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The wealth of experimental data accumulated in recent years on low-energy pion-nuclear interactions [1], from pionic atoms at zero pion kinetic energy E_{kin} to elastic, inelastic and charge exchange reactions at $E_{kin} \leq 50$ MeV, is generally well described by a theoretically motivated phenomenological optical potential [2]. However, the construction of this potential, essentially by using the nonrelativistic Impulse Approximation $V \sim t\rho$ where t is the πN t matrix and ρ is the nuclear density, fails to produce a central pion-nucleus repulsion of about 30 MeV as required by fitting to the data [3]. The underlying isoscalar πN s -wave interaction is exceptionally small at threshold (with a scattering length $b_0 = -0.010$ m $^{-1}$, m being the pion mass), its double-scattering modification [4] is relatively ineffective, and the dispersive contribution accompanying the imaginary πNN absorption term is believed to be attractive [5]; if this latter contribution were to blame, it would have to be *repulsive* and of magnitude of up to five times its imaginary counterpart, depending on the details of the potential used.

In a very recent paper [6], Birbrair and Gridnev (BG) have made the straightforward yet stunning observation that, to the extent that Nuclear Dirac Phenomenology provides a valid and meaningful description of the gross properties of nuclei at low energies, it naturally offers a *spin-independent* coupling of the spin dependent terms of the πN t matrix to the spin dependence which the bound-nucleon lower Dirac components imply. BG showed, within what we call the Relativistic Impulse Approximation (RIA), that such a coupling could become responsible for the central isoscalar repulsion missing in the conventional nonrelativistic Impulse Approximation. More specifically, one starts from a zero-range πN scattering amplitude F specified in the cm system by the scattering length b_0 and the scattering volumes c_0 and d_0 (only the isoscalar terms are here considered explicitly),

$$F = b_0 + c_0 \vec{k}'_{cm} \cdot \vec{k}_{cm} + id_0 \vec{\sigma} \cdot \vec{k}'_{cm} \times \vec{k}_{cm}, \quad (1)$$

and embeds it within an off-shell relativistic description [7]

$$F = \frac{M}{s^{1/2}} \bar{u}(p') T u(p), \quad T = A(s, t, u) + B(s, t, u) \not{Q}, \quad Q = \frac{1}{2} (k + k'), \quad (2)$$

where A and B are Lorentz-invariant amplitudes expressed in terms of the Mandelstam variables s, t, u , M is the free-nucleon mass and $\not{Q} = \gamma^\mu Q_\mu$. Furthermore, k and k' are pion initial- and final four momenta respectively, and $u(p)$ and $u(p')$ are free-nucleon four spinors appropriate to four momenta p and p' respectively. The RIA expression for the pion-nucleus optical potential,

$$2\mu U_{RIA}^{(1)} = -4\pi (\bar{\Psi}_{g.s.} T \Psi_{g.s.}) \quad (3)$$

appropriate to a LS closed-shell nuclear target, is then given by [6]

$$2\mu U_{RIA}^{(1)} = 2\mu U_{IA}^{(1)} - 4\pi \frac{m}{M} d_0 \left(\frac{M^2}{M^2(r)} - 1 \right) \rho_k(r) \quad (4)$$

where μ is the pion reduced mass with respect to the nucleus and the subscript IA stands for the conventional nonrelativistic Impulse Approximation result [2]

Does the Relativistic Impulse Approximation produce sufficient s -wave repulsion for pionic atoms?

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Ambiguities due to off-shell extrapolation of the πN T matrix in the application of the Relativistic Impulse Approximation to low-energy pion-nucleus interactions are demonstrated. The extent to which the "missing" s -wave repulsion for pionic atoms is produced by the introduction of Nuclear Dirac Phenomenology is shown to be strongly model dependent by performing a large scale fit to pionic atom data.

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$$2\mu U_{IA}^{(1)} = -4\pi \left\{ \left(1 + \frac{m}{M}\right) b_0 \rho(r) + \left(1 + \frac{m}{M}\right)^{-1} c_0 \left[\frac{m}{2M} (\Delta \rho(r)) + \frac{m^2}{M^2} \rho(r) \right] - \left(1 + \frac{m}{M}\right)^{-1} c_0 \vec{\nabla} \rho(r) \cdot \vec{\nabla} \right\}. \quad (5)$$

In eqs. (4), (5), $\rho_E(r)$ is the squared nucleon-momentum distribution given in the local density approximation by

$$\rho_E(r) = \frac{3}{5} \left(\frac{3}{2} \pi^2 \right)^{2/3} \rho^{5/3}(r). \quad (6)$$

The new, repulsive ($d_0 = -0.190 \text{ m}^{-3}$) term in eq. (4) vanishes in the limit where the effective nucleon mass $M(r) \rightarrow M$, i.e. in the limit of weak scalar and vector potentials for the bound nucleon. It is due to the use of *in-medium* four spinors, parameterized by the effective mass $M(r)$, that the RIA correction term in eq. (4) arises. We know of no fundamental requirement invalidating it. However, this result is not unique.

The form of T in eq. (2), using a 4×4 scalar I and a four vector γ^μ is neither unique nor particularly reflecting of pion nucleon dynamics. A scalar term A violates the requirement of chirality for pion energy $\omega \rightarrow 0$ and this leads to unnatural relationships that must be imposed on the partial-wave amplitudes of A and B in order to satisfy empirically the near vanishing of b_0 . A more natural choice of extrapolating the pion nucleon dynamics away from on-shell measurements would be to replace T in eq. (2) by

$$T' = C \not{Q} + \frac{i}{2M} D \sigma^{\lambda\mu} k'_\lambda k_\mu, \quad \sigma^{\lambda\mu} = \frac{i}{2} [\gamma^\lambda, \gamma^\mu]. \quad (7)$$

In general one could have any linear combination of I , γ^μ and $\sigma^{\lambda\mu}$ terms. This leaves one coefficient not determined by the free pion nucleon data since in that case they are related by the Gordon decomposition (see eq. (8) below). For the purpose of demonstration we take A to be zero. As noted previously this makes b_0 automatically small without severe cancellations between the different terms.

To express C and D in terms of A and B , write down the Gordon decomposition

$$\bar{u}(p') \gamma^\lambda u(p) = \bar{u}(p') \left[\frac{(p + p')^\lambda}{2M} + \frac{i \sigma^{\lambda\mu} (p' - p)_\mu}{2M} \right] u(p), \quad (8)$$

multiply by Q_λ (cf. eq. (2)) and replace $p' - p$ on shell by $k - k'$:

$$\bar{u}(p') \not{Q} u(p) = \bar{u}(p') \left[\nu + \frac{i \sigma^{\lambda\mu} k'_\lambda k_\mu}{2M} \right] u(p), \quad \nu = \frac{s - u}{4M}. \quad (9)$$

It is then straightforward to obtain

$$A = -\nu D, \quad B = C + D \quad (10)$$

as the necessary and sufficient conditions for $T' = T$ on shell. Applying to T' the same procedure as that applied [6] to T , one obtains the following RIA optical potential

$$2\mu U_{RIA}^{(1)} = 2\mu U_{IA}^{(1)} - 4\pi \frac{m}{2M} d_0 \left[\vec{\nabla} \left(\frac{M}{M(r)} - 1 \right) \cdot \vec{\nabla} \rho(r) \right]. \quad (11)$$

In Fig. 1 we compare the new d_0 term above, essentially a surface term, to the BG d_0 (volume) term of eq. (4) for ^{40}Ca . One expects therefore only a slight insignificant RIA effect by going off shell according to eq. (7) rather than according to eq. (2).

To test one's expectation more quantitatively, we briefly report results of a large scale fit to pionic-atom shifts and widths data, from the 1s state in ^{16}O to the 4f state in ^{209}Bi , 25 states in all. The conventional form of optical potentials [2] used is

$$U = \tilde{U}_{IA}^{(1)} + U^{(2)},$$

$$2\mu U^{(2)} = -4\pi \left(1 + \frac{m}{2M} \right) 4B_0 \rho(r) \rho_n(r) + 4\pi \left(1 + \frac{m}{2M} \right)^{-1} 4C_0 \vec{\nabla} \rho(r) \rho_n(r) \cdot \vec{\nabla} + 4\pi^{2/3} \left(1 + \frac{m}{M} \right) (b_0^2 + 2b_1^2) \left(\frac{3}{2} \rho(r) \right)^{4/3}, \quad (12)$$

where $\tilde{U}_{IA}^{(1)}$ is obtained from $U_{IA}^{(1)}$ of eq. (5) by (i) dropping off the m^2/M^2 term; (ii) introducing isovector terms in the s -wave and p -wave terms:

$$\begin{aligned} b_0 \rho(r) &\rightarrow (b_0 + b_1) \rho_n(r) + (b_0 - b_1) \rho_p(r), \\ c_0 \rho(r) &\rightarrow (c_0 + c_1) \rho_n(r) + (c_0 - c_1) \rho_p(r); \end{aligned} \quad (13)$$

(iii) allowing for the EELL renormalization effect in the p -wave terms:

$$\vec{\nabla} A(r) \cdot \vec{\nabla} \rightarrow \vec{\nabla} \frac{A(r)}{1 + \frac{1}{3} \xi A(r)} \cdot \vec{\nabla}, \quad A(r) = 4\pi [(c_0 + c_1) \rho_n(r) + (c_0 - c_1) \rho_p(r)]. \quad (14)$$

The two-nucleon component $U^{(2)}$ of the optical potential, except for the double-scattering correction (the $\rho^{4/3}$ term [4]), is a purely phenomenological construct to account for absorption ($Im B_0, Im C_0$) and dispersion ($Re B_0, Re C_0$). The common theoretical expectation [5] is that the dispersive effects lead to attraction of a magnitude scaled by the absorption, $Re B_0 \simeq Im B_0$ and $Re C_0 \simeq Im C_0$. The recent J4 potential (Meirav *et al.* [2]), for example, keeps the coefficients b_0, b_1, c_0, c_1 of the one-nucleon terms in $\tilde{U}_{IA}^{(1)}$ as close as possible to the free, on shell values. The parameters varied in the fit, thus, are the five parameters B_0, C_0 (complex ones) and ξ . Varying all or most of the nine parameters of the optical potential (12) would lead to undesired strong correlations between some of the fitted parameters (R. Seki and K. Masutani [2]).

The results of the fit are given in Table 1 under Conventional Potential. Chi-squared per degree of freedom (χ^2/F) about 2-3 represent good fits in this trade. The very small errors for B_0 indicate that it is well determined, yielding a *repulsive* dispersive term,

$ReB_0 \simeq -ImB_0$; even stronger repulsion, $ReB_0 \simeq -4ImB_0$, is found in very recent fits (Konijn *et al.* [3]) to a most extensive (70 states in all) data base. If one tries to bring ReB_0 closer to ImB_0 , the χ^2/F rises very sharply, exceeding the value 20 already for $ReB_0 = 0$. We note that ImB_0 remains well determined also for the other "good" fits of the Table, assuming approximately a constant value. Similarly, ImC_0 shares these qualities with ImB_0 . However, ReC_0 and ξ , as indicated by their larger errors, are correlated with each other. In the subsequent fits, therefore, we chose to constrain one of these. The next two columns in the Table correspond to adding the RIA correction of eq. (4), with d_0 and d_1 , the latter added in analogy with eq. (13). Following BG, we have parameterized $M/M(r) = 1 + a\rho(r)$. ReB_0 now becomes positive (attractive) and of a similar magnitude to ImB_0 , as expected theoretically. ReB_0 and a are correlated with each other as indicated by their errors, and if one imposes $ReB_0 \simeq ImB_0$, then a perfectly good fit ($\chi^2/F = 2.5$) is found with $a = 3.17 \pm 0.13 \text{ fm}^3$, corresponding to an effective mass $M(0)/M = 0.65$ in the nuclear center, in agreement with the thesis of BG [6, 8]. Imposing $ReC_0 \simeq ImC_0$ too, the same fit gives rise to a value $\xi = 1.7 \pm 0.1$ in good agreement with current theoretical expectations (for a recent review of the EELL correction see ref. [9]). The last two columns of the Table correspond to adding the RIA version of eq. (11) (again with d_1 added as above), exemplifying the point of this paper with regard to the ambiguity of the RIA corrections. Comparing the two fits with the constraint $ReB_0 = ReC_0 = 0.055$ in pion-mass units (a fit for each of the RIA versions discussed here), one concludes that the version of eq. (11) is unable to produce a fit to the data ($\chi^2/F > 100$) by failing to account for the s -wave repulsion which the data implies and which BG's version eq. (4) is capable of producing. Removing the constraint on ReB_0 , in the last column of the Table, brings it back to a repulsive value close to that given by the Conventional Potential, whereas a becomes - quite unphysically - negative.

In summary, we have confirmed by performing a large scale pionic-atom fit, and in agreement with the conjecture of BG [6] and their further analysis [8], that the RIA is capable of resolving the missing central isoscalar repulsion in low-energy pion-nuclear interactions. We stress, however, that this RIA to the pion-nucleus optical potential is not constrained by any dynamical principle such as the imposition of the One Boson Exchange underlying dynamics on the RIA construction of the nucleon-nucleus optical potential [10]. In fact the RIA is subject to severe problems with off shell ambiguities. We have shown how a different off-shell relativistic extrapolation yields RIA correction terms which have a negligible effect on low-energy pion-nucleus interactions and, hence, does not resolve the missing repulsion problem. Precisely the same result obtains if the amplitude T is traded off for a new one T'' made out of a Lorentz scalar I and the tensor $\sigma^{\lambda\mu}$ [11]. It is clear that more theoretical work on low energy πN dynamics, in particular for the nonresonant channels (other than the P_{33}), is called for.

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Parameters^{a)} of pion-nucleus potentials as determined from fits to pionic atom data.

	Conventional Potential [eq.(12)]	Relativistic			
		eqs. (2),(4),(12)		eqs. (7),(11),(12)	
$ReB_0(m^{-4})$	-0.063 ± 0.006	<u>0.055</u>	0.094 ± 0.040	<u>0.055</u>	-0.10 ± 0.02
$ImB_0(m^{-4})$	0.058 ± 0.003	0.057 ± 0.003	0.056 ± 0.003	0.029 ± 0.009	0.064 ± 0.005
$a(\text{fm}^3)$	—	3.17 ± 0.13	4.0 ± 1.0	0.9 ± 1.1	-0.85 ± 0.42
$ReC_0(m^{-6})$	0.073 ± 0.043	<u>0.055</u>	0.054 ± 0.009	<u>0.055</u>	0.00 ± 0.03
$ImC_0(m^{-6})$	0.049 ± 0.005	0.051 ± 0.004	0.053 ± 0.004	0.052 ± 0.019	0.064 ± 0.009
ξ	1.9 ± 0.5	1.7 ± 0.1	<u>1.7</u>	3.4 ± 1.5	<u>1.7</u>
χ^2/F	3.3	2.5	2.5	117	2.9

^{a)} underlined quantities were held fixed; the scattering lengths and scattering volumes are fixed everywhere at $b_0 = -0.009 \text{ m}^{-1}$, $b_1 = -0.094 \text{ m}^{-1}$, $c_0 = 0.231 \text{ m}^{-3}$, $c_1 = 0.152 \text{ m}^{-3}$; the values of spin-flip scattering volumes used for the four columns headed "relativistic" are $d_0 = -0.190 \text{ m}^{-3}$, $d_1 = -0.069 \text{ m}^{-3}$.

Fig. 1. Comparison between different RIA corrections to the isoscalar optical potential of pions for ^{40}Ca . The solid curve shows the term $(M^2/M^2(r) - 1)\rho(r)$ of eqs. (4), (6) as appropriate to the RIA version of eq. (2). The dashed curve shows the term $\vec{\nabla} \cdot (M/M(r) - 1) \cdot \vec{\nabla}\rho(r)/2$ of eq. (11) as appropriate to the RIA version of eq. (7). The density $\rho(r)$ is a two parameter Fermi distribution with half radius $c = 3.51 \text{ fm}$ and diffusivity $a = 0.563 \text{ fm}$, and is shown by the dot-dashed curve. The nucleon effective mass $M(r)$ is taken here, for the purpose of demonstration, in the form $M(r) = M(1 - 0.3\rho(r)/\rho(0))$. The units of the ordinate are fm^{-5} for the solid and dashed curves, and fm^{-3} for the dot-dashed curve.

Fig. 1