and vertical phase space for a chaotic particle

a) The horizontal and b) the vertical amplitude respectively over the lifetime of a chaotic particle of some 65,000 turns. At 12,000 turns an exchange of energy takes place between the two planes, as the four-dimensional amplitude (c) does not show any particular variation at that turn number. Moreover this four-dimensional amplitude is not constant, which we interpret as an energy exchange between the transverse motion and the longitudinal motion. The positions of the points indicated as A,B,C,D,E, and F correspond to those of the tune diagram of Fig. 11.

Figure 11: Evolution of the transverse tunes for a chaotic particle

For the same chaotic particle as in *Fig. 10* the evolution of the transverse tunes is depicted. After starting at point A, and then being locked briefly on a resonance of order 23 (point B), the particle reaches a difference resonance (point C), which produces an energy exchange; then the motion is locked to two main nests of resonances for some more time (points D and E); after coming back to the same region of difference resonances (point F), the particle is extracted by a certain sum resonance, which cannot be identified owing to the speed of the loss.

Figure 12: Evolution of the transverse tunes for a companion particle

The transverse tunes of a companion of the particle shown in *Fig. 11* follow the same path in the tune diagram for about 4000 turns (point B); then the particle 'decides' to enter some other regime in the tune diagram (point C), which leads to a particle loss due to another resonance (point D) and at a different time.