CM-P00057031

P00000505

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PARAMETERS IN THE ELECTROWEAK THEORY
III. MASSES OF THE TOP QUARK AND THE HIGGS BOSON

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ABSTRACT

From the eeH coupling in the electroweak theory, it is found that the one-loop correction is finite (i. e., not logarithmically divergent) when a certain quadratic mass relation is satisfied. When this relation is used together with the one given previously, and the lepton masses are neglected compared with those of the W and the Z , it is found that

$$m_H^2 = 9m_Z^2 - 6m_W^2.$$

* Work supported in part by the Norwegian Research Council for Science and the Humanities.

§ Work supported in part by the U.S. Department of Energy under Grant DE-FG02-84ER40158.

1. Introduction

It is well established experimentally that there are three generations, each of which consists of a charge $+2/3$ quark, a charge $-1/3$ quark, a lepton, a massless neutrino and their anti-particles [1]. In this list of the particles for the three generations, the only one as yet not observed is the top quark. Therefore, in the Standard Model [2] with these three generations, the only parameters that are not known directly are the masses m_t and m_H of the top quark and the Higgs [3].

In the preceding two papers, referred to as Paper I and Paper II, it is first proposed to use point splitting as a method of regularization [4]. When this method is used to study the quadratic divergences in the Standard Model, it is found that all quadratic divergences disappear [5] when the following mass relation is satisfied [6]:

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2. \quad (1.1)$$

Actually, this mass relation is valid only to the zeroth order in the coupling constants, i.e., there are radiative corrections to this mass relation. These radiative corrections are difficult to compute, and will not be considered here. Without taking such radiative corrections into account, (1.1) gives one linear relation between m_t^2 and m_H^2 . In order to be able to determine these two unknowns m_t and m_H , it is necessary to have a second relation between these two quantities. It is the purpose of this paper to find this second relation.

When (1.1) is satisfied, it has been emphasized in Paper II that, with point-splitting regularization, to one loop not only there is no quadratic divergence, but also the logarithmic divergence in the electron self mass can be removed. This makes it possible to study the logarithmic divergences in the vertices. In the Standard Model, there are two distinct types of vertices, namely those due to the gauge couplings and those due to the Yukawa couplings. In this paper, the desired relation is to be determined from the Yukawa coupling. For the present purpose, it is sufficient to consider the simplest case of the one-loop eeH vertex. This has the nice property of not involving any particle with strong couplings.

It has been seen in Paper II that the quadratic divergences in the Standard Model are sensitive to the regularization used. In particular, point-splitting regularization and dimensional regularization [7] give different answers. In this respect, the situation is much simpler for the eeH vertex to be studied in this paper. Without regularization, the divergences encountered here are only logarithmic. Such divergences are insensitive to the regularization used: they are of the order of $\ln |\delta|$ in point-splitting regularization, $1/(4-d)$ in dimensional regularization, and $\ln M$ in Pauli-Villars regularization [8] for example. Thus

the mass relation to be derived here can be obtained with any regularization, or even no regularization at all by simply picking out the logarithmically divergent terms.

There are many Feynman diagrams for the one-loop correction to the eeH vertex in the Standard Model. Let q be the momentum of the incoming electron line, and q' that of the outgoing electron line, then

$$p = q' - q \quad (1.2)$$

is the momentum of the incoming Higgs line.

In Sec. 2, the proper eeH vertex diagrams are treated, and their “logarithmically divergent” parts extracted. As expected, the sum of these logarithmically divergent parts is not gauge invariant. In Sec. 3 and Sec. 4, the corresponding contributions from one-loop radiative corrections to the electron line and the Higgs line are found. In Sec. 5 these three contributions are added together, and the basic relation

$$m_H^2 = 9m_Z^2 - 6m_W^2 \quad (1.3)$$

is derived by setting the sum of these logarithmically divergent parts to zero and using (1.1). Finally, some of the consequences of the Higgs mass are listed in Sec. 6 together with an overview of the present procedure.

2. Proper Vertex Diagrams

Consider one-loop eeH proper vertex diagrams. Since the electron does not appear in any four-vertex, such diagrams are all of the form shown generically in fig. 1. [As discussed in Paper I, there are peculiar vertices involving e^+e^- and two gauge particles. However, since such vertices involve a factor of δ , no one-loop eeH proper vertex diagram can lead to a $\ln|\delta|$ term if it contains a peculiar vertex.] Let such a diagram be labelled by the particles in the triangle, 1, 2 and 3 as shown in the figure.

In the Standard Model, there are several three-vertices involving the Higgs. Therefore the lines 1 and 2 can be:

$$ee, uu, dd, ZZ, Z\phi_0, \phi_0 Z, \phi_0\phi_0, WW, W\phi, \phi W, \phi\phi, HH, \eta^+\eta^+, \eta^-\eta^-, \eta^Z\eta^Z. \quad (2.1)$$

Here ϕ denotes the charged $\phi^\pm$, and only one generation (u, d, ν, e) is shown explicitly. Since the quarks (u and d) and the Faddeev-Popov ghosts [9] (η^+ , η^- and η^Z) do not couple directly to the electron, these five cases can be eliminated. If $12 = ee$, then there are four possible choices for 3, namely γ , Z , ϕ_0 and H . Therefore there is a total of thirteen proper vertex diagrams as shown in fig. 2.

The contributions of these 13 diagrams are respectively

$$J_{ee\gamma} = \frac{1}{(2\pi)^4} \int d^4k (-ie\gamma_\nu) \frac{i}{\not{q}' - \not{k} - m_e + i\epsilon} \left(-\frac{igm_e}{2m_W} \right) \frac{i}{\not{q} - \not{k} - m_e + i\epsilon} (-ie\gamma_\mu) \\ \times \frac{i}{k^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 + i\epsilon} \right], \quad (2.2)$$

$$J_{eeZ} = \frac{1}{(2\pi)^4} \int d^4k \frac{i}{k^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m_Z^2 + i\epsilon} \right] \left(\frac{-ig}{4\cos\theta_W} \right)^2 \left(-\frac{igm_e}{2m_W} \right) \\ \times \gamma_\nu [1 - 4\sin^2\theta_W - \gamma_5] \frac{i}{\not{q}' - \not{k} - m_e + i\epsilon} \frac{i}{\not{q} - \not{k} - m_e + i\epsilon} \gamma_\mu [1 - 4\sin^2\theta_W - \gamma_5], \quad (2.3)$$

$$J_{ee\phi_0} = \frac{1}{(2\pi)^4} \int d^4k (-i) \left(\frac{gm_e}{2m_W} \right)^3 \frac{i}{k^2 - \xi m_Z^2 + i\epsilon} \gamma_5 \frac{i}{\not{q}' - \not{k} - m_e + i\epsilon} \frac{i}{\not{q} - \not{k} - m_e + i\epsilon} \gamma_5, \quad (2.4)$$

$$J_{eeH} = \frac{1}{(2\pi)^4} \int d^4k \left(-\frac{igm_e}{2m_W} \right)^3 \frac{i}{k^2 - m_H^2 + i\epsilon} \frac{i}{\not{q}' - \not{k} - m_e + i\epsilon} \frac{i}{\not{q} - \not{k} - m_e + i\epsilon}, \quad (2.5)$$

$$J_{ZZe} = \frac{1}{(2\pi)^4} \int d^4k \left(ig \frac{m_Z^2}{m_W} g_{\alpha\beta} \right) \left(\frac{-ig}{4\cos\theta_W} \right)^2 \gamma_\nu [1 - 4\sin^2\theta_W - \gamma_5] \frac{i}{\not{k} - m_e + i\epsilon} \\ \times \gamma_\mu [1 - 4\sin^2\theta_W - \gamma_5] \frac{i}{(q-k)^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\alpha} + \frac{(1-\xi)(q-k)^\mu (q-k)^\alpha}{(q-k)^2 - \xi m_Z^2 + i\epsilon} \right] \\ \times \frac{i}{(q'-k)^2 - m_Z^2 + i\epsilon} \left[-g^{\nu\beta} + \frac{(1-\xi)(q'-k)^\nu (q'-k)^\beta}{(q'-k)^2 - \xi m_Z^2 + i\epsilon} \right], \quad (2.6)$$

$$J_{Z\phi_0e} = \frac{1}{(2\pi)^4} \int d^4k \left(\frac{gm_e}{2m_W} \right) \gamma_5 \frac{i}{\not{k} - m_e + i\epsilon} \left(\frac{-ig}{4\cos\theta_W} \right) \gamma_\mu [1 - 4\sin^2\theta_W - \gamma_5] \\ \times \frac{i}{(q'-k)^2 - \xi m_Z^2 + i\epsilon} \frac{i}{(q-k)^2 - m_Z^2 + i\epsilon} \\ \times \left[-g^{\mu\alpha} + \frac{(1-\xi)(q-k)^\mu (q-k)^\alpha}{(q-k)^2 - \xi m_Z^2 + i\epsilon} \right] \left(\frac{-g}{2\cos\theta_W} \right) (2q' - q - k)_\alpha, \quad (2.7)$$

$$J_{\phi_0Ze} = \frac{1}{(2\pi)^4} \int d^4k \left(\frac{-ig}{4\cos\theta_W} \right) \gamma_\nu [1 - 4\sin^2\theta_W - \gamma_5] \frac{i}{\not{k} - m_e + i\epsilon} \left(\frac{gm_e}{2m_W} \right) \gamma_5 \\ \times \frac{i}{(q-k)^2 - \xi m_Z^2 + i\epsilon} \frac{i}{(q'-k)^2 - m_Z^2 + i\epsilon} \\ \times \left[-g^{\nu\beta} + \frac{(1-\xi)(q'-k)^\nu (q'-k)^\beta}{(q'-k)^2 - \xi m_Z^2 + i\epsilon} \right] \left(\frac{-g}{2\cos\theta_W} \right) (q' - 2q + k)_\beta, \quad (2.8)$$

$$J_{\phi_0\phi_0e} = \frac{1}{(2\pi)^4} \int d^4k \left(\frac{gm_e}{2m_W} \right)^2 \gamma_5 \frac{i}{\not{k} - m_e + i\epsilon} \gamma_5 \frac{i}{(q-k)^2 - \xi m_Z^2 + i\epsilon} \frac{i}{(q'-k)^2 - \xi m_Z^2 + i\epsilon}$$

$$\times \left(-\frac{1}{2}ig \frac{m_H^2}{m_W} \right), \quad (2.9)$$

$$\begin{aligned} J_{WW\nu} &= \frac{1}{(2\pi)^4} \int d^4k \left(\frac{ig}{2\sqrt{2}} \right)^2 \gamma_\nu (1 - \gamma_5) \frac{i}{\not{k} + i\epsilon} \gamma_\mu (1 - \gamma_5) (igm_W g_{\alpha\beta}) \\ &\times \frac{i}{(q-k)^2 - m_W^2 + i\epsilon} \left[-g^{\mu\alpha} + \frac{(1-\xi)(q-k)^\mu(q-k)^\alpha}{(q-k)^2 - \xi m_W^2 + i\epsilon} \right] \\ &\times \frac{i}{(q'-k)^2 - m_W^2 + i\epsilon} \left[-g^{\nu\beta} + \frac{(1-\xi)(q'-k)^\nu(q'-k)^\beta}{(q'-k)^2 - \xi m_W^2 + i\epsilon} \right], \end{aligned} \quad (2.10)$$

$$\begin{aligned} J_{W\phi\nu} &= \frac{1}{(2\pi)^4} \int d^4k \left(\frac{ig}{2\sqrt{2}} \right)^2 \left(\frac{-m_e}{m_W} \right) (1 - \gamma_5) \frac{i}{\not{k} + i\epsilon} \gamma_\mu (1 - \gamma_5) \left(\frac{-ig}{2} (2q' - q - k)_\alpha \right) \\ &\times \frac{i}{(q-k)^2 - m_W^2 + i\epsilon} \left[-g^{\mu\alpha} + \frac{(1-\xi)(q-k)^\mu(q-k)^\alpha}{(q-k)^2 - \xi m_W^2 + i\epsilon} \right] \frac{i}{(q'-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \quad (2.11)$$

$$\begin{aligned} J_{\phi W\nu} &= \frac{1}{(2\pi)^4} \int d^4k \left(\frac{ig}{2\sqrt{2}} \right)^2 \left(\frac{-m_e}{m_W} \right) \gamma_\nu (1 - \gamma_5) \frac{i}{\not{k} + i\epsilon} (1 + \gamma_5) \left(\frac{ig}{2} (q' - 2q + k)_\beta \right) \\ &\times \frac{i}{(q'-k)^2 - m_W^2 + i\epsilon} \left[-g^{\nu\beta} + \frac{(1-\xi)(q'-k)^\nu(q'-k)^\beta}{(q'-k)^2 - \xi m_W^2 + i\epsilon} \right] \frac{i}{(q-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \quad (2.12)$$

$$\begin{aligned} J_{\phi\phi\nu} &= \frac{1}{(2\pi)^4} \int d^4k \left(\frac{-igm_e}{2\sqrt{2}m_W} \right)^2 (1 - \gamma_5) \frac{i}{\not{k} + i\epsilon} (1 + \gamma_5) \left(\frac{-igm_H^2}{2m_W} \right) \\ &\times \frac{i}{(q-k)^2 - \xi m_W^2 + i\epsilon} \frac{i}{(q'-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \quad (2.13)$$

$$\begin{aligned} J_{HHe} &= \frac{1}{(2\pi)^4} \int d^4k \left(\frac{-igm_e}{2m_W} \right)^2 \left(-\frac{3}{2}ig \frac{m_H^2}{m_W} \right) \frac{i}{\not{k} - m_e + i\epsilon} \\ &\times \frac{i}{(q-k)^2 - m_H^2 + i\epsilon} \frac{i}{(q'-k)^2 - m_H^2 + i\epsilon}. \end{aligned} \quad (2.14)$$

Strictly speaking, each of the integrands should have an extra factor of $e^{ik \cdot \delta}$. As discussed in Sec. 1, not writing this factor explicitly does not cause any confusion.

Of these thirteen integrals, the following five are convergent:

$$J_{ZZe}, \quad J_{\phi_0\phi_0e}, \quad J_{WW\nu}, \quad J_{\phi\phi\nu}, \quad \text{and} \quad J_{HHe}.$$

The other eight are logarithmically divergent; the task is to extract the logarithmically divergent parts of these eight integrals.

Since the integrals are only logarithmically divergent, the masses in the denominators of the propagators are irrelevant. For example, with the I_4 defined by eq. (C.2) of Paper I,

the explicit answer (C.6) shows that the $\ln |\delta|$ part of I_4 does not depend on the value of M . Indeed the presence of a positive value of M merely serves the purpose of preventing any infrared divergence, which is of no relevance here. Therefore, we can ignore all the masses in the denominators, provided that the convention of ignoring all infrared divergences is followed.

Let the symbol $\Rightarrow$ be used to denote the retention of logarithmically divergent terms only. In other words, the two sides of this $\Rightarrow$ symbol may differ by some finite quantity. With these conventions, the integral $J_{ee\gamma}$ is found to be given by

$$\begin{aligned} J_{ee\gamma} &\Rightarrow \frac{e^2}{(2\pi)^4} \frac{gm_e}{2m_W} \int d^4k \gamma_\nu \frac{1}{\not{k}} \frac{1}{\not{k}} \gamma_\mu \frac{1}{k^2} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2} \right] \\ &= \frac{e^2}{(2\pi)^4} \frac{gm_e}{2m_W} \int d^4k \frac{1}{k^4} [-4 + (1-\xi)] \\ &\Rightarrow -\frac{e^2 gm_e}{2m_W} i\Lambda(3+\xi), \end{aligned} \quad (2.15)$$

where Λ is formally

$$i\Lambda = \frac{1}{(2\pi)^4} \int \frac{d^4k}{k^4}, \quad (2.16)$$

with the infrared divergent part omitted. Somewhat less formally, it is

$$i\Lambda = \frac{1}{(2\pi)^4} \int d^4k \frac{1}{(k^2 - M^2 + i\epsilon)^2}, \quad (2.17)$$

with any fixed positive value of M . With point-splitting regularization, it is

$$i\Lambda \Rightarrow \frac{1}{(2\pi)^4} I_4, \quad (2.18)$$

where the I_4 is given explicitly by eq. (C.6) of Paper I as

$$\begin{aligned} \Lambda &\Rightarrow \begin{cases} -\frac{1}{8\pi^2} \left[\ln \frac{M\delta}{2} + \gamma \right] & \delta \text{ space-like,} \\ -\frac{1}{8\pi^2} \left[\ln \frac{M\delta}{2} + \gamma + \frac{i\pi}{2} \right] & \delta \text{ time-like} \end{cases} \\ &\Rightarrow -\frac{1}{8\pi^2} \ln |\delta|. \end{aligned} \quad (2.19)$$

Similarly

$$J_{eeZ} \Rightarrow \frac{1}{(2\pi)^4} \frac{g^3 m_e}{2m_W} \frac{1}{(4 \cos \theta_W)^2} \int d^4k \gamma_\nu [1 - 4 \sin^2 \theta_W - \gamma_5] \frac{1}{\not{k}} \frac{1}{\not{k}} \gamma_\mu [1 - 4 \sin^2 \theta_W - \gamma_5]$$

$$\begin{aligned}
& \times \frac{1}{k^2} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2} \right] \\
& = \frac{1}{(2\pi)^4} \frac{g^3 m_e}{2m_W} \frac{1}{(4 \cos \theta_W)^2} [(1 - 4 \sin^2 \theta_W)^2 - 1] \int d^4 k \frac{1}{k^4} [-4 + (1 - \xi)] \\
& \Rightarrow -\frac{g^3 m_e}{2m_W} \frac{(1 - 4 \sin^2 \theta_W)^2 - 1}{(4 \cos \theta_W)^2} i\Lambda(3 + \xi), \tag{2.20}
\end{aligned}$$

$$J_{ee\phi_0} \Rightarrow -i\Lambda \left(\frac{gm_e}{2m_W} \right)^3, \tag{2.21}$$

$$J_{eeH} \Rightarrow i\Lambda \left(\frac{gm_e}{2m_W} \right)^3, \tag{2.22}$$

$$\begin{aligned}
J_{Z\phi_0 e} & \Rightarrow \frac{1}{(2\pi)^4} \frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 \int d^4 k \gamma_5 \frac{1}{\not{k}} \gamma_\mu [1 - 4 \sin^2 \theta_W - \gamma_5] \\
& \times \frac{1}{k^4} \left[-g^{\mu\alpha} + \frac{(1-\xi)k^\mu k^\alpha}{k^2} \right] (-k_\alpha) \\
& = \frac{1}{(2\pi)^4} \frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 \int d^4 k \frac{\xi}{k^4} \gamma_5 \frac{1}{\not{k}} \not{k} [1 - 4 \sin^2 \theta_W - \gamma_5] \\
& \Rightarrow -\frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 i\Lambda \xi [1 - (1 - 4 \sin^2 \theta_W) \gamma_5], \tag{2.23}
\end{aligned}$$

$$\begin{aligned}
J_{\phi_0 Z e} & \Rightarrow \frac{1}{(2\pi)^4} \frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 \int d^4 k \gamma_\nu [1 - 4 \sin^2 \theta_W - \gamma_5] \frac{1}{\not{k}} \gamma_5 \\
& \times \frac{1}{k^4} \left[-g^{\nu\beta} + \frac{(1-\xi)k^\nu k^\beta}{k^2} \right] k_\beta \\
& = \frac{1}{(2\pi)^4} \frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 \int d^4 k \frac{-\xi}{k^4} \not{k} [1 - 4 \sin^2 \theta_W - \gamma_5] \frac{1}{\not{k}} \gamma_5 \\
& \Rightarrow -\frac{gm_e}{m_W} \left(\frac{g}{4 \cos \theta_W} \right)^2 i\Lambda \xi [1 + (1 - 4 \sin^2 \theta_W) \gamma_5], \tag{2.24}
\end{aligned}$$

$$\begin{aligned}
J_{W\phi\nu} & \Rightarrow \frac{-1}{(2\pi)^4} \frac{g^3 m_e}{16m_W} \int d^4 k (1 - \gamma_5) \frac{1}{\not{k}} \gamma_\mu (1 - \gamma_5) (-k_\alpha) \frac{1}{k^4} \left[-g^{\mu\alpha} + \frac{(1-\xi)k^\mu k^\alpha}{k^2} \right] \\
& = \frac{-1}{(2\pi)^4} \frac{g^3 m_e}{16m_W} \int d^4 k (1 - \gamma_5) \frac{1}{\not{k}} (\xi \not{k}) (1 - \gamma_5) \frac{1}{k^4} \\
& \Rightarrow -\frac{\xi g^3 m_e}{8m_W} i\Lambda(1 - \gamma_5), \tag{2.25}
\end{aligned}$$

$$J_{\phi W \nu} \Rightarrow \frac{-1}{(2\pi)^4} \frac{g^3 m_e}{16m_W} \int d^4 k \gamma_\nu (1 - \gamma_5) \frac{1}{\not{k}} (1 + \gamma_5) (-k_\beta) \frac{1}{k^4} \left[-g^{\nu\beta} + \frac{(1-\xi)k^\nu k^\beta}{k^2} \right]$$

$$\begin{aligned}
&= \frac{-1}{(2\pi)^4} \frac{g^3 m_e}{16m_W} \int d^4 k (\xi \not{k}) (1 - \gamma_5) \frac{1}{\not{k}} (1 + \gamma_5) \frac{1}{\not{k}} \\
&\Rightarrow -\frac{\xi g^3 m_e}{8m_W} i\Lambda(1 + \gamma_5).
\end{aligned} \tag{2.26}$$

It is interesting to note that the γ_5 terms cancel between $J_{Z\phi_0 e}$ and $J_{\phi_0 Z e}$, and also between $J_{W\phi\nu}$ and $J_{\phi W\nu}$.

It only remains to add up the thirteen contributions:

$$\begin{aligned}
J = & J_{ee\gamma} + J_{eeZ} + J_{ee\phi_0} + J_{eeH} + J_{ZZe} + J_{Z\phi_0 e} + J_{\phi_0 Z e} + J_{\phi_0\phi_0 e} \\
& + J_{WW\nu} + J_{W\phi\nu} + J_{\phi W\nu} + J_{\phi\phi\nu} + J_{HHe},
\end{aligned} \tag{2.27}$$

to get

$$\begin{aligned}
J \Rightarrow & -i\Lambda \frac{gm_e}{2m_W} \left\{ e^2(3 + \xi) + \left(\frac{g}{4 \cos \theta_W} \right)^2 [(1 - 4 \sin^2 \theta_W)^2 - 1](3 + \xi) \right. \\
& \left. + 4\xi \left(\frac{g}{4 \cos \theta_W} \right)^2 + g^2 \frac{\xi}{2} \right\}.
\end{aligned} \tag{2.28}$$

Using $e = g \sin \theta_W$, this can be simplified slightly to

$$J \Rightarrow -i\Lambda \frac{3g^3 m_e}{8m_W} \left[\frac{2 + \xi}{\cos^2 \theta_W} - 2 \right]. \tag{2.29}$$

Note that this result depends on ξ , and hence is not gauge invariant. It is slightly better to use $\cos \theta_W = m_W/m_Z$ to rewrite (2.29) as

$$J \Rightarrow \frac{3g^3 m_e}{8m_W^3} i\Lambda [-(2 + \xi)m_Z^2 + 2m_W^2]. \tag{2.30}$$

3. Electron Wave-Function Renormalization

The twelve diagrams with radiative corrections on the external electron lines are shown in fig. 3. There are also many diagrams with a Higgs tadpole on the external electron lines. However, these diagrams do not lead to any dependence on q or q' , and hence do not contribute to the eeH vertex renormalization. They are therefore omitted.

In this fig. 3, the radiative corrections appear on the incoming electron line of momentum q for the upper row of six diagrams, and on the outgoing electron line of momentum q' for the lower row of six diagrams. It is sufficient to study only the upper row of diagrams. In these diagrams, the six self-energy insertions are shown in fig. 4 and designated by $I_{e\gamma}$,

I_{eH} , I_{eZ} , $I_{e\phi_0}$, $I_{\nu W}$ and $I_{\nu\phi^+}$ respectively, where the indices indicate the internal lines of the loop. They are explicitly

$$I_{e\gamma} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} (-ie\gamma_\nu) \frac{i(\not{q} - \not{k} + m_e)}{(q-k)^2 - m_e^2 + i\epsilon} (-ie\gamma_\mu) \frac{i}{k^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 + i\epsilon} \right], \quad (3.1)$$

$$I_{eH} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} \frac{i}{k^2 - m_H^2 + i\epsilon} \left(-\frac{igm_e}{2m_W} \right)^2 \frac{i(\not{q} - \not{k} + m_e)}{(q-k)^2 - m_e^2 + i\epsilon}, \quad (3.2)$$

$$I_{eZ} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} \frac{i}{k^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m_Z^2 + i\epsilon} \right] \\ \times \left(\frac{-ig}{4\cos\theta_W} \right)^2 \gamma_\nu (1 - 4\sin^2\theta_W - \gamma_5) \frac{i(\not{q} - \not{k} + m_e)}{(q-k)^2 - m_e^2 + i\epsilon} \gamma_\mu (1 - 4\sin^2\theta_W - \gamma_5), \quad (3.3)$$

$$I_{e\phi_0} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} \left(\frac{gm_e}{2m_W} \right)^2 \frac{i}{k^2 - \xi m_Z^2 + i\epsilon} \gamma_5 \frac{i(\not{q} - \not{k} + m_e)}{(q-k)^2 - m_e^2 + i\epsilon} \gamma_5, \quad (3.4)$$

$$I_{\nu W} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} \frac{i}{k^2 - m_W^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m_W^2 + i\epsilon} \right] \\ \times \left(\frac{ig}{2\sqrt{2}} \right)^2 \gamma_\nu (1 - \gamma_5) \frac{i(\not{q} - \not{k})}{(q-k)^2 + i\epsilon} \gamma_\mu (1 - \gamma_5), \quad (3.5)$$

$$I_{\nu\phi^+} = \frac{1}{(2\pi)^4} \int d^4k e^{ik\cdot\delta} \frac{i}{k^2 - \xi m_W^2 + i\epsilon} \left(\frac{-igm_e}{2\sqrt{2}m_W} \right)^2 (1 - \gamma_5) \frac{i(\not{q} - \not{k})}{(q-k)^2 + i\epsilon} (1 + \gamma_5), \quad (3.6)$$

where point-splitting regularization is used.

Let I_e be the sum of these six terms,

$$I_e = I_{e\gamma} + I_{eH} + I_{eZ} + I_{e\phi_0} + I_{\nu W} + I_{\nu\phi^+}; \quad (3.7)$$

it must have the form

$$I_e = A_e + B_e \not{q} + D_e \not{q} \gamma_5. \quad (3.8)$$

When this form (3.8) is used in the eeH vertex diagrams of fig. 3, the result is

$$\frac{1}{2} \left(\frac{-igm_e}{2m_W} \right) \frac{i}{\not{q} - m_e + i\epsilon} [B_e(\not{q} - m_e) + D_e \not{q} \gamma_5] \\ + \frac{1}{2} [B_e(\not{q}' - m_e) + D_e \not{q}' \gamma_5] \frac{i}{\not{q}' - m_e + i\epsilon} \left(\frac{-igm_e}{2m_W} \right) \\ = \frac{gm_e}{4m_W} \left[2B_e + D_e \gamma_5 \frac{\not{q}}{\not{q} + m_e + i\epsilon} - D_e \frac{\not{q}'}{\not{q}' + m_e + i\epsilon} \gamma_5 \right], \quad (3.9)$$

where the factors of $\frac{1}{2}$ are the usual ones from wave-function renormalization. As $\not{q}$ and $\not{q}'$ are both replaced by m_e , the second and third terms cancel, and the above expression is simply

$$\frac{gm_e}{2m_W} B_e. \quad (3.10)$$

The task is to extract the B_e part from eqs. (3.1)–(3.6). More precisely, only the $\ln |\delta|$ part of B_e is needed for small splitting δ .

It is straightforward to simplify (3.1)–(3.6) to the following:

$$I_{e\gamma} = \frac{e^2}{(2\pi)^4} \int d^4k \frac{e^{ik\cdot\delta}}{k^2 + i\epsilon} \left\{ -\frac{2\not{k} + 2m_e}{(q-k)^2 - m_e^2 + i\epsilon} + \frac{2}{(q-k)^2 - m_e^2 + i\epsilon} (\not{q} - m_e) \right. \\ \left. + \frac{1-\xi}{k^2 + i\epsilon} \left[(\not{q} - m_e) \frac{1}{\not{q} - \not{k} - m_e + i\epsilon} (\not{q} - m_e) - (\not{q} - m_e) \right] \right\}, \quad (3.11)$$

$$I_{eH} = \frac{1}{(2\pi)^4} \frac{g^2}{4} \frac{m_e^2}{m_W^2} \int d^4k e^{ik\cdot\delta} \frac{(2m_e - \not{k}) + (\not{q} - m_e)}{[k^2 - m_H^2 + i\epsilon][(q-k)^2 - m_e^2 + i\epsilon]}, \quad (3.12)$$

$$I_{eZ} = \frac{1}{(2\pi)^4} \left(\frac{g}{4\cos\theta_W} \right)^2 \int d^4k \frac{e^{ik\cdot\delta}}{k^2 - m_Z^2 + i\epsilon} \left\{ -2(1 - 4\sin^2\theta_W)^2 \frac{\not{k} - \not{q} + 2m_e}{(q-k)^2 - m_e^2 + i\epsilon} \right. \\ + (1 - 4\sin^2\theta_W)\gamma_5 \frac{4(\not{q} - \not{k})}{(q-k)^2 - m_e^2 + i\epsilon} - 2 \frac{\not{k} - \not{q} - 2m_e}{(q-k)^2 - m_e^2 + i\epsilon} \\ \left. + \frac{1-\xi}{k^2 - \xi m_Z^2 + i\epsilon} \left[(1 - 4\sin^2\theta_W)^2 \left((\not{q} - m_e) \frac{1}{\not{q} - \not{k} - m_e + i\epsilon} (\not{q} - m_e) - (\not{q} - m_e) \right) \right. \right. \\ \left. + (1 - 4\sin^2\theta_W)\gamma_5 \left((\not{q} - m_e) \frac{1}{\not{q} - \not{k} - m_e + i\epsilon} - 1 \right) (\not{q} - m_e) \right. \\ \left. - (1 - 4\sin^2\theta_W)(\not{q} - m_e) \left(\frac{1}{\not{q} - \not{k} - m_e + i\epsilon} (\not{q} - m_e) - 1 \right) \gamma_5 \right. \\ \left. \left. + (\not{q} + m_e) \frac{\not{q} - \not{k} - m_e}{(q-k)^2 - m_e^2 + i\epsilon} (\not{q} + m_e) - (\not{q} + m_e) \right] \right\}, \quad (3.13)$$

$$I_{e\phi_0} = \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k \frac{e^{ik\cdot\delta}}{k^2 - \xi m_Z^2 + i\epsilon} \frac{\not{q} - \not{k} - m_e}{(q-k)^2 - m_e^2 + i\epsilon}, \quad (3.14)$$

$$I_{\nu W} = \frac{1}{(2\pi)^4} \frac{g^2}{4} \int d^4k \frac{e^{ik\cdot\delta}}{k^2 - m_W^2 + i\epsilon} \\ \times \left\{ \frac{2(\not{q} - \not{k})}{(q-k)^2 + i\epsilon} + \frac{1-\xi}{k^2 - \xi m_W^2 + i\epsilon} \left(\not{q} \frac{1}{\not{q} - \not{k} + i\epsilon} - 1 \right) \not{q} \right\} (1 - \gamma_5), \quad (3.15)$$

$$I_{\nu\phi} = \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k \frac{e^{ik\cdot\delta}}{k^2 - \xi m_W^2 + i\epsilon} \frac{\not{q} - \not{k}}{(q-k)^2 + i\epsilon} (1 + \gamma_5). \quad (3.16)$$

There are two kinds of contributions to B_e , from terms in the integrands that are explicitly proportional to $\not{q}$, and also from those that are proportional to $\not{k}$. We note that

$$\int d^4k e^{ik \cdot \delta} \frac{1}{k^2 - m^2 + i\epsilon} \frac{\not{k}}{(q - k)^2 - m_e^2 + i\epsilon} = B_0 \not{q}, \quad (3.17)$$

with

$$B_0 = \frac{1}{q^2} \int d^4k e^{ik \cdot \delta} \frac{1}{k^2 - m^2 + i\epsilon} \frac{k \cdot q}{(q - k)^2 - m_e^2 + i\epsilon}, \quad (3.18)$$

where we have neglected terms proportional to δ . Equivalently, we may substitute in eqs. (3.11)–(3.16)

$$\not{k} \rightarrow \frac{\not{q}}{q^2} (k \cdot q). \quad (3.19)$$

Consequently the contributions from (3.11)–(3.16) to B_e are respectively

$$(B_e)_{e\gamma} = \frac{e^2}{(2\pi)^4} \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 + i\epsilon][(q - k)^2 - m_e^2 + i\epsilon]} \times \left\{ \frac{2}{q^2} [q \cdot (q - k)] + \frac{1 - \xi}{k^2 + i\epsilon} \left[-k^2 + (k \cdot q) - m_e^2 \frac{k \cdot q}{q^2} \right] \right\}, \quad (3.20)$$

$$(B_e)_{eH} = \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_H^2 + i\epsilon][(q - k)^2 - m_e^2 + i\epsilon]} \frac{1}{q^2} [q \cdot (q - k)], \quad (3.21)$$

$$(B_e)_{eZ} = \frac{1}{(2\pi)^4} \left(\frac{g}{4 \cos \theta_W} \right)^2 \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_Z^2 + i\epsilon][(q - k)^2 - m_e^2 + i\epsilon]} \times [(1 - 4 \sin^2 \theta_W)^2 + 1] \times \left\{ \frac{2}{q^2} [q \cdot (q - k)] + \frac{1 - \xi}{k^2 - \xi m_Z^2 + i\epsilon} \left[-k^2 + (k \cdot q) - m_e^2 \frac{k \cdot q}{q^2} \right] \right\}, \quad (3.22)$$

$$(B_e)_{e\phi_0} = \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - \xi m_Z^2 + i\epsilon][(q - k)^2 - m_e^2 + i\epsilon]} \frac{1}{q^2} [q \cdot (q - k)], \quad (3.23)$$

$$(B_e)_{\nu W} = \frac{1}{(2\pi)^4} \frac{g^2}{4} \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_W^2 + i\epsilon][(q - k)^2 + i\epsilon]} \times \left\{ \frac{2}{q^2} [q \cdot (q - k)] + \frac{1 - \xi}{k^2 - \xi m_W^2 + i\epsilon} [-k^2 + (k \cdot q)] \right\}, \quad (3.24)$$

$$(B_e)_{\nu\phi} = \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - \xi m_W^2 + i\epsilon][(q - k)^2 + i\epsilon]} \frac{1}{q^2} [q \cdot (q - k)]. \quad (3.25)$$

For small δ , these integrals are all of the form

$$\text{const.} \frac{q \cdot \delta}{q^2 \delta^2} + \text{const.} \ln |\delta| + \mathcal{O}(1). \quad (3.26)$$

See for example Appendix C of Paper I. If δ is replaced by $-\delta$, the first term reverses sign; this first term thus does not contribute if we adopt the convention of averaging over δ and $-\delta$ [10]. It is therefore convenient to generalize the notation $\Rightarrow$ of Sec. 2 to mean that the $\ln |\delta|$ piece is the same.

As an example, consider

$$\begin{aligned} & \int d^4 k e^{ik \cdot \delta} \frac{k_\mu}{[k^2 - m^2 + i\epsilon][(q - k)^2 - m^2 + i\epsilon]} \\ &= \int_0^1 d\alpha \int d^4 k e^{ik \cdot \delta} \frac{k_\mu}{[k^2 - 2\alpha(q \cdot k) + \alpha q^2 - m^2 + i\epsilon]^2} \\ &= \int_0^1 d\alpha \int d^4 k' e^{i(k' + \alpha q) \cdot \delta} \frac{k'_\mu + \alpha q_\mu}{[k'^2 + \alpha(1 - \alpha)q^2 - m^2 + i\epsilon]^2} \\ &= -2\pi^2 \frac{\delta_\mu}{\delta^2} + \int_0^1 \alpha d\alpha \int d^4 k' e^{ik' \cdot \delta} \frac{q_\mu}{[k'^2 + \alpha(1 - \alpha)q^2 - m^2 + i\epsilon]^2} + \mathcal{O}(1) \\ &\Rightarrow \frac{1}{2} \int d^4 k' e^{ik' \cdot \delta} \frac{q_\mu}{[k'^2 - m^2 + i\epsilon]^2}. \end{aligned} \quad (3.27)$$

In other words, the k_μ in the numerator can be replaced by $\frac{1}{2}q_\mu$. This factor of $\frac{1}{2}$ can be understood as follows. By using the averaging over δ and $-\delta$, and letting $\bar{k} = q - k$, we have

$$\begin{aligned} & \int d^4 k e^{ik \cdot \delta} \frac{k_\mu}{[k^2 - m^2 + i\epsilon][(q - k)^2 - m^2 + i\epsilon]} \\ &= \int d^4 \bar{k} e^{iq \cdot \delta} e^{-i\bar{k} \cdot \delta} \frac{q_\mu - \bar{k}_\mu}{[(q - \bar{k})^2 - m^2 + i\epsilon][\bar{k}^2 - m^2 + i\epsilon]} \\ &\Rightarrow \int d^4 \bar{k} e^{i\bar{k} \cdot \delta} \frac{q_\mu - \bar{k}_\mu}{[(q - \bar{k})^2 - m^2 + i\epsilon][\bar{k}^2 - m^2 + i\epsilon]}, \end{aligned} \quad (3.28)$$

which implies immediately

$$\int d^4 k e^{ik \cdot \delta} \frac{2k_\mu - q_\mu}{[k^2 - m^2 + i\epsilon][(q - k)^2 - m^2 + i\epsilon]} \Rightarrow 0. \quad (3.29)$$

With (3.27) or (3.29), the six B_e 's given by (3.20)–(3.25) lead to

$$(B_e)_{e\gamma} \Rightarrow \frac{e^2}{(2\pi)^4} \xi \int d^4 k \frac{e^{ik \cdot \delta}}{[k^2 + i\epsilon][(q - k)^2 - m_e^2 + i\epsilon]} \Rightarrow e^2 i \Lambda \xi, \quad (3.30)$$

$$\begin{aligned}
(B_e)_{eH} &\Rightarrow \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \frac{1}{2} \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_H^2 + i\epsilon][(q-k)^2 - m_e^2 + i\epsilon]} \\
&\Rightarrow \frac{1}{2} \left(\frac{gm_e}{2m_W} \right)^2 i\Lambda,
\end{aligned} \tag{3.31}$$

$$\begin{aligned}
(B_e)_{eZ} &\Rightarrow \frac{1}{(2\pi)^4} \left(\frac{g}{4 \cos \theta_W} \right)^2 \xi \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_Z^2 + i\epsilon][(q-k)^2 - m_e^2 + i\epsilon]} [(1 - 4 \sin^2 \theta_W)^2 + 1] \\
&\Rightarrow \left(\frac{g}{4 \cos \theta_W} \right)^2 [(1 - 4 \sin^2 \theta_W)^2 + 1] i\Lambda \xi,
\end{aligned} \tag{3.32}$$

$$\begin{aligned}
(B_e)_{e\phi_0} &\Rightarrow \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \frac{1}{2} \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - \xi m_Z^2 + i\epsilon][(q-k)^2 - m_e^2 + i\epsilon]} \\
&\Rightarrow \frac{1}{2} \left(\frac{gm_e}{2m_W} \right)^2 i\Lambda,
\end{aligned} \tag{3.33}$$

$$(B_e)_{\nu W} = \frac{1}{(2\pi)^4} \frac{g^2}{4} \xi \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - m_W^2 + i\epsilon][(q-k)^2 + i\epsilon]} \Rightarrow \frac{g^2}{4} i\Lambda \xi, \tag{3.34}$$

$$\begin{aligned}
(B_e)_{\nu\phi} &= \frac{1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \frac{1}{2} \int d^4k \frac{e^{ik \cdot \delta}}{[k^2 - \xi m_W^2 + i\epsilon][(q-k)^2 + i\epsilon]} \\
&\Rightarrow \frac{1}{2} \left(\frac{gm_e}{2m_W} \right)^2 i\Lambda.
\end{aligned} \tag{3.35}$$

Adding them together gives

$$B_e \Rightarrow i\Lambda \left\{ e^2 \xi + \left(\frac{g}{4 \cos \theta_W} \right)^2 [(1 - 4 \sin^2 \theta_W)^2 + 1] \xi + \frac{g^2}{4} \xi + \frac{3}{2} \left(\frac{gm_e}{2m_W} \right)^2 \right\}. \tag{3.36}$$

This can be simplified slightly to

$$B_e \Rightarrow i\Lambda g^2 \left\{ \frac{(3 + 2 \sin^2 \theta_W) \xi}{8 \cos^2 \theta_W} + \frac{3}{2} \left(\frac{m_e}{2m_W} \right)^2 \right\}, \tag{3.37}$$

and the substitution into (3.10) gives the following contribution to the eeH coupling constant renormalization:

$$\begin{aligned}
&\frac{g^3 m_e}{2m_W} i\Lambda \left[\frac{(3 + 2 \sin^2 \theta_W) \xi}{8 \cos^2 \theta_W} + \frac{3}{2} \left(\frac{m_e}{2m_W} \right)^2 \right] \\
&= \frac{g^3 m_e}{8m_W^3} i\Lambda \left[\frac{5}{2} \xi m_Z^2 - \xi m_W^2 + \frac{3}{2} m_e^2 \right].
\end{aligned} \tag{3.38}$$

4. Higgs Wave-Function Renormalization

Similar to the calculation of the electron wave-function renormalization of Sec. 3, there are fourteen diagrams with radiative corrections on the external Higgs line, as shown in

fig. 5. Once again, the diagrams with a Higgs tadpole on the external Higgs line do not contribute and are not shown. There are also many other one-loop diagrams, as shown in fig. 6, that do not contribute.

If the point-splitting regularization of Paper I is used, there are, as already mentioned, in addition a number of diagrams involving one peculiar vertex. Such vertices are proportional to δ , the distance of point splitting. Because of this factor of δ , these additional diagrams again do not contribute to the logarithmically divergent terms, i. e., they do not contain any term proportional to $\ln|\delta|$. This situation is entirely the same as that discussed in connection with (3.26) and the example (3.27).

Each of the diagrams of fig. 5 is of the form of a horizontal electron line with an attached Higgs self-energy diagram. Let I_{ee} etc. denote the integral corresponding to the Higgs self-energy diagrams, i. e., the diagrams of fig. 5 with the eeH vertex and the Higgs propagator deleted. These I 's are given explicitly by

$$I_{ee} = \frac{-1}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} \left(-\frac{1}{2}ig \frac{m_e}{m_W} \right)^2 \text{Tr} \frac{i(\not{k} + \frac{1}{2}\not{p} + m_e)}{(k + \frac{1}{2}p)^2 - m_e^2 + i\epsilon} \frac{i(\not{k} - \frac{1}{2}\not{p} + m_e)}{(k - \frac{1}{2}p)^2 - m_e^2 + i\epsilon}, \quad (4.1)$$

$$I_{uu} = \frac{-3}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} \left(-\frac{1}{2}ig \frac{m_u}{m_W} \right)^2 \text{Tr} \frac{i(\not{k} + \frac{1}{2}\not{p} + m_u)}{(k + \frac{1}{2}p)^2 - m_u^2 + i\epsilon} \frac{i(\not{k} - \frac{1}{2}\not{p} + m_u)}{(k - \frac{1}{2}p)^2 - m_u^2 + i\epsilon}, \quad (4.2)$$

$$I_{dd} = \frac{-3}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} \left(-\frac{1}{2}ig \frac{m_d}{m_W} \right)^2 \text{Tr} \frac{i(\not{k} + \frac{1}{2}\not{p} + m_d)}{(k + \frac{1}{2}p)^2 - m_d^2 + i\epsilon} \frac{i(\not{k} - \frac{1}{2}\not{p} + m_d)}{(k - \frac{1}{2}p)^2 - m_d^2 + i\epsilon}, \quad (4.3)$$

$$\begin{aligned} I_{WW} = \frac{1}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} (igm_W)^2 \\ \times \frac{i}{(\frac{1}{2}p + k)^2 - m_W^2 + i\epsilon} \left[-g_{\mu\nu} + \frac{(1-\xi)(\frac{1}{2}p + k)_\mu(\frac{1}{2}p + k)_\nu}{(\frac{1}{2}p + k)^2 - \xi m_W^2 + i\epsilon} \right] \\ \times \frac{i}{(\frac{1}{2}p - k)^2 - m_W^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)(\frac{1}{2}p - k)^\mu(\frac{1}{2}p - k)^\nu}{(\frac{1}{2}p - k)^2 - \xi m_W^2 + i\epsilon} \right], \end{aligned} \quad (4.4)$$

$$I_{\phi^+\phi^-} = \frac{1}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} \left(-\frac{1}{2}ig \frac{m_H^2}{m_W} \right)^2 \frac{i}{(\frac{1}{2}p + k)^2 - \xi m_W^2 + i\epsilon} \frac{i}{(\frac{1}{2}p - k)^2 - \xi m_W^2 + i\epsilon}, \quad (4.5)$$

$$\begin{aligned} I_{W^+\phi^-} = \frac{1}{(2\pi)^4} \int d^4k e^{ik \cdot \delta} (\frac{1}{2}ig(\frac{3}{2}p - k)_\mu)(\frac{1}{2}ig(\frac{3}{2}p - k)_\nu) \\ \times \frac{i}{(\frac{1}{2}p + k)^2 - m_W^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi)(\frac{1}{2}p + k)^\mu(\frac{1}{2}p + k)^\nu}{(\frac{1}{2}p + k)^2 - \xi m_W^2 + i\epsilon} \right] \frac{i}{(\frac{1}{2}p - k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \quad (4.6)$$

$$\begin{aligned}
I_{\phi^+ W^-} &= \frac{1}{(2\pi)^4} \int d^4 k e^{ik \cdot \delta} \left(\frac{1}{2} i g \left(\frac{3}{2} p + k \right)_\mu \right) \left(\frac{1}{2} i g \left(\frac{3}{2} p + k \right)_\nu \right) \\
&\times \frac{i}{\left(\frac{1}{2} p + k \right)^2 - \xi m_W^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - m_W^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi) \left(\frac{1}{2} p - k \right)^\mu \left(\frac{1}{2} p - k \right)^\nu}{\left(\frac{1}{2} p - k \right)^2 - \xi m_W^2 + i\epsilon} \right],
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
I_{ZZ} &= \frac{1}{(2\pi)^4} \frac{1}{2} \int d^4 k e^{ik \cdot \delta} \left(i g \frac{m_Z^2}{m_W} \right)^2 \\
&\times \frac{i}{\left(\frac{1}{2} p + k \right)^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi) \left(\frac{1}{2} p + k \right)_\mu \left(\frac{1}{2} p + k \right)_\nu}{\left(\frac{1}{2} p + k \right)^2 - \xi m_Z^2 + i\epsilon} \right] \\
&\times \frac{i}{\left(\frac{1}{2} p - k \right)^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi) \left(\frac{1}{2} p - k \right)_\mu \left(\frac{1}{2} p - k \right)_\nu}{\left(\frac{1}{2} p - k \right)^2 - \xi m_Z^2 + i\epsilon} \right],
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
I_{\phi_0 \phi_0} &= \frac{1}{(2\pi)^4} \frac{1}{2} \int d^4 k e^{ik \cdot \delta} \left(-\frac{1}{2} i g \frac{m_H^2}{m_W} \right)^2 \frac{i}{\left(\frac{1}{2} p + k \right)^2 - \xi m_Z^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - \xi m_Z^2 + i\epsilon},
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
I_{Z\phi_0} &= \frac{1}{(2\pi)^4} \int d^4 k e^{ik \cdot \delta} \left(\frac{g}{2 \cos \theta_W} \left(\frac{3}{2} p - k \right)_\mu \right) \left(-\frac{g}{2 \cos \theta_W} \left(\frac{3}{2} p - k \right)_\nu \right) \\
&\times \frac{i}{\left(\frac{1}{2} p + k \right)^2 - m_Z^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{(1-\xi) \left(\frac{1}{2} p + k \right)_\mu \left(\frac{1}{2} p + k \right)_\nu}{\left(\frac{1}{2} p + k \right)^2 - \xi m_Z^2 + i\epsilon} \right] \frac{i}{\left(\frac{1}{2} p - k \right)^2 - \xi m_Z^2 + i\epsilon},
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
I_{HH} &= \frac{1}{(2\pi)^4} \frac{1}{2} \int d^4 k e^{ik \cdot \delta} \left(-\frac{3}{2} i g \frac{m_H^2}{m_W} \right)^2 \frac{i}{\left(\frac{1}{2} p + k \right)^2 - m_H^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - m_H^2 + i\epsilon},
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
I_{\eta^+ \eta^+} &= \frac{-1}{(2\pi)^4} \int d^4 k e^{ik \cdot \delta} \left(-\frac{1}{2} i g \xi m_W \right)^2 \frac{i}{\left(\frac{1}{2} p + k \right)^2 - \xi m_W^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - \xi m_W^2 + i\epsilon},
\end{aligned} \tag{4.12}$$

$$\begin{aligned}
I_{\eta^- \eta^-} &= \frac{-1}{(2\pi)^4} \int d^4 k e^{ik \cdot \delta} \left(-\frac{1}{2} i g \xi m_W \right)^2 \frac{i}{\left(\frac{1}{2} p + k \right)^2 - \xi m_W^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - \xi m_W^2 + i\epsilon},
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
I_{\eta^z \eta^z} &= \frac{-1}{(2\pi)^4} \int d^4 k e^{ik \cdot \delta} \left(-\frac{1}{2} i g \xi \frac{m_Z^2}{m_W} \right)^2 \frac{i}{\left(\frac{1}{2} p + k \right)^2 - \xi m_Z^2 + i\epsilon} \frac{i}{\left(\frac{1}{2} p - k \right)^2 - \xi m_Z^2 + i\epsilon}.
\end{aligned} \tag{4.14}$$

In connection with eqs. (4.1)–(4.14), the following points should be noted. First, it may be argued that some of the factors of $e^{ik \cdot \delta}$ due to point-splitting regularization should

be $e^{2ik\cdot\delta}$ as in Paper I. Since the interesting terms, as already seen in Sec. 2 and Sec. 3, are those with a factor of $\ln|\delta|$, it makes no difference here, in contrast to Paper I, whether $e^{ik\cdot\delta}$ or $e^{2ik\cdot\delta}$ is used. Secondly, it may be further argued that, when the internal lines are both gauge particles, there should not be any factor of $e^{ik\cdot\delta}$ at all even when point-splitting regularization is used. Fortunately, as will be seen later in this section, I_{WW} and I_{ZZ} do not contribute. Thirdly, although it is natural to choose the momenta of the internal lines to be $\frac{1}{2}p+k$ and $\frac{1}{2}p-k$, it is arbitrary which one of these two momenta is assigned to which line. For this reason, strictly speaking it is necessary to take these two possible assignments and average over the two resulting integrals. Because of the factor $e^{ik\cdot\delta}$, reversing the sign of k is the same as reversing the sign of δ [10]. This prescription is therefore equivalent to averaging over δ and $-\delta$, in agreement with the discussion immediately after (3.26). As an explicit example, a comparison of (4.6) and (4.7) shows that $I_{W^+ \phi^-}$ and $I_{\phi^+ W^-}$ differ only by $\delta \rightarrow -\delta$.

Similar to (3.26), here each of the fourteen integrals are, for small δ , of the form

$$\text{const.} \frac{1}{\delta^2} + \text{const.} \ln|\delta| + \mathcal{O}(1). \quad (4.15)$$

Terms of the form $(p \cdot \delta)/\delta^2$ are omitted because they give zero when averaged over δ and $-\delta$, while terms of the form $(p \cdot \delta)^2/\delta^2$ are omitted explicitly because they are of order 1. The first term, $\text{const.}/\delta^2$ of (4.15) contributes only to mass renormalization, and hence is irrelevant to the present study of the eeH coupling constant renormalization. For this present purpose, we are interested only in a part of the second term, namely, terms of the form

$$\text{const.} p^2 \ln|\delta|. \quad (4.16)$$

This p^2 , which is actually $p^2 - m_H^2$, is needed to cancel the denominator of the internal Higgs propagator present in all diagrams of fig. 5. It should be mentioned that the const. in (4.16) can only depend on the coupling constants and the masses of the various particles in the standard model, while the second const. in (4.15) may depend on p^2 in addition. We shall use in this section the symbol $\Rightarrow$ to mean the extraction of terms of the form (4.16) from an integral that behaves as (4.15).

Consider first the I_{ee} of (4.1). By taking the trace explicitly, it is given by

$$\begin{aligned} I_{ee} &= \frac{-1}{(2\pi)^4} \left(\frac{gm_e}{2m_W} \right)^2 \int d^4k e^{ik\cdot\delta} 4 \frac{k^2 - \frac{1}{4}p^2 + m_e^2}{[(k + \frac{1}{2}p)^2 - m_e^2 + i\epsilon][(k - \frac{1}{2}p)^2 - m_e^2 + i\epsilon]} \\ &= \frac{-1}{(2\pi)^4} \left(\frac{gm_e}{m_W} \right)^2 \int d^4k e^{ik\cdot\delta} \left\{ \frac{1}{2[(k + \frac{1}{2}p)^2 - m_e^2 + i\epsilon]} + \frac{1}{2[(k - \frac{1}{2}p)^2 - m_e^2 + i\epsilon]} \right\} \end{aligned}$$

$$+ \frac{-p^2 + 4m_e^2}{2[(k + \frac{1}{2}p)^2 - m_e^2 + i\epsilon][(k - \frac{1}{2}p)^2 - m_e^2 + i\epsilon]}. \quad (4.17)$$

Each of the first two terms in the braces gives $e^{\pm \frac{1}{2}ip \cdot \delta} I^{(1)}$, with $I^{(1)}$ defined by (A.1) of Paper I. From (A.7) and (A.14) of Paper I, they contain terms proportional to $1/\delta^2$, $m_e^2 \ln(m_e \delta)$, $(p \cdot \delta)^2/\delta^2$ and terms of order 1 for small δ . None of these terms can contribute to the eeH coupling constant renormalization. Therefore

$$\int d^4k \frac{e^{ik \cdot \delta}}{(k \pm \frac{1}{2}p)^2 - m_e^2 + i\epsilon} \Rightarrow 0, \quad (4.18)$$

and hence

$$\begin{aligned} I_{ee} &\Rightarrow \frac{-1}{(2\pi)^4} \left(\frac{gm_e}{m_W} \right)^2 \int d^4k e^{ik \cdot \delta} \frac{-p^2}{2[(k + \frac{1}{2}p)^2 - m_e^2 + i\epsilon][(k - \frac{1}{2}p)^2 - m_e^2 + i\epsilon]} \\ &\Rightarrow 2p^2 \left(\frac{gm_e}{2m_W} \right)^2 i\Lambda. \end{aligned} \quad (4.19)$$

Similarly, it follows from (4.2) and (4.3) that

$$I_{uu} \Rightarrow 6p^2 \left(\frac{gm_u}{2m_W} \right)^2 i\Lambda, \quad (4.20)$$

$$I_{dd} \Rightarrow 6p^2 \left(\frac{gm_d}{2m_W} \right)^2 i\Lambda. \quad (4.21)$$

The extraction of the desired terms from the $I_{Z\phi_0}$ is similar but somewhat more lengthy:

$$\begin{aligned} I_{Z\phi_0} &= \frac{1}{(2\pi)^4} \left(\frac{g}{2 \cos \theta_W} \right)^2 \int d^4k \frac{e^{ik \cdot \delta}}{[(k + \frac{1}{2}p)^2 - m_Z^2 + i\epsilon][(k - \frac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon]} \\ &\quad \times \left\{ -(k - \frac{3}{2}p)^2 + \frac{(1 - \xi)[(k - \frac{3}{2}p) \cdot (k + \frac{1}{2}p)]^2}{(k + \frac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon} \right\} \\ &= \frac{1}{(2\pi)^4} \left(\frac{g}{2 \cos \theta_W} \right)^2 \int d^4k e^{ik \cdot \delta} \left\{ \frac{-2}{(k + \frac{1}{2}p)^2 - m_Z^2 + i\epsilon} + \frac{1}{(k - \frac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon} \right. \\ &\quad + \frac{-2p^2 - 2\xi m_Z^2 + m_Z^2}{[(k + \frac{1}{2}p)^2 - m_Z^2 + i\epsilon][(k - \frac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon]} \\ &\quad \left. + \frac{1 - \xi}{[(k + \frac{1}{2}p)^2 - m_Z^2 + i\epsilon][(k + \frac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon]} \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left[(k - \tfrac{1}{2}p)^2 + \xi m_Z^2 - 2p^2 + \frac{p^4 + \xi^2 m_Z^4 - 2p^2 \xi m_Z^2}{(k - \tfrac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon} \right] \Big\} \\
& \Rightarrow \frac{1}{(2\pi)^4} \left(\frac{g}{2 \cos \theta_W} \right)^2 \int d^4 k e^{ik \cdot \delta} \left\{ \frac{-2p^2}{[(k + \tfrac{1}{2}p)^2 - m_Z^2 + i\epsilon][(k - \tfrac{1}{2}p)^2 - \xi m_Z^2 + i\epsilon]} \right. \\
& \quad \left. + \frac{1 - \xi}{(k^2 - m_Z^2 + i\epsilon)(k^2 - \xi m_Z^2 + i\epsilon)} e^{-\frac{1}{2}p \cdot \delta} [(k - p)^2 - 2p^2] \right\} \\
& \Rightarrow \left(\frac{g}{2 \cos \theta_W} \right)^2 i\Lambda [-2p^2 - (1 - \xi)p^2] \\
& = p^2 \left(\frac{g}{2 \cos \theta_W} \right)^2 i\Lambda (-3 + \xi). \tag{4.22}
\end{aligned}$$

Similarly, it follows from (4.6) and (4.7) that

$$I_{W^+\phi^-} \Rightarrow p^2 \left(\frac{g}{2} \right)^2 i\Lambda (-3 + \xi), \tag{4.23}$$

and

$$I_{\phi^+W^-} \Rightarrow p^2 \left(\frac{g}{2} \right)^2 i\Lambda (-3 + \xi). \tag{4.24}$$

On the other hand, the other eight integrals do not contribute. For example, consider the I_{ZZ} of (4.8). Without any regularization, this integral is only logarithmically divergent, and furthermore this logarithmically divergent part has no dependence on p . Therefore I_{ZZ} does not contain any term of the form (4.16); in the present notation, this is

$$I_{ZZ} \Rightarrow 0. \tag{4.25}$$

Similarly, from (4.4), we get

$$I_{WW} \Rightarrow 0. \tag{4.26}$$

These are the results referred to earlier in this section. For precisely the same reason, the remaining six integrals also do not contribute:

$$I_{\phi^+\phi^-} \Rightarrow 0,$$

$$I_{\phi_0\phi_0} \Rightarrow 0,$$

$$I_{HH} \Rightarrow 0,$$

$$I_{\eta^+\eta^+} \Rightarrow 0,$$

$$I_{\eta^-\eta^-} \Rightarrow 0,$$

and

$$I_{\eta^z \eta^z} \Rightarrow 0. \quad (4.27)$$

It only remains to add together the preceding nine formulas. Let

$$\begin{aligned} I_H = & I_{ee} + I_{uu} + I_{dd} + \text{two more generations} + I_{WW} + I_{\phi^+ \phi^-} + I_{W^+ \phi^-} \\ & + I_{\phi^+ W^-} + I_{ZZ} + I_{\phi_0 \phi_0} + I_{Z \phi_0} + I_{HH} + I_{\eta^+ \eta^+} + I_{\eta^- \eta^-} + I_{\eta^z \eta^z}, \end{aligned} \quad (4.28)$$

then

$$\begin{aligned} I_H \Rightarrow p^2 i \Lambda \frac{g^2}{2m_W^2} [m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ - (3 - \xi)(\frac{1}{2}m_Z^2 + m_W^2)]. \end{aligned} \quad (4.29)$$

Finally, the contribution to the eeH coupling constant renormalization is given by

$$\frac{1}{2} I_H \frac{i}{p^2} \left(-\frac{1}{2} i g \frac{m_e}{m_W} \right),$$

where $\frac{1}{2}$ is again the usual factor for wave-function renormalization, i/p^2 comes from the internal Higgs line, and $-\frac{1}{2} i g m_e/m_W$ is again the bare eeH vertex. Thus the divergent part of this contribution is explicitly

$$\begin{aligned} \frac{g^3 m_e}{8m_W^3} i \Lambda [m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ - (3 - \xi)(\frac{1}{2}m_Z^2 + m_W^2)]. \end{aligned} \quad (4.30)$$

5. Determination of the Masses of the Top Quark and Higgs

When the contributions (2.30) from the proper vertex diagram, (3.38) from the electron wave-function renormalization, and (4.30) from the Higgs wave-function renormalization are added together, the result is

$$\begin{aligned} \frac{g^3 m_e}{8m_W^3} i \Lambda \left[m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \right. \\ \left. - \frac{15}{2} m_Z^2 + 3m_W^2 + \frac{3}{2} m_e^2 \right]. \end{aligned} \quad (5.1)$$

Note that ξ indeed does not appear in the sum.

If this calculation had been carried out for the $\mu\mu H$ vertex, the result would have differed from (5.1) in two respects: first, the m_e in the overall factor is replaced by m_μ , and secondly the last term in the bracket is $\frac{3}{2} m_\mu^2$ instead. In other words, if we want

to make the eeH coupling constant renormalization finite by setting the quantity in the bracket of (5.1) to zero, we would have to set the same quantity with the last term $\frac{3}{2}m_e^2$ replaced by $\frac{3}{2}m_\mu^2$ also to zero by considering the $\mu\mu H$ vertex. Since this does not make sense, we have to argue that we make use of the smallness of the masses of e , μ and τ to drop the last term. We therefore get the second mass relation

$$\begin{aligned} m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ = \frac{15}{2}m_Z^2 - 3m_W^2. \end{aligned} \quad (5.2)$$

This is the desired result.

Unlike the first mass relation (1.1), in (5.2) the only mass that is not known experimentally is m_t . Solving for m_t gives

$$m_t = \left[\frac{5}{2}m_Z^2 - m_W^2 - \frac{1}{3}(m_e^2 + m_\mu^2 + m_\tau^2) - (m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2) \right]^{\frac{1}{2}}. \quad (5.3)$$

With m_t given by (5.3), the substitution into (1.1) gives the elegant relation (1.3).

6. Conclusion and Discussion

Since the masses of the Z and the W are known experimentally [11], eq. (1.3) gives the mass of the Higgs in the Standard Model:

$$m_H = 190 \text{ GeV}. \quad (6.1)$$

Such a Higgs boson has the following properties:

(1) its width is

$$\Gamma_H = 1 \text{ GeV}, \quad (6.2)$$

(2) its dominant decay modes are W^+W^- , ZZ and $b\bar{b}$ with branching ratios

$$\text{BR}(H \rightarrow W^+W^-) = 77\%$$

$$\text{BR}(H \rightarrow ZZ) = 22\%$$

and

$$\text{BR}(H \rightarrow b\bar{b}) = 1\%, \quad (6.3)$$

(3) it is easy to detect at SSC and LHC even without enhanced luminosity.

Similarly, it follows from (5.3) that the mass of the top quark is

$$m_t = 120 \text{ GeV}. \quad (6.4)$$

This is in the range favored by existing phenomenology [12], [13]. Of course there are radiative corrections to both (6.1) and (6.4).

We conclude with a few brief comments. Because of the point-splitting regularization, the first mass relation (1.1) guarantees that all one-loop mass corrections in the Standard Model are finite. So far as the three-point vertices are concerned, only eeH together with the essentially identical $\mu\mu H$ and $\tau\tau H$ have been studied. These are the only one-loop three-point fermion-fermion-Higgs Yukawa vertices that do not involve the gluon, which has complications due to confinement. Such vertices with gluons remain to be studied. Contrary to the situation with the eeH vertex, the gauge vertices such as $ee\gamma$, eeZ etc. do not lead to any condition such as (5.3).

Looking beyond the Standard Model, we believe that a similar analysis for the Supersymmetric Standard Model [14] is most worthwhile. Clearly a number of relations can be obtained for this case. Since there are many more unknown parameters, it is less clear whether various masses, such as that of the top quark or those of the Higgs bosons, can be determined from existing experimental data. We look forward to these and other related developments.

Acknowledgments

We are indebted to Dr. Conrad Newton for checking our calculations. For helpful discussions, it is a pleasure to thank Professors Carlo Becchi, John Ellis, Raoul Gatto, Howard Georgi, Maurice Jacob, Kenneth Johnson, and especially Professors Harry Lehmann and Chen Ning Yang. One of us (TTW) wishes to thank the Theory Division of CERN for its kind hospitality.

References

- [1] G. S. Abrams et al., Phys. Rev. Lett. 63 (1989) 724;
 B. Adeva, et al. (L3 Collaboration) Phys. Lett. B231 (1989) 509;
 D. Decamp, et al. (ALEPH Collaboration) Phys. Lett. B231 (1989) 519;
 M. Z. Akrawy, et al. (OPAL Collaboration) Phys. Lett. B231 (1989) 530;
 P. Aarnio, et al. (DELPHI Collaboration) Phys. Lett. B231 (1989) 539
- [2] S. L. Glashow, Nucl. Phys. 22 (1961) 579;
 S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
 A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367;
 S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, 1285 (1970)
- [3] F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321;
 P. W. Higgs, Phys. Lett. 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508;
 G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. 13 (1964) 585;
 P. W. Higgs, Phys. Rev. 145 (1966) 1156;
 T. W. B. Kibble, Phys. Rev. 155 (1967) 1554
- [4] P. Osland and T. T. Wu, "Paper I", CERN-TH.6385, to be published
- [5] P. Osland and T. T. Wu, "Paper II", CERN-TH.6386, to be published
- [6] M. Veltman, Acta Physica Polonica B12 (1981) 437
- [7] C. G. Bollini and J. J. Giambiagi, Phys. Lett. B40 (1972) 566;
 G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189;
 J. F. Ashmore, Nuovo Cim. Lett. 4 (1972) 289;
 G. M. Cicuta and E. Montaldi, Nuovo Cim. Lett. 4 (1972) 329
- [8] W. Pauli and F. Villars, Rev. Mod. Phys. 21 (1949) 434
- [9] L. D. Faddeev and V. N. Popov, Phys. Lett. 25B (1967) 29
- [10] K. Johnson, in *Lectures on Particles and Field Theory*, Lectures at Brandeis University 1964, eds. S. Deser and K. W. Ford, vol. 2 (Prentice-Hall, Inc., 1965) p. 1
- [11] Review of Particle Properties, Phys. Lett. B239 (1990) 1
- [12] J. Ellis and G. L. Fogli, Phys. Lett. B231 (1989) 189; B232 (1989) 139;
 P. Langacker, Phys. Rev. Lett. 63 (1989) 1920;
 G. Altarelli, CERN preprint CERN-TH-6206-91;
 for a recent review, see J. Ellis, CERN-TH.6193, to appear in *Proc. 1991 LP-HEP Conference*, Geneva 1991
- [13] The LEP Collaborations – ALEPH, DELPHI, L3 and OPAL, CERN-PPE/91-232 (1991)
- [14] P. Fayet, Phys. Lett. 69B (1977) 489;
 S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;

H.-P. Nilles, Phys. Rep. C110 (1984) 1;
H. E. Haber and G. L. Kane, Phys. Rep. C117 (1985) 75

Figure captions

- Fig. 1. Generic one-loop eeH proper vertex diagram.
- Fig. 2. *a)* and *b)*: The thirteen one-loop eeH proper vertex diagrams.
- Fig. 3. One-loop diagrams where the radiative correction is on the external electron lines.
- Fig. 4. The six one-loop electron self-energy diagrams.
- Fig. 5. One-loop diagrams where the radiative correction is on the external Higgs line.
- Fig. 6. One-loop diagrams that do not contribute to the coupling constant renormalization.

Figure 1

Figure 2a

Figure 2b

Figure 3

Figure 4

Figure 5

Figure 6