PRECISE MEASUREMENT OF THE TOPONIUM MASS FROM THE OBSERVATION OF ITS TWO-PHOTON DECAY AT THE LHC

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Abstract

We propose to measure the toponium mass using an electromagnetic calorimeter of the "Crystal Ball" type at the LHC to detect $t\bar{t}$ bound states (toponium). If such a heavy system exists, the $J^{PC} = 0^{-+}$ state will dominate production at the LHC and could be observed above a relatively low background in the two-photon decay channel. The toponium mass could be measured for instance with a precision of 3×10^{-4} for $m_{t\bar{t}} = 200 \text{ GeV}/c^2$. This method provides an accuracy comparable with what will ultimately be obtained with future dedicated e^+e^- top-quark factories.

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1. INTRODUCTION

If the top quark exists as expected, the precise determination of its mass is of great importance as a step in the understanding of fermion masses, as a test of the Standard Model relating the top-quark mass to the Higgs mass through radiative corrections and, more importantly, as additional information on the mysterious mechanism of CP violation. We first discuss here some of the unusual features of $t\bar{t}$ bound states that are due to the sixth quark being extraordinarily heavy; we then describe a method to measure its mass accurately. Within the framework of the Standard Model, the present data (mainly from LEP [1]) constrain the top-quark mass, m_t , to be between 92 and 147 GeV/c² [2] at 68% confidence level. We show that under most of the allowed mass range, hadron colliders, in particular the LHC [3], will offer the possibility to observe toponium bound states and allow a precise mass measurement. Subsequently, m_t can be extracted with an accuracy depending on the knowledge of the QCD potential and on the precision in the value of the strong coupling constant α_s .

2. DECAY OF THE TOP QUARK AND OF THE TOPONIUM

As m_t is now known to be above the real W threshold for the decay $t \rightarrow W^+b$ [4], the top-quark width is a steeply increasing function of its mass [5], [6] ($\Gamma_t \sim m_t^3$ for $m_t \gg m_W$); it ranges from 90 MeV/c² for $m_t = 100$ GeV/c² to 2.4 GeV/c² for $m_t = 200$ GeV/c². The corresponding lifetimes, of 7×10^{-24} s and 3×10^{-25} s respectively, are shorter than the typical strong interaction time scale, the time needed for a top quark to hadronize into $t\bar{q}$ or tqq hadrons. The top quark therefore behaves as a quasi-free particle [7]. In the Standard Model range for the top-quark mass, the toponium decay is dominated by the decay of individual top quarks and the toponium width is given by $\Gamma_{t\bar{t}} \sim 2\Gamma_t$ (Fig. 1) to a very good approximation. The toponium is therefore expected to be a very peculiar system where constituents tend to decay before the bound state does.

3. TOPONIUM PRODUCTION AT ELECTRON-POSITRON COLLIDERS

Electron-positron colliders have proved to be ideally suited for the precise study of the spectroscopy of quark bound states, in particular for $c\bar{c}$ and $b\bar{b}$. If m_t is near the lower end of its allowed range, $m_t \sim 100$ GeV/c², the study of its various bound states can provide a unique test of QCD [6]. However, the most recent fit to electroweak data suggests $m_t \sim 120$ GeV/c² [2] and, as can be seen in Fig. 1, it is about the value above which the toponium spectroscopy disappears, since the toponium width reaches the value of 1S to 2S energy-level splitting. If a top-quark factory with a luminosity in excess

of $10^{33} \text{ cm}^{-2}\text{s}^{-1}$ can be built, as proposed for instance in Ref. [8], one may be able to measure m_t with a precision of 200 to 500 MeV/c² [9]. While waiting for such an instrument, we must consider other possibilities to determine m_t with precision. However, we should note that for m_t larger than $\sim 130 \text{ GeV}/c^2$, there will be no $t\bar{t}$ spectroscopy to be studied at future e^+e^- colliders.

4. TOPONIUM PRODUCTION AT HADRON COLLIDERS

The measurement of the $t\bar{t}$ continuum production cross-section ($\sigma_{t\bar{t}}$) at the LHC can provide a measurement of m_t with an error of about 15 GeV/c² [10]. If one uses the $t\bar{t} \rightarrow WWb\bar{b} \rightarrow \ell\nu \text{ jet-jet } b\bar{b}$ channel and reconstructs either the complete $t \rightarrow \text{jet-jet } b$ decay or the $t \rightarrow \ell\nu b$ decay, it is estimated that a precision of about 8 GeV/c² can be reached [10]. In the two-lepton channel, $t \rightarrow Wb \rightarrow \ell\nu b \rightarrow \ell\nu\ell'v'c$, the $\ell\ell'$ invariant-mass distribution is sensitive to the parent top-quark mass. The precision in this case could reach about 5 GeV/c² [10]. The study of open top-quark production does not provide a precision better than about 4%, and it is worthwhile exploring other methods.

Charm [11] and beauty [12] were discovered in hadron collisions from the production and decay of (1^{--}) bound states. A rough estimate of the cross-section for producing a $t\bar{t}$ (1^{--}) state at the LHC can be obtained from the expected scaling behaviour of the Υ cross-section, noting that $\sqrt{\tau} (= m/\sqrt{s})$ for a 200 GeV/c² $t\bar{t}$ state at the LHC is about the same as $\sqrt{\tau}$ for Υ at the CERN proton-antiproton collider, where $\sigma[\text{Br}(p\bar{p} \rightarrow \Upsilon, \Upsilon', \Upsilon'' \rightarrow \mu^+\mu^-)]$ was measured by UA1 to be 0.98 nb [13]. The corresponding $t\bar{t}$ cross-section, for $m_t = 100 \text{ GeV}/c^2$ at the LHC is about 5 pb. For the decay into e^+e^- , estimates of the branching ratio range from 7×10^{-5} to 6×10^{-6} [6] for toponium masses of 200 GeV/c² to 400 GeV/c² respectively. For $m_t = 100 \text{ GeV}/c^2$ one would only collect about 30 events in one year of running under a background of several thousand Drell-Yan events in a bin of 2.3 GeV/c². Unless the (1^{--}) cross-section turns out to be at least one order of magnitude larger than our naive estimate, it seems very unlikely that toponium (1^{--}) can be observed in the lepton-pair channel at the LHC.

However, it has been pointed out that at the LHC energies, and for heavy top quarks, the production of the (0^{-+}) toponium state which proceeds through gluon-gluon fusion is of the order of 190 pb [7] for a toponium mass of 200 GeV/c². As seen in Fig. 1 the toponium states, if they exist, are characterized by a narrow width, so that the production cross-section at hadron colliders, for a state of mass M , can be written as :

$$\sigma[pp \rightarrow \eta(0^{-+}) + X] = \pi^2 \Gamma(\eta \rightarrow gg) \zeta(3) (\tau dL/d\tau)_{gg} / 8M^3$$

where $\Gamma(\eta \rightarrow gg)$ is the decay width of toponium into two gluons, $\tau(dL/d\tau)_{gg}$ is the gluon-gluon luminosity function, and $\zeta(3)$ is the Riemann zeta function obtained by summing over all bound states. The toponium partial width for the two gluon decay is :

$$\Gamma(\eta \rightarrow gg) = 32 \pi \alpha_s^2(m_t^2) |\Psi(0)|^2 / 3M^2,$$

where $\Psi(0)$ is the total wave function at the origin [6] :

$$|\Psi(0)|^2 = 8(m_t \alpha_s(Q_s^2))^3 / 27\pi.$$

In the previous expression for $\Psi(0)$, the strong coupling constant α_s is evaluated at a momentum transfer $Q_s^2 = m_t(E^2 + \Gamma_t^2)^{1/2}$, which is the scale appropriate to the non-relativistic treatment of the Coulombic bound state (E is the energy of the lowest lying bound state given in this case by $E = -4 m_t \alpha_s^2/9$). The total cross-section obtained as a function of the proton-proton centre-of-mass energy is shown in Fig. 2 for top-quark masses of interest and using the same set of structure functions (EHLQ1 [14]) as in Fadin et al. [7]. By comparison, the Higgs production cross-section at a mass of 200 GeV/c² is about a factor of 6 smaller (Fig. 3). Such a toponium state can decay into a photon pair at a substantial rate. In the Born approximation [6] its $\gamma\gamma$ width is given by :

$$\Gamma(0^- \rightarrow \gamma\gamma) = 12 e_t^4 \alpha^2 |\Psi(0)|^2 / M^2$$

where e_t is the top-quark electric charge, α is the fine structure constant, $\Psi(0)$ is the total wave function at the origin discussed above, and M is the toponium mass. Figure 4 shows the branching ratio for the decay of 0^- toponium into two photons and a comparison with the Standard Model Higgs. For what follows, and to be conservative, we choose to use the HMRSB [15] set of structure functions, which gives a cross-section about 25% smaller than does EHLQ1. For a toponium of 200 GeV/c² mass one expects that 2300 events are produced in the two-photon channel in one year of running at the LHC. There are two types of backgrounds : (i) A reducible background which comes from two jets fluctuating into π^0 's and which may be indistinguishable from photons. This is discussed in Ref. [16] for the case of background to Higgs signal in the two-photon channel. It is assumed that as for the case of the Higgs search it can be reduced to an acceptable level, provided the rejection per jet is better than 10^{-4} . (ii) An irreducible source of background from genuine two-photon production. The three main processes contributing, gg , $q\bar{q}$ and bremsstrahlung, have been calculated [17] and included in the total background curve shown in Fig. 5. The cross-sections shown are evaluated with the following cuts : a) $|y| \leq 2$, where the cut is applied to the rapidity of each photon; b) each photon is required to have a transverse momentum larger than 70 GeV/c. In addition, for the background coming from the bremsstrahlung process $qg \rightarrow q\gamma\gamma$ [18] the following cuts have been used : (i) a balance cut on the photon transverse momenta : $P_{t1}/(P_{t1} + P_{t2}) < 0.7$ where $P_{t1} > P_{t2}$; (ii) an isolation cut which requires that no track with $P_t > 3$ GeV/c and no photon with $P_t > 3$ GeV/c is within a distance of 0.3 in (η, ϕ) space. A very clear signal can be observed over a broad mass range (Fig. 5). Note that the toponium peaks shown have been smeared with the expected experimental resolution discussed below in Section 5.

Another physics background comes, in principle, from the Higgs decaying into two photons. For Higgs masses of interest here, above 180 GeV/c², where the two- Z^0 channel is open, the branching ratio in the two-photon mode is very small, 4.5×10^{-5} and 7×10^{-6} for m_H of 200 and 300 GeV/c² respectively [19]. If the Standard Model prediction for Higgs production and decay is correct, the Higgs boson will not constitute a

background to the detection of toponium at the LHC. If a peak is observed in the two-photon channel, the measurement of the cross-section and of the mass should provide a definite test of the consistency of the signal with the toponium 0^{-+} hypothesis. The scalar character of that state can be tested in studying for instance the photons angular distribution. In addition the toponium mass observed should be consistent with the top-quark mass obtained from the study of open $t\bar{t}$ production.

We also considered the possibility of using the channel $0^{-+} \rightarrow Z^0\gamma$: unfortunately the rate [6] seems to be too low to hope to extract a signal from events with a photon and a charged lepton pair at the Z^0 mass.

One should note that the type of detector we propose to use would be equally well suited to a search for the production of a Higgs boson either in the electron ($H \rightarrow ZZ^* \rightarrow 4\ell^\pm$) or in the photon channel ($H \rightarrow \gamma\gamma$ or $WH \rightarrow e\nu\gamma\gamma$) depending on the Higgs mass.

5. MEASUREMENT OF THE TOPONIUM MASS

In order to detect photons with an energy between about 100 GeV and 150 GeV, we propose a "Crystal Ball" type of calorimeter, using for instance crystals of the type CeF_3 or BaF_2 . This should allow an excellent energy resolution given by $\Delta E/E = 2\%/\sqrt{E} \oplus 0.5\%$ [20]. In the energy range considered here, the resolution is dominated by the constant term. We assume that the shower position or direction measurements are precise enough ($\Delta\theta \leq 3 \times 10^{-3}$ rad) not to add significantly to the mass error coming from the energy resolution. If a vertex detector exists, the vertex associated to toponium may be reconstructed precisely from charged tracks of recoiling jets, provided the toponium transverse momentum is sufficiently large. In that case the photon direction determination would not rely entirely on the calorimetric measurement. Table 1 shows that under these conditions the observed half-width is dominated by the experimental resolution. For a toponium mass of $200 \text{ GeV}/c^2$ the observed half-width is $765 \text{ MeV}/c^2$. Counting data in a mass bin of $2.3 \text{ GeV}/c^2$, the ratio of signal to background is 39%, which leads to an 18 standard-deviation signal. One can estimate that the corresponding error on the position of the $\gamma\gamma$ peak, from statistics alone, is of the order of $30 \text{ MeV}/c^2$. Assuming that the energy scale is calibrated precisely with the Z^0 peak, resulting in an error of $50 \text{ MeV}/c^2$ at the toponium mass and estimating other systematic errors to about $50 \text{ MeV}/c^2$, the toponium mass can be determined with a global error of about $80 \text{ MeV}/c^2$; this corresponds to a precision of 4×10^{-4} . If one uses $m(0^{-+}) = 2m_t - 4/9 m_t \alpha_s^2$, the corresponding error on m_t , which is dominated by the uncertainty on the value of α_s , is $200 \text{ MeV}/c^2$ for an error of 5% on α_s . However, given the difficulty of defining the mass, even for a quasi-free quark, we leave it to others to eventually extract a top-quark mass from the toponium mass. Figure 5 shows the cross-section expected for two-photon events, for the $\gamma\gamma$ background (the jet-jet background is assumed to be negligible),

and for 0^{-+} states with masses of 200, 220, 240, and 260 GeV/c², which covers most of the expected range for a Standard Model top quark. The comparison with a Higgs of 100 GeV/c² mass is also shown. Figure 6 shows the significance of the $\gamma\gamma$ signal, expressed in standard deviations, assuming mass windows centered on the peak with a width of ± 1.5 times the effective half-width of the toponium according to Table 1. If we require a 4 standard-deviation signal, the toponium state is observable in the two-photon channel for masses in the range : $180 \text{ GeV/c}^2 \leq m_{t\bar{t}} \leq 240 \text{ GeV/c}^2$, which covers most of the remaining part of the mass range where a $t\bar{t}$ bound state still exists. It is possible that, with more sophisticated cuts to reject the two-photon background, the observable mass range could be extended slightly. However, since for masses above $\sim 130 \text{ GeV/c}^2$ the bound state disappears, we cover here all of the relevant mass range. We should also point out that, because of the unambiguous signature, the search for the two-photon decay of the toponium can be done at the LHC at the highest possible luminosity, while at the same time using a relatively moderate cost dedicated detector.

6. CONCLUSION

Because of the very large top-quark mass, the $t\bar{t}$ bound state, if it exists, will have unique properties of great interest. Using an electromagnetic calorimeter with an energy resolution $\Delta E/E = 2\%/\sqrt{E} \oplus 0.5\%$ one can study the toponium (0^{-+}) at the highest luminosity at the LHC, where its relatively large production cross-section makes it possible to observe its decay into two photons over a relatively low background (provided it has sufficient electromagnetic shower angular resolution $\Delta\theta \leq 3 \times 10^{-3} \text{ rad}$). In one year of running at the LHC with a "Crystal Ball" type calorimeter, the mass of the toponium (0^{-+}) can be determined with a precision of 4×10^{-4} for a toponium mass of 200 GeV/c². The corresponding precision on m_t is more than one order of magnitude better than from other methods proposed at hadron colliders and is comparable with the precision that future dedicated electron-positron colliders will offer.

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Table 1. The effective toponium half-width, given in terms of the equivalent Gaussian σ , is shown with its contributions from experimental resolution [σ (exp)] and from the natural width [σ (nat)] of the bound state, for four toponium masses.

$m_{t\bar{t}}$	σ (exp)	σ (nat)	Effective σ
(GeV/c ²)	(MeV/c ²)	(MeV/c ²)	(MeV/c ²)
200	760	80	765
260	975	415	1060
300	1120	755	1350
400	1470	2080	2550

FIGURE CAPTIONS

Fig. 1 Toponium width compared with the energy splitting between the 1S and 2S states as a function of the top-quark mass. The 1S to 2S splitting is calculated for Richardson [21], Coulomb and QCD [22] potentials. The Coulomb potential used is $V_c = -4\alpha_s/3r$. The two lines for the Richardson potential correspond to the two parametrizations in the two references given.

Fig. 2 Cross-section for the production of a 0^{-+} toponium bound state in proton-proton collisions as a function of the centre-of-mass energy and for top-quark masses of 100, 130, 150, and 200 GeV/c². The set of structure functions used here is EHLQ1.

Fig. 3 Production cross-section for the Standard Model Higgs boson and for toponium 0^{-+} bound state at LHC as a function of mass. The calculation is done for two different sets of structure functions : HMRSB (full lines) and EHLQ1 (dashed lines).

Fig. 4 Branching ratios for Higgs and 0^{-+} toponium decay into two photons as a function of mass.

Fig. 5 Differential cross-section versus two-photon invariant mass for background ($q\bar{q}$, gluon-gluon and bremsstrahlung added together) and (1) 200, (2) 220, (3) 240 and (4) 260 GeV/c² toponium 0^{-+} states decaying into two photons. The corresponding Higgs signal in the two-photon channel is also shown for the selection described in Ref. [16] and for a Higgs mass of 100 GeV/c².

Fig. 6 Significance of the signal expressed in number of standard deviations (full line) for toponium decay into two photons at the LHC for 10^5 pb⁻¹, as a function of the toponium mass. The horizontal dashed lines indicate the 3 and 4 standard-deviation levels. The grey area corresponds to the present 68% confidence-level mass range from Ref. [2].

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6