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DYNAMIC APERTURE OF THE CERN LARGE HADRON COLLIDER WITH INJECTION OPTICS

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** On leave of absence from IHEP, Beijing, China*

Abstract

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Abstract

In a hadron collider, the stability of the particle motion is basically determined by the field-shape imperfections of the superconducting dipoles and quadrupoles, especially during the injection flat bottom, when the effect of the persistent currents is maximum and the transverse size of the beam is large. Some precautions can make the motion of the particles less sensitive to the non-linear components of the guiding fields. Correcting multipoles can be foreseen in the regular cells, to reduce the non-linear tune-shift caused by the systematic components of the field errors. The variations of the orbit functions can be limited along the insertions. The closed orbit and the linear coupling can be corrected sufficiently well. Finally the ripple of the power supplies can be reduced as much as possible. Most of these concepts have been embedded in the design of the LHC and their beneficial effects on the dynamic aperture have been extensively evaluated by computer simulations. Cost optimization implies that the dynamic aperture of the LHC has to be mainly limited by the field-shape imperfections in the dipoles rather than by the errors in the quadrupoles or by the mechanical limitation represented by the primary collimator, inserted to collect the halo particles before they hit the cold bore.

I. LATTICE MODEL

The layout adopted is that of Ref. [1]. It consists of eight arcs interleaved with eight insertions. Each arc contains 25 regular FODO cells 99 m long, with 90° phase advance. The insertions have a general purpose (GP) design, except those in point Nos. 3 and 7, dedicated to the beam dump and the halo cleaning respectively. The range of values of the β -functions in the insertions with the injection optics is shown in Table 1.

Table 1
Values of β^* and β -peak in the LHC insertions
with the injection optics

	GP No. 1, 2, 5	GP No. 4, 6, 8	Cleaning No. 7	Dump No. 3
β^* [m]	8.0	8.0	15	220
β -peak [m]	318	327	620	630

The length and the phase advance of the regular cells result from a compromise to obtain a large focusing strength, a dense packing of the dipoles, and a sufficiently high dispersion to keep the strength of the chromaticity sextupoles low: β -peak values of 165 m, a D-max value of 1.9 m and a collision energy of 7.7 TeV per beam are reached with bending fields of 10 T and focussing fields of 250 T·m.

A thin-lens version of the lattice has been used for computer tracking simulations. To introduce synchrotron oscillations an RF cavity has been added. Relative momentum deviations of 10^{-3} are generally used, corresponding to two times the r.m.s. momentum dispersion of the bunch. The beams are separated vertically by $\pm 3\sigma$ in all the IPs, and cross horizontally at an angle of $\pm 100 \mu\text{rad}$ in all the GP insertions.

In general, an aperture limitation of 25 mm, as large as the dipole inner coil radius, has been assumed to compute the dynamic aperture. A tighter limitation was used when the primary collimator

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required to protect the superconducting magnets from the halo particles was also considered. The nominal value suggested for the position of the collimator in both the horizontal and the vertical directions was 6 or 8 mm for dipoles with inner coil diameter of 50 or 56 mm respectively [2].

The choice of the nominal working point $Q_H = 70.28$, $Q_V = 70.31$, has been based on the optimization of the medium-term ($\sim 10^4$ turns) dynamic aperture. An increase by one unit of Q_H has been considered in order to reduce the linear coupling resonances. Larger horizontal-vertical tune separations have not been retained since they imply either an unacceptable mismatch of the optical functions in the insertions, or a heavy modification of the lattice layout.

II. COMPUTATIONAL TOOLS

Simulations over many turns have been performed to estimate the range of amplitudes for which the particle trajectories are still confined. The tracking codes MAD and SIXTRACK in their vectorized versions [3,4] have been run on the Cray-XMP-48 at CERN. They have approximately the same computational speed, namely $2 \cdot 10^5$ s of CPU for 20 particles followed over 10^6 turns. The effects of the rounding errors with single precision on the Cray have been estimated tolerable for the numerical evaluation of the dynamic aperture over few 10^6 turns, as well as for the detection of the onset of chaos with the Lyapunov exponent, which was estimated by following the trajectories of two particles starting at an initial distance of 10^{-7} m.

III. IMPERFECTIONS

Positioning errors of the magnetic elements with random Gaussian distributions, truncated at 3σ , and with an r.m.s. value of 0.5 mm for the misalignment and of 1 mrad for the tilt are envisaged in the LHC. The reading errors of the beam position monitors are assumed to have an r.m.s. value of 0.6 mm.

Recent estimates of the field-shape imperfections [5], based on the measurements of the HERA magnets, have been used for the LHC magnets. The appropriate scaling law to take into account the change of the inner coil diameter, of the size of the filament and of the excitation level at injection have been applied, and the systematic imperfections due to the two-in-one design have been added.

Dipoles with inner coil diameter of 50 mm and 56 mm respectively have been considered to compare their performances. Their field-shape imperfections at injection are listed in Table 2.

Table 2
Dipole field errors at injection in units of 10^{-4}
at $R_f = 1$ cm.

	Inner coil diameter of 50 mm.				Inner coil diameter of 56 mm.			
	Systematic		Random		Systematic		Random	
n	b_n	a_n	b_n	a_n	b_n	a_n	b_n	a_n
1	5	-	-	-	5	-	-	-
2	-1.217	0.80	0.416	1.373	-1.150	0.756	0.372	1.227
3	-2.564	0.10	1.116	0.235	-2.192	0.086	0.882	0.186
4	-0.148	0.03	0.08	0.272	-0.111	0.023	0.055	0.186
5	0.521	0.03	0.14	0.069	0.344	0.02	0.083	0.041
6	-0.003	-	0.027	0.043	-0.001	-	0.014	0.022
7	0.044	0.02	0.027	0.024	0.023	0.01	0.012	0.011
8	0.003	-	0.012	0.012	0.002	-	0.005	0.005
9	0.012	0.001	0.008	0.011	0.005	-	0.003	0.004
10	-	-	0.006	0.006	-	-	0.002	0.002
11	0.004	-	0.004	0.005	0.001	-	0.001	0.001

Quadrupoles with a inner coil diameter of 56 mm have been assumed everywhere, except in the inner triplet of the GP insertions and in a limited number of locations of the machine insertions, where quadrupoles with a special design are likely to be required. The field-shape imperfections used at injection are listed in Table 3. In the single bore quadrupoles, the systematic components b_3 to b_6 have been reduced by a factor of two, while the inner coil radius R_C of ten quadrupoles in the cleaning and the dump insertions, in which the value of B is in excess of 600 m, has been assumed as a free parameter in the present optimization of the dynamic aperture and each non-linear component of order n has been reduced proportionally to R_C^{-n+1} (see section VII).

Table 3
Quadrupole field errors at injection in units of 10^{-4}
at $R_T = 1$ cm
Inner coil diameter of 56 mm.

n	Two-in-one quadrupoles				Inner triplet single bore quadrupoles			
	Systematic		Random		Systematic		Random	
	b_n	a_n	b_n	a_n	b_n	a_n	b_n	a_n
1	-	-	-	-	-	-	-	-
2	5	-	-	-	5	-	-	-
3	0.020	0.280	0.454	0.478	0.010	0.280	0.454	0.478
4	0.19	0.01	0.133	0.188	0.10	0.01	0.133	0.188
5	0.05	0.01	0.065	0.093	0.025	0.01	0.065	0.093
6	-0.6	-	0.053	0.069	-0.3	-	0.053	0.069
7	-	-	0.012	0.013	-	-	0.012	0.013
8	-	-	0.006	0.007	-	-	0.006	0.007
9	-	-	0.007	0.004	-	-	0.007	0.004
10	0.020	-	0.005	0.003	0.020	-	0.005	0.003
11	-	-	0.002	0.002	-	-	0.002	0.002

The field of an LHC magnet follows a well defined point-like symmetry because of the two-in-one design with horizontal separation. The same kind of symmetry is likely to be followed also by the systematic part of the field shape imperfections, which are due to the iron saturation as well as to geometrical imperfections of the coils and of the yoke.

When going from an octant to the next one, through an interaction point, the beam trajectories pass from the inner to the outer channel of the main magnets or vice-versa. As a consequence the signs of the systematic multipolar components of the magnetic errors follow a regular pattern: in two contiguous arcs, the normal multipolar components b_n have equal sign if n is odd, and opposite sign if n is even; whilst the skew multipolar components a_n have opposite sign if n is odd, and equal sign if n is even. A practical implication of this fact is that the quadrupolar and the octupolar systematic imperfections are approximately self-compensated over one turn.

IV. CORRECTORS

The two LHC rings are treated individually as far as corrections are concerned. Orbit correctors and monitors are located next to each twin lattice quadrupole. Directional monitors are installed near the inner triplets of the GP insertions, where the counter-rotating beams are almost collinear. Additional correctors and monitors are added in the insertions to take care of the vertical separation and of the crossing angle at the IPs, as well as of the possible orbit mismatch at the outer ends of the dispersion suppressors. The integrated strength of the correcting dipoles is 1.5 T·m in the arcs, and 2.2 T·m in the insertions, where the orbit functions are more irregular and a better correction is required.

To decouple the horizontal and vertical motions a global correction is implemented: 92 skew quadrupoles, powered in four families, are installed in the insertions, twelve in each GP insertion, and ten in each machine insertion. Two of the families are almost orthogonal in phase for the main

diagonal Q_H - Q_V . The correcting magnets are 0.72 m long, with a gradient of 120 T/m, and are of the same design as the arc tuning quadrupoles.

Sextupole, octupole and decapole correctors à la Neuffer [7] are used in the regular cells only, as shown in Fig. 1.

V. ORBIT AND LINEAR COUPLING CORRECTION

The correction of the orbit in the LHC can be very efficiently performed with the MICADO algorithm [6], using the full set of correctors. Nevertheless, to have a realistic representation of a not fully optimized machine, we decided to perform simulations in presence of somewhat large orbit deviations. The effect of the quadrupole misalignments was thus compensated with only a reduced number of correctors in the arcs. The resulting r.m.s. orbit deviation over the full ring was about 1.5 mm in both planes.

At injection, the linear coupling of the LHC is mainly due to the systematic and random skew quadrupole component a_2 in the field of the main dipoles. The other sources of coupling, i.e. the tilt of the main quadrupoles, and the vertical orbit in the chromaticity sextupoles, are at least an order of magnitude weaker. This is also true for the collision optics, although in this case the tilt of the inner triplet of the experimental insertions is no longer negligible, and possible axial fields in the experimental devices will need to be locally corrected. With realistic errors, and the nominal setting of the main quadrupole gradients, the strength of the main diagonal resonance is as large as almost half a unit in tune, which will make it impossible to operate an uncorrected machine. With one unit separation of the horizontal and vertical tunes, this strength is reduced to 0.02; it can be further diminished by more than a factor of two with two families of skew quadrupoles orthogonal in phase, by minimizing the value δQ of the closest tune approach with numerical methods.

There are two other approaches to globally decouple the machine with the four families of skew quadrupoles, assuming a perfect knowledge of the magnetic imperfections a_2 in all the dipoles. In the first approach, the strength of the correctors is adjusted to cancel the off-diagonal coupling terms of the one-turn transfer matrix computed from IP1. Of course, the results will depend on the choice of the starting point of the computation, but we found this dependence to be almost negligible in practice. In the second approach, the first-order skew resonances $Q_H \pm Q_V = p$ are compensated. At the nominal working points, 70.28/70.31 and 71.28/70.31, with both systematic and random a_2 , the two methods lead to a decoupled machine, with $\delta Q \approx 10^{-3}$, with almost equal compensating strengths.

The proposed decoupling strategies are inadequate to cancel the vertical dispersion around the accelerator: they leave residues of about 0.6 m in the arcs and less than 4 cm at the IPs for both the injection and the collision optics. Other correction schemes which could compensate this effect are under investigation.

VI. SETTING OF THE NON-LINEAR CORRECTORS AND OF THE WORKING POINT

The systematic multipolar imperfections of the LHC magnets mainly produce a large amplitude- and momentum-dependent tune shift which results in a sizeable reduction of the dynamic aperture. Our aim is to find a compensation for sextupolar, octupolar, and decapolar components, by which the tune shift is limited to below 0.005 units up to half of the vacuum chamber aperture (9 mm) for on-momentum particles, and up to one third of the vacuum chamber aperture (6 mm) for off-momentum particles with $\delta = 10^{-3}$, i.e. 2σ of the momentum distribution, a criterion inspired by the operational experience of the proton-antiproton collider at CERN. The most efficient arrangement with a small number of correcting elements seems to be that proposed in Ref. 7, which has been implemented in the regular cells of the LHC accordingly to the scheme of Fig. 1. Three different strategies have been employed to set the strengths of the lumped multipoles [8]: two minimize the amplitude-dependent tune shift with computer tracking and with normal forms evaluations at 11th order respectively, and the third minimizes the momentum-dependent tune shift over the full bucket area. In spite of the different starting points, the three approaches have fairly similar performances and give adequate corrections. The quality of the corrections remains sufficient if the octupole component of the central corrector is omitted.

The choice of the working point and the difference of 0.03 units between the horizontal and the vertical tune has been discussed in Ref. 8. Simulations over $2 \cdot 10^4$ turns were performed with the

assignment of the random errors corresponding to the intermediate seed and dipoles of 50 mm inner coil diameter, in order to validate our choice. The medium-term dynamic aperture and the chaotic boundary are shown in Fig. 2 for different values of the tunes sitting along a parallel to the main diagonal in the working diagram. The LHC working point is in a safe area between the third and the fourth order resonances. The effect of a_2 and of the correcting skew quadrupoles is evident in Fig. 3 where the medium-term dynamic aperture is presented as a function of the tune for machines without linear coupling and with corrected coupling. In the realistic case with coupling the third order skew resonance $3Q_V = 211$ is slightly larger than when a_2 is neglected. Should this be too limiting for the operation of the LHC, we could envisage to move the working point to just above the integer in the working diagram.

VII. APERTURE OF THE INSERTION QUADRUPOLES

The short-term dynamic aperture has been first evaluated by tracking particles of different starting amplitudes for 10^3 turns. This was fast and well suited for a first exploration of the space parameters, and was also sufficient to reveal the most important features of the non-linear phase space. Ten different seeds were used to fix the test samples of the random errors. Off-momentum particles with a value of the relative momentum deviation fixed to $\delta = 10^{-3}$ were always considered. Both the chromaticity and the non-linear detuning were corrected using the criteria described in section VI.

We first tried to disentangle the effects of the dipole imperfections from those of the quadrupole imperfections in a machine with a perfect closed orbit and no linear coupling. The results are shown in Fig. 4. In presence of the quadrupole errors only, the aperture limit is peaked between 9.5 and 10.5 mm, whilst, with only the dipoles errors, the aperture is considerably more dependent on the seed chosen. If the dipole inner coil diameter is increased from 50 to 56 mm, the aperture is increased by more than 2 mm for all the considered seeds. By choosing three representative seeds in each distribution, i.e. one with the smallest, one with the largest, and one with an average value of the aperture, with the precaution that the same seeds are selected for the dipoles with different diameter, one can easily check the combined effect of the dipole and the quadrupole errors. In the results of Fig. 5, the predominance of the quadrupole errors is evident. The short-term dynamic aperture is almost independent of the diameter of the dipoles; its spread is made large by the presence of the dipole errors, and its maximum value is limited by the effect of the quadrupole errors. This behavior is only slightly changed when a finite closed orbit is added, as in the results of Fig. 6. To find how the situation can be improved, the seed No. 2 of a machine with only quadrupole errors has been studied in more detail, by assuming more linear magnetic fields in the quadrupoles. With the nominal quadrupole errors, the aperture is 9.6 mm. When all the arc quadrupoles are assumed perfect the aperture becomes 9.8 mm; when all the insertion quadrupoles are supposed free of errors the aperture is 18 mm; with perfect quadrupoles everywhere, the aperture is of course that of the bare machine, chromaticity corrected, i.e. 20.4 mm, in presence of the physical limitation at 25 mm. The aperture is thus determined by the imperfections of the insertion quadrupoles. In fact the leading errors are those of the quadrupoles in the straight parts of the machine insertions for dump and for cleaning, in which the value of the β -functions are in excess of 600 m. When these errors are set to zero the aperture becomes 14.2 mm; if only the multipoles up to the order 6 are neglected the aperture is 10.2 mm and increases to 11 mm if also b_{10} is cancelled. These results suggest a simple way to improve the aperture of the LHC by just increasing the inner coil radius R_c of the most critical quadrupoles in the machine insertions, and by assuming that the multipolar error components of order n are reduced proportionally to R_c^{-n+1} . The case where only the more critical quadrupoles of the machine insertions have a larger bore, has been compared to the case in which all the insertion quadrupoles have a larger bore. A third case has also been considered in which, for cost considerations, only some of the critical quadrupoles are improved. The results are shown in Fig. 7. Satisfactory results are achieved when all the critical quadrupoles of the machine insertions have coils of 75 mm inner diameter, as shown in Figs. 8 to 10. In this case, the aperture is no longer dominated by the quality of the insertion quadrupoles, at least if the dipoles have the nominal size of 50 mm. When 56 mm dipoles are used, the aperture increases by 0.4 to 1.4 mm, depending on the seed chosen, but this beneficial effect is still limited by the errors of the insertion quadrupoles. To avoid such a limitation, the field-shape imperfections have to be reduced in all the insertion quadrupoles. For instance, if it would be possible to use enlarged quadrupoles with 85

mm inner coil diameter in all the insertions the dynamic aperture would again be dominated by the dipole errors, as shown in Figs. 11 and 12. Unfortunately this situation is likely to be incompatible with the geometrical and the optical constraints of the LHC.

It should be mentioned that enlarging the diameter of the ten quadrupoles in the machine insertions to 75 mm may entail a reduction of their gradient below the nominal value of 250 T·m. In this case these quadrupoles have to be lengthened, and we assume that by changing the insertion layout it is still possible to keep the values of the β -functions below 630 m.

VIII. DYNAMIC APERTURE

There are many parameters that limit the long-term dynamic aperture of the LHC, therefore our first task was to identify the most important ones, in order to reduce to a reasonable amount the enormous need of computing time required for an exhaustive set of simulations. In our study, we routinely included synchrotron motion with $\delta = 10^{-3}$, and considered transversely round beams. In assigning the random errors, we routinely considered three combinations of seeds over a set of several, characterized by the worst, the medium and the best short-term dynamic aperture respectively. Two cases in which the inner diameter of the dipole coils was 50 or 56 mm respectively were systematically evaluated and compared. Quadrupoles with 75 mm inner coil diameter were used in the machine insertions at the locations with β -values in excess of 600 m. A particular care was taken to assess the effects on the stability of the particle motion of the linear coupling, of a residual but small value of the chromaticity, and of a ripple in the main quadrupole power supplies. The dynamic aperture in presence of a primary collimator was also evaluated, by finding the border of the starting amplitudes for which a circulating particle was performing at least $5 \cdot 10^5$ turns without being intercepted by the collimator. A summary of our results is shown in Tables 4 and 5.

In Table 4, realistic values of the closed orbit deviations and of a_2 were assumed and a global decoupling was performed with four families of skew quadrupoles as in Ref. 9, whilst the mechanical aperture was as large as 25 mm. A fourth seed was also studied in which the effect of the closed orbit on the short-term dynamic aperture was particularly pronounced. By enlarging the dipoles, the amplitude range of non chaotic particles was increased from $6.25 + 7.25$ mm to $7 + 8$ mm, depending on the seed, whilst the dynamic aperture over $5 \cdot 10^5$ turns was improved from $7 + 8.25$ mm to $8.75 + 9.5$ mm. To have indications on the behavior of the LHC in slightly more perturbed conditions, with parameters not precisely set to the optimum value, the case of the central seed was more extensively considered, both with 50 and/or 56 mm dipoles. A reduction in size of the region of stable motion was found when the systematic values of b_7 or b_9 were increased by a factor of three and of two respectively. The same result was found when two families of skew quadrupoles were used instead of four to compensate the linear coupling by means of the closest tune approach technique, when a small fraction ($Q' = 3$) of the horizontal and the vertical chromaticities was left uncorrected, or when a combination of the two effects was considered. On the contrary, the dynamic aperture was practically unaffected by the suppression of the central octupolar correctors in the regular cells, and was only slightly reduced in presence of a modest amount of tune modulation, made of a mixture of 50 and 200 Hz frequencies with amplitudes similar to those measured in the SPS quadrupoles, of $5 \cdot 10^{-4}$ and 10^{-4} respectively.

Particles above the stable region are expected to diffuse towards the vacuum pipe at a speed strongly increasing with the transverse amplitude. A set of collimators [10] is used to absorb them before they hit the magnets and provoke an unwanted deposition of energy in the superconducting coils. The transverse position of the primary collimator defines the mechanical aperture of the accelerator, and is basically fixed taking into account the mechanical tolerances of the cold bore and of the thermal screen (liner) of the main dipoles, the expected peak-values of the closed-orbit, of the dispersion, and of the β -function modulation, and, of course, the optimum value of the separation between the primary and the secondary collimators. Realistic estimates [2] suggest that the aperture will be limited at 6 mm with 50 mm dipoles and at 8 mm with 56 mm dipoles. Tracking simulations were made to study the effect of the primary collimator in a simplified model of the LHC, with $a_2 = 0$ and no closed orbit deviations, the results of which are shown in Table 5. Without the primary collimator, for the three typical seeds, the dynamic aperture was always larger with larger dipoles. With the primary collimator located at distances up to about 8 mm from the axis of the vacuum chamber, the dynamic aperture was practically independent of the inner coil diameter of the dipoles, at least for $5 \cdot 10^5$ turns, for the representative case of the central seed to which we restricted this kind of simulations, whilst, above 8 mm, an increasingly stronger dependence on the design of the dipole was found (the effect of the orbit

was checked and found to be small). On the other hand, with 50 mm dipoles and a collimator position of 6 mm as well as with 56 mm dipoles and a collimator position of 8 mm (i.e. with the collimator in the recommended nominal positions) the dynamic aperture was unchanged when the systematic values of b_7 or b_9 were increased by a factor of three and of two respectively. The correlation between the dynamic and the mechanical apertures is made even more evident when the results of Table 5 are plotted, as in Fig. 13, together with the aperture of a linear machine, strictly determined by the position of the collimator, and of a bare machine chromaticity corrected. Up to a broad limit of about 8 mm, the four curves are almost parallel and close to each other, whilst above this limit a divergent pattern starts to appear. Our qualitative interpretation of this is the following: with a small mechanical aperture, trajectories with small amplitude oscillations only are allowed, which are weakly perturbed even in presence of large field-shape imperfections, whilst, with larger mechanical apertures, and larger amplitude oscillations, the non-linear perturbation becomes larger and the size of the magnetic errors start to play a leading role for the dynamic aperture. On the other hand, we believe that particles with amplitudes up to the chaotic boundary are stable, although the non-linear perturbations induce a finite smear of their trajectories, whilst particles with larger amplitudes may become unstable after a sufficiently large number of turns. Ideally, the chaotic limit should be equal to the dynamic aperture corresponding to the collimator position, in which case only the unstable particles will be intercepted by the collimator. In this respect the mechanical size and the field quality are well matched in the case of the 56 mm dipoles.

In all our results, the aperture has been found to be larger than 4.5σ of the beam with the nominal emittance, i.e. 5.2 mm at $\beta = 165$ m, which is considered the absolute minimum needed for a safe operation of the LHC. Nevertheless, with 50 mm dipoles, we found situations in which the computed dynamic aperture is just at the edge of the safe area, so that no margin is left for additional destabilizing phenomena, not yet sufficiently well known to be included in our model, as for instance the imperfect setting of the machine parameters during the initial periods of setting-up, or the drastic change of the regime of the persistent currents at the rise of the magnetic field. Even if the chaotic boundary could be enlarged by reducing the magnetic errors, the stable area would still be limited to below 6 mm by the primary collimator. Our personal feeling is that a value of 6σ , i.e. of 7.2 mm, for the needed aperture would be a wise choice. In this hypothesis, the data presented here show that 56 mm would be the most appropriate size of the inner coil diameter of the LHC dipoles, in order to ensure both the mechanical and the dynamical apertures required.

IX. CONCLUSION

The dynamic behavior of the LHC in presence of field-shape imperfections scaled from the measurement of the HERA magnets meets the minimum requirements, even when the dipoles have the nominal inner coil diameter of 50 mm, provided that the quadrupoles located in the machine insertions with β -function values in excess of 600 m are enlarged to 75 mm inner coil diameter or improved in an equivalent way. This remains true even in presence of large orbit distortions, of an imperfect correction of the linear coupling and of the chromaticity, and of a small ripple in the power supplies of the main quadrupoles. With such a configuration the ensured aperture is only slightly in excess of 4.5σ , whilst it seems to be more prudent to have some additional margin, to take into account also our imperfect modelling of the accelerator. Our recommendation is to choose dipoles with 56 mm inner coil diameter which seem adequate to ensure an 8 mm mechanical aperture of the primary collimator and a 6σ dynamic aperture in the routine operation of the LHC.

X. ACKNOWLEDGEMENTS

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Fig. 1 LHC regular half-cell with multipolar correctors

Fig. 2 Short-term dynamic aperture and chaotic border vs the working point:
case of 50 mm dipoles

Fig. 3 Short-term dynamic aperture and chaotic border vs the working point
with and without linear coupling:
solid line: $a_2 = 0$
dashed line: $a_2 \neq 0$, with correcting skew quadrupoles

Fig. 4 Short-term dynamic aperture with different configurations of the magnetic errors

Fig. 5 Short-term dynamic aperture with imperfections in all magnets for different dipole inner coil diameters

Fig. 6 Short-term dynamic aperture with imperfections in all magnets and closed orbit distortions for different dipole inner coil diameters

Fig. 7 Short-term dynamic aperture vs the inner coil diameter of the machine insertion quadrupoles

Fig. 8 Short-term dynamic aperture with enlarged quadrupoles in the machine insertions

Fig. 9 Short-term dynamic aperture with imperfections in all magnets and with enlarged quadrupoles in the machine insertions

Fig. 10 Short-term dynamic aperture with imperfections in all magnets and closed orbit distortions with enlarged quadrupoles in the machine insertions

Fig. 11 Short-term dynamic aperture with enlarged quadrupoles in all the insertions

Fig. 12 Short-term dynamic aperture with imperfections in all magnets and with enlarged quadrupoles in all the insertions

Figure 13. LHC long term (10^5 turns) aperture vs. collimator position for four cases:

- no errors,
 - linear machine (quadrupoles only)
 - quadrupoles and chromaticity sextupoles
- field errors (medium seed) in quadrupoles (inner coil dimension 56 mm, enlarged to 75 mm in dump and cleaning insertion) and dipoles, with corresponding multipole corrections
 - for 50 mm dipole inner coil dimension
 - for 56 mm dipole inner coil dimension

The horizontal dashed lines show the *Lyapunov chaotic boundaries* (average of 4 seeds) in the presence of orbit distortion and field errors corresponding to the two coil dimensions. The arrows indicate the *optimum collimator positions* and the resulting *dynamic aperture* for these two cases.