A.D. /

CERN-PRE 92-012

su 9216

Preprint n.864
February 11, 1992

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Università di Roma "La Sapienza",
I.N.F.N. - Sezione di Roma

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CM-P00063088

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"Improved Renormalization Constants for B Decay and $B\bar{B}$ Mixing"

Improved Renormalization Constants for B Decay and $B\bar{B}$ Mixing

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Abstract

In this paper we discuss the improvement of lattice matrix elements involving heavy quarks. We then perform the new one-loop matching between continuum and improved lattice operators relevant for the evaluation of B meson decay constant and $B\bar{B}$ mixing amplitude.

*Supported by Fondazione Angelo Della Riccia.

1 Introduction

In recent years much theoretical and experimental effort has been devoted to the search for t quark, and to the estimate of its mass, m_t . Being members of the same weak isospin doublet as t , b quarks can be an important channel of information; they also are a possible source for evidence of CP-violation, through measurement of the imaginary part of the Cabibbo-Kobayashi-Maskawa mixing matrix.

The evaluation of the B meson decay constant and $B\bar{B}$ mixing amplitude involves weak matrix elements between hadronic states, which depend non-perturbatively on strong interactions. Although lattice QCD has been successfully used for the evaluation of several non-perturbative quantities, the case of B meson requires a different approach. In present lattice calculation the typical range of values for the inverse lattice spacing is $a^{-1} \simeq 2 \div 3$ GeV; thus, owing to the fact that m_b is of the order of 5 GeV, b quarks are too heavy to propagate on such lattices, and cannot be treated in the same way as lighter ones.

A solution to this problem has been suggested in ref.[1] and then developed by Eichten and Hill [2], who proposed an effective theory in which m_b is considered infinite, and thus it is analytically removed from the lattice action. The use of the expansion in inverse power of the heavy quark mass has drawn much attention in successive works [3].

To compare the results of lattice simulations involving heavy quarks to the experimental values, it is necessary to relate lattice operators in the effective theory to the corresponding renormalized continuum ones. The renormalization constants calculated with the effective lattice lagrangian for b -quarks and with light quarks treated as Wilson fermions, have been computed at one-loop in ref.[4, 5, 6] for B decay and in ref.[7] for $B\bar{B}$ mixing.

Lattice simulations, however, are affected by systematic errors due to the finiteness of the lattice spacing. A practical method to reduce the cut-off dependence of the lattice hadronic matrix elements involving Wilson fermions has been presented in refs.[8, 9]. In this paper we apply the method of improvement to lattice matrix elements involving heavy quarks and we evaluate at one-loop the improved lattice renormalization constants of the relevant two and four-fermion operators.

We will consider the renormalization of the generic heavy-light bilinear

$$O_\Gamma = \bar{b}\Gamma q, \quad (1)$$

where Γ is one of the 16 Dirac matrices. We apply our results to the interesting case of the time component of the axial current, which is used to compute on the lattice the pseudoscalar B meson decay constant f_B , defined as:

$$\langle 0 | O_{\gamma_\mu \gamma_5} | B^0 \rangle = i p_\mu f_B, \quad (2)$$

where p_μ is the B meson momentum. We will also consider the renormalization of the $\Delta B = 2$ four-fermion operator

$$O_L = \frac{1}{2} \bar{b} \gamma_\mu^L q \bar{b} \gamma_\mu^L q, \quad (3)$$

where we have defined

$$\gamma_\mu^L = \gamma_\mu \frac{(1 - \gamma_5)}{2}. \quad (4)$$

This operator enters in the effective hamiltonian for $B\bar{B}$ mixing, whose amplitude is usually expressed in terms of the B -parameter, defined as

$$\langle \bar{B}^0 | O_L | B^0 \rangle = \frac{8}{3} B_B M_B^2 f_B^2, \quad (5)$$

where M_B is the B -meson mass.

The main results of this work - which have already been presented in [10] - are the following: at the values $\beta = 6.0$, i.e. $\alpha_s^{LATT} = 0.080^1$

$$\mu = a^{-1} = 2 \text{ GeV}, \quad m_b = 5 \text{ GeV}, \quad r = 1 \quad (6)$$

and using the one-loop continuum value of the strong coupling constant $\alpha_s = 0.33$, corresponding to $\Lambda_{QCD} = 200 \text{ MeV}$ for four active quarks, we find that the relation between the value of f_B measured on such a lattice, using the improved theory, and the physical value is

$$f_B^{PHYS} = (0.90) f_B^{LATT}. \quad (7)$$

¹Recently the possibility of large failures of perturbation theory resulting from the use of the bare lattice coupling constant as an expansion parameter, has been investigated [11].

In the unimproved case the corresponding factor would be (0.85). In eq.(6): $\beta = 6/g_0^2$, g_0^2 is the squared lattice coupling constant, μ is the continuum renormalization point and r is the Wilson parameter.

For the combination $f_B^2 B_B$, neglecting the mixing with operators with different chirality, the value computed with the same lattice parameters, is related to the physical one by

$$(f_B^2 B_B)^{PHYS} = (0.78)(f_B^2 B_B)^{LATT}; \quad (8)$$

in the unimproved case the factor would be (0.68). For the correction to the B_B parameter alone, we find

$$B_B^{PHYS} = (0.98)B_B^{LATT}; \quad (9)$$

in the unimproved case the factor would be (0.99).

In our calculations we assume that $\log(m_b a)$ is not a “large logarithm”, so that the two-loop corrections [12] are small. For a recent two-loop calculation of the anomalous dimension of the axial current, see [13].

The paper is organized as follows. In sect.2 we discuss the method of improvement in general. In sect.3 we briefly review the $1/m_b$ expansion and the effective field theory. In sect.4 we show how to apply the improvement to matrix elements involving heavy quarks; we explicitly show in appendix A that on-shell matrix elements of improved two-fermion heavy-light operators at one-loop do not have order a corrections. We present in sect.5 and 6 our results respectively for two and four-fermion heavy-light operators, comparing them with previous calculations; technical details and analytic formulas will be given in appendix B.

2 Improvement for Light Quarks

In ref.[8, 9] it has been shown that it is possible to eliminate all terms which are effectively of order a , from lattice hadronic matrix elements with Wilson fermions. In perturbation theory, since in the scaling limit the squared lattice coupling constant g^2 is proportional to $1/\log a$, these are all the terms of the form $(g^2)^n a \log^n a$. For Monte Carlo simulations, the practical method to be used can be summarized as follows: one must work with the improved action

with nearest-neighbour interactions only [14]

$$S_F^I = S_F^W - a^4 \sum_{x,\mu\nu} [ig \frac{ar}{4} \bar{\psi}(x) \sigma_{\mu\nu} P_{\mu\nu}(x) \psi(x)], \quad (10)$$

where S_F^W is the “standard” Wilson action:

$$\begin{aligned} S_F^W = & a^4 \sum_{x,\mu} \left\{ -\frac{1}{2a} [\bar{\psi}(x)(r - \gamma_\mu)U_\mu(x)\psi(x + \mu) + \right. \\ & + \bar{\psi}(x + \mu)(r + \gamma_\mu)U_\mu^\dagger(x)\psi(x)] \} \\ & + a^4 \sum_x [\bar{\psi}(x)(m_0 + \frac{4r}{a})\psi(x)]; \end{aligned} \quad (11)$$

m_0 is the bare light fermion mass and $P_{\mu\nu}$ is the lattice definition of the continuum field strength tensor. We define [15]:

$$P_{\mu\nu} \equiv \frac{1}{4a^2} \sum_{i=1}^4 \frac{1}{2ig} (U_i - U_i^\dagger), \quad (12)$$

where the sum is over the four plaquettes in the $\mu - \nu$ plane, stemming from the point x and taken counterclockwise. At the same time, the quark fields in fermion operators must undergo the transformation²

$$\begin{aligned} \psi & \rightarrow \psi' = \left(1 - a \frac{r}{2} \overrightarrow{D}\right) \psi, \\ \bar{\psi} & \rightarrow \bar{\psi}' = \bar{\psi} \left(1 + a \frac{r}{2} \overleftarrow{D}\right), \end{aligned} \quad (13)$$

where the lattice covariant derivative is defined as:

$$\begin{aligned} a \overrightarrow{D}_\mu \psi(x) &= \frac{1}{2} [U_\mu(x) \psi(x + \mu) - U_\mu^\dagger(x - \mu) \psi(x - \mu)], \\ a \bar{\psi}(x) \overleftarrow{D}_\mu &= \frac{1}{2} [\bar{\psi}(x + \mu) U_\mu^\dagger(x) - \bar{\psi}(x - \mu) U_\mu(x - \mu)]. \end{aligned} \quad (14)$$

Thus, for the two-fermion operator of the kind

$$O_\Gamma = \bar{\psi} \Gamma \psi, \quad (15)$$

²There is actually a full family of transformations, that can be described by an arbitrary real parameter z discussed in detail in ref.[16, 17], which are equivalent on-shell. In this paper we consider only the transformation defined in eq.(13).

the corresponding improved operator is defined as [16]

$$O_{\Gamma}^I = O_{\Gamma} + a \frac{r}{2} \bar{\psi}(x) \overleftarrow{\mathcal{D}} \Gamma \psi(x) - a \frac{r}{2} \bar{\psi}(x) \Gamma \overrightarrow{\mathcal{D}} \psi(x). \quad (16)$$

Analogously, for the four-fermion operators

$$O_{\pm} = \frac{1}{2} [(\bar{\psi}_1 \gamma_{\mu}^L \psi_2)(\bar{\psi}_3 \gamma_{\mu}^L \psi_4) \pm (2 \leftrightarrow 4)], \quad (17)$$

the improved operator is [17]

$$\begin{aligned} O_{\pm}^I &= \frac{1}{2} \{ (L_{12} \times L_{34}) + [(L_{12}^{\otimes} \times L_{34}) \\ &\quad + (L_{12} \times L_{34}^{\otimes})] \pm (2 \leftrightarrow 4) \}. \end{aligned} \quad (18)$$

To shorten the notation, in eq.(18) we have defined

$$L_{12} \equiv \bar{\psi}_1(x) \gamma_{\mu}^L \psi_2(x) \quad (19)$$

and the “rotated” operator is defined as

$$L_{12}^{\otimes} \equiv \frac{ar}{2} \bar{\psi}_1(x) (\overleftarrow{\mathcal{D}} \gamma_{\mu}^L - \gamma_{\mu}^L \overrightarrow{\mathcal{D}}) \psi_2(x). \quad (20)$$

On-shell lattice matrix elements of the improved operators (16) and (18) do not have effective $O(a)$ corrections.

3 The Effective Field Theory for b Quarks

Non-perturbative hadronic matrix elements can be estimated by numerical lattice calculations. As already mentioned in the introduction, in the case of f_B and B_B , since

$$m_b \gg a^{-1}, \quad (21)$$

to treat b quarks on the lattice it is necessary to use an approach different from the usual one.

We will only consider the renormalization of the operators in eqs.(1) and (3) at lowest order in $1/m_b$. At this order in the continuum theory, expanding the heavy quark propagator in powers of $1/m_b$ leads to [1]

$$S_b(\vec{x}, t) = P \begin{pmatrix} t \\ 0 \end{pmatrix} \delta(\vec{x}) \left[\Theta(t) e^{-m_b^0 t} \frac{1 + \gamma_0}{2} + \Theta(-t) e^{m_b^0 t} \frac{1 - \gamma_0}{2} \right], \quad (22)$$

where m_b^0 is the bare b -quark mass and

$$P \begin{pmatrix} t \\ 0 \end{pmatrix} = T \exp \left[ig \int_0^t dt' A_0^a(0, t') \tau^a \right]. \quad (23)$$

A_0^a is the time component of an external gauge field, T denotes time ordering and τ^a are the color matrices normalized by $\text{tr}(\tau^a \tau^b) = \delta^{ab}/2$. Except for the factor $e^{-m_b^0 t}$ ($e^{m_b^0 t}$), the heavy quark (antiquark) has been replaced by a static color source. Eichten and Hill [2] have proposed an effective action from which the propagator (22) can be obtained. A fixed momentum $p_\mu \equiv (m_b^0, \vec{0})$ is subtracted from the momentum of the heavy quark in order to eliminate the remaining trivial dependence on m_b^0 . In this theory, heavy quarks and anti-quarks are considered as independent fields, because at infinite m_b , $b\bar{b}$ pair creation cannot happen. In the 4-component formalism the heavy quark spinors have the form

$$b = \begin{pmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{pmatrix} \quad (24)$$

and are projected out by $(1 + \gamma_0)/2$. The antiquark spinors are the row vectors

$$(0 \quad 0 \quad \bar{b} = (\bar{b}_1 \quad \bar{b}_2)) \quad (25)$$

and are projected out by $(1 - \gamma_0)/2$. The continuum effective lagrangian for b quarks in the Euclidean space-time is

$$L_b = b^\dagger (i\partial_0 + gA_0) b \quad (26)$$

and the one for anti-quarks is

$$L_{\bar{b}} = \bar{b} (i\partial_0 + gA_0) \bar{b}^\dagger. \quad (27)$$

Using this formalism, we can treat heavy quarks as 2-component spinors, as long as we remember that γ_0 applied to a b field is $+1$ and to a $\bar{b}$ field is -1 .

On the lattice we use the same discretized actions as in ref.[4]

$$\begin{aligned} S_b &= ia^3 \sum_n b^\dagger(n) \left[b(n) - U_0^\dagger(n - \hat{0}) b(n - \hat{0}) \right], \\ S_{\bar{b}} &= ia^3 \sum_n \bar{b}(n) \left[\bar{b}^\dagger(n) - U_0(n - \hat{0}) \bar{b}^\dagger(n - \hat{0}) \right], \end{aligned} \quad (28)$$

where $\hat{0}$ is the unit versor of the time component. With this choice the free heavy quark propagator in momentum space is

$$\frac{1}{\frac{-i}{a}(e^{ip_0 a} - 1) + i\epsilon}. \quad (29)$$

One uses the definition (28) to avoid the doubling problem, that would occur with a symmetric derivative.

4 Improvement for Heavy Quarks

It is possible to show that for order a improvement of on-shell matrix elements, no modification of the static quark propagator is needed [18]. Let us consider the usual lattice heavy quark propagator [1] for $t > 0$ in the static approximation

$$S_b^{LAT}(\vec{x}, t) = \frac{1 + \gamma_0}{2} H(\vec{x}, t), \quad (30)$$

where we have defined

$$H(\vec{x}, t) = P_{LAT} \begin{pmatrix} t \\ 0 \end{pmatrix} \delta(\vec{x}) \theta(t) \quad (31)$$

and

$$P_{LAT} \begin{pmatrix} t \\ 0 \end{pmatrix} = U_0(t) U_0(t-1) \dots U_0(1). \quad (32)$$

The quantity $H(\vec{x}, t)$ is the solution of the discretized differential equation

$$\frac{i}{a} \left[H(\vec{x}, t) - U_0^\dagger(\vec{x}, t-1) H(\vec{x}, t-1) \right] = \delta(\vec{x}). \quad (33)$$

This equation differs from the continuum one by terms of order a , as can be seen from the free case; in momentum space we find

$$H_0^{-1}(p_0) = \frac{-i}{a} (e^{ip_0 a} - 1) \sim p_0 + \frac{i}{2} p_0^2 a + O(a^2). \quad (34)$$

In order to eliminate the order a terms, we can add next-to-nearest neighbour interactions and modify eq.(33) as follows

$$\begin{aligned} \frac{1}{a} [H(\vec{x}, t) &- \frac{4}{3} U_0^\dagger(\vec{x}, t-1) H(\vec{x}, t-1) \\ &+ \frac{1}{3} U_0^\dagger(\vec{x}, t-2) H(\vec{x}, t-2)] = \frac{2}{3} \delta(x). \end{aligned} \quad (35)$$

In this case we get

$$H_0^{-1}(p_0) = \frac{-i}{a} (2e^{ip_0 a} - \frac{1}{2}e^{2ip_0 a} - \frac{3}{2}) \sim p_0 + O(a^2). \quad (36)$$

In this way eq.(35) and the corresponding improved free propagator coincide with the continuum up to terms of order a . The key observation is that the solution of eq.(35) is of the form

$$H(\vec{x}, t) = P_{LATT} \begin{pmatrix} t \\ 0 \end{pmatrix} \delta(\vec{x}) \theta(t) f(t), \quad (37)$$

where $f(t)$ tends exponentially to 1 for $t \rightarrow \infty$. We thus conclude that, for on-shell matrix elements, i.e. $t \rightarrow \infty$, the improved heavy quark propagator coincides with the unimproved one.

On the other end, in the effective field theory formalism, we show explicitly in appendix A that, following the prescriptions of sect.3 for light quarks only, without modifying the heavy sector of the theory, the on-shell matrix elements of any heavy-light bilinear do not have effective order a contributions at one-loop.

Summing up, in order to eliminate order a terms from on-shell matrix elements of the heavy-light operators of eqs.(1) and (3), the recipe is the following:

- i) use the nearest-neighbour improved action, eq.(10), for light quarks;
- ii) use the usual effective actions, eq.(28), for heavy quarks;

iii) “rotate” only the light-quark fields appearing in the operators according to eq.(13), i.e. use the improved operators

$$O_{\Gamma}^I = O_{\Gamma} - a \frac{r}{2} (\bar{b} \Gamma \vec{D} q) \quad (38)$$

and

$$\begin{aligned} O_L^I &= O_L - a \frac{r}{2} \left[\frac{1}{2} (\bar{b} \gamma_{\mu}^L \vec{D} q) (\bar{b} \gamma_{\mu}^L q) \right. \\ &\quad \left. + \frac{1}{2} (\bar{b} \gamma_{\mu}^L q) (\bar{b} \gamma_{\mu}^L \vec{D} q) \right]. \end{aligned} \quad (39)$$

5 Renormalization of Improved Two-Fermion Heavy-Light Operators

In this section we present the one-loop perturbative calculation of the lattice renormalization constants of the improved heavy-light bilinears defined in eq.(38). Following refs.[2] and [4], we perform the calculation in two steps, first relating the generic heavy-light bilinear in the continuum full theory, eq.(1), to the corresponding operator in the effective theory

$$O_{\Gamma}^{\text{BFP}} = b^{\dagger} (1 \quad 0) \Gamma q, \quad (40)$$

which has the same tree level matrix element between an incoming light quark and an outgoing heavy quark. The second step is the matching of the continuum effective theory to the lattice effective one.

All the continuum quantities have already been calculated and the result of the first step matching, in the $\overline{MS}$ renormalization scheme and with the definition of always anticommuting γ_5 , is [2]

$$O_{\Gamma} = \left\{ 1 + \frac{g^2}{16\pi^2} C_F \left[\left(\frac{1}{4} H^2 - \frac{5}{2} \right) \log \frac{\mu^2}{m_b^2} - \frac{1}{2} H G + \frac{3}{4} H^2 - H H' - 4 \right] \right\} O_{\Gamma}^{\text{BFP}}, \quad (41)$$

where

$$C_F = \frac{N^2 - 1}{2N} = \frac{4}{3}. \quad (42)$$

We use the same notations of ref.[2], defining

$$\begin{aligned} H\Gamma &= \gamma_\mu \Gamma \gamma_\mu, \\ G\Gamma &= \gamma_0 \Gamma \gamma_0. \end{aligned} \quad (43)$$

H' is the derivative of H with respect to d in d dimensions. As for the second step, for the unimproved case we obtain, in agreement with ref.[4]

$$O_\Gamma^{\text{BFF}} = \left\{ 1 + \frac{g^2}{16\pi^2} C_F \left[\frac{3}{2} \log \mu^2 a^2 + \frac{5}{4} - A_\Gamma \right] \right\} O_\Gamma^{\text{LATT}}, \quad (44)$$

where the constant A_Γ , expressed in terms of the quantities defined in ref.[4], is (see also appendix B)

$$A_\Gamma = d_1 + d_2 G + \frac{1}{2}(e + f). \quad (45)$$

In the improved case, we must first consider the diagrams in fig.1 with improved Feynman rules for the light quark-gluon vertex. Beside these diagrams we have to compute those of fig.2, corresponding to the “rotated” term of the operator³. Summing all the contributions, the result of the second step in the improved case is

$$O_\Gamma^{\text{BFF}} = \left[1 + \frac{g^2}{16\pi^2} C_F \left(\frac{3}{2} \log \mu^2 a^2 + \frac{5}{4} - A_\Gamma - A_\Gamma^I \right) \right] (O_\Gamma^I)^{\text{LATT}}. \quad (46)$$

The analytical expression of the new constant A_Γ^I is given in appendix B.

In the interesting case of the time component of the axial current, used to determine f_B , at $\mu = a^{-1}$, we obtain

$$\begin{aligned} O_{\gamma_0 \gamma_5} &= \left\{ 1 + \frac{\alpha_S^{\text{CONT}}(a^{-1})}{3\pi} \left[-\frac{3}{2} \log \frac{\mu^2}{m_b^2} - 2 \right] \right. \\ &\quad \left. + \frac{\alpha_S^{\text{LATT}}(a^{-1})}{3\pi} \left[\frac{5}{4} - A_{\gamma_0 \gamma_5} - A_{\gamma_0 \gamma_5}^I \right] \right\} (O_{\gamma_0 \gamma_5}^I)^{\text{LATT}}. \end{aligned} \quad (47)$$

Eq.(47) contains the contributions of the two steps and gives the final relation between lattice effective and continuum full theories.

We have computed numerically the constants $A_{\gamma_0 \gamma_5}$ and $A_{\gamma_0 \gamma_5}^I$ with an error of less than 5% and their numerical value, as a function of the Wilson parameter r , is given in table 1.

³In fig.2, 4, 5, with the symbol $\otimes$ we denote the $-a \frac{r}{2} \Gamma \overrightarrow{D}$ insertion.

r	1.00	0.75	0.50	0.25	0
$A_{\gamma_0\gamma_b}$	31.60	31.21	30.65	29.36	24.30
$A_{\gamma_0\gamma_b}^I$	-5.23	-5.30	-4.77	-3.07	0

Table 1: Numerical values of the finite renormalization constants $A_{\gamma_0\gamma_b}$ and $A_{\gamma_0\gamma_b}^I$ for several values of the Wilson parameter r .

Usually [19], f_B is extracted from numerical simulations by fitting

$$\langle \text{tr} \left[\frac{1-\gamma_0}{2} S_q(\vec{0}, t) U_0(t) U_0(t-1) \dots U_0(1) \right] \rangle_{\text{LATT}} = C \exp(-\Delta E_B t) \quad (48)$$

where C is a constant, while ΔE_B contains linear divergencies

$$\lim_{a \rightarrow 0} a \Delta E_B \simeq -X. \quad (49)$$

In perturbation theory the constant X in eq.(49) coincides with the coefficient of the linearly divergent term in the self-energy, $\Sigma(p_0)$, of the heavy quark (including the tad-pole)

$$X = ai\Sigma(p_0 = 0) = \frac{g^2}{16\pi^2} C_F i\Sigma_0, \quad (50)$$

where

$$i\Sigma_0 = (19.95). \quad (51)$$

For the analytical expression of $i\Sigma_0$ see appendix B. When using a fit of the form (48), the heavy quark self-energy contributes to $A_{\gamma_0\gamma_b}$ with a reduced value - see [4, 6]

$$e^{(R)} = e - i\Sigma_0 = 4.53. \quad (52)$$

In this case, as anticipated in the introduction, we find that at $\beta = 6.0$, i.e. $\alpha_s^{\text{LATT}} = 0.080$,

$$\mu = a^{-1} = 2 \text{ GeV}, \quad m_b = 5 \text{ GeV}, \quad r = 1 \quad (53)$$

and using the one-loop continuum values of the strong coupling constant $\alpha_s^{\text{CONT}} = 0.33$, corresponding to $\Lambda_{QCD} = 200 \text{ MeV}$ for four active quarks, the lattice value of f_B is related to the physical one by

$$f_B^{\text{PHYS}} = Z_A f_B^{\text{LATT}}, \quad (54)$$

where

$$Z_A = 0.90. \quad (55)$$

6 Renormalization of Improved Four-Fermion Operator $\Delta B = 2$

We now consider the four-fermion operator of eq.(3)

$$O_L = \frac{1}{2} (\bar{b} \gamma_\mu^L q) (\bar{b} \gamma_\mu^L q). \quad (56)$$

To be definite we focus on the matrix element

$$\langle \bar{B}^0 | O_L | B^0 \rangle, \quad (57)$$

with a B^0 ($\bar{b}q$) in the initial state and a $\bar{B}^0$ ($\bar{q}b$) in the final one. As in sect.5, we proceed in two steps: the first one is the matching between continuum full and continuum effective theory; again, the result of this first step is the same as in previous calculations [7] (we use the same notation)

$$O_L^{\text{FULL}} = \left[1 + \frac{g^2}{16\pi^2} \left(-6 \log \frac{\mu^2}{m^2} + C_L \right) \right] O_L^{\text{EFF}} + \frac{g^2}{16\pi^2} C_S O_S^{\text{EFF}}, \quad (58)$$

where

$$O_S = \frac{1}{2} (\bar{b} P_L q) (\bar{b} P_L q) \quad (59)$$

is a new operator that mixes with O_L at one-loop; we define

$$P_{R,L} = \frac{1 \pm \gamma^5}{2}. \quad (60)$$

The constants in eq.(58) are

$$C_L = -14, \quad C_S = -8. \quad (61)$$

The continuum effective operators are defined as

$$O_L^{\text{EFF}} = (\bar{b}^\dagger (1, 0) \gamma_L^\mu q) (\bar{b} (0, 1) \gamma_L^\mu q) \quad (62)$$

and

$$O_S^{\text{BFF}} = (b^\dagger (1, 0) P_L q) (\bar{b} (0, 1) P_L q). \quad (63)$$

In reproducing this result, special care must be taken of the renormalization convention used; we follow the choice of ref.[7] and work in the $\overline{MS}$ naive dimensional renormalization scheme, with fully anticommuting γ_5 . Note, however, that Fierz transformations are not well defined for $d \neq 4$: for the correct projection of divergent diagrams on physical operators, see ref.[20].

As we have seen, new operators appear in the continuum when we renormalize O_L . The same happens on the lattice, where we find the mixing with the operators

$$O_R = \frac{1}{2} (\bar{b} \gamma_\mu^R q) (\bar{b} \gamma_\mu^R q) \quad (64)$$

and

$$\begin{aligned} O_N = & \frac{1}{2} [(\bar{b} \gamma_\mu^R q) (\bar{b} \gamma_\mu^L q) + (\bar{b} \gamma_\mu^L q) (\bar{b} \gamma_\mu^R q) \\ & + 2 (\bar{b} P_R q) (\bar{b} P_L q) + 2 (\bar{b} P_L q) (\bar{b} P_R q)]. \end{aligned} \quad (65)$$

In the unimproved case the graphs considered are those in fig.3; the result, in agreement with ref.[7], is

$$\begin{aligned} O_L^{\text{BFF}} = & \left[1 + \frac{g^2}{16\pi^2} (4 \log \mu^2 a^2 + D_L) \right] O_L^{\text{LATT}} \\ & + \frac{g^2}{16\pi^2} D_N O_N^{\text{LATT}} + \frac{g^2}{16\pi^2} D_R O_R^{\text{LATT}}; \end{aligned} \quad (66)$$

D_L, D_R, D_N , are the same constants defined in ref.[7] (see also appendix B).

In the improved theory, we compute the same diagrams calculated with the improved light quark-gluon vertex. Along with these, we have new graphs (figs.4 and 5) corresponding to the rotated part of the operator. The result for the improved lattice operator corresponding to the continuum effective O_L^{BFF} is:

$$\begin{aligned} O_L^{\text{BFF}} = & \left[1 + \frac{g^2}{16\pi^2} (4 \log \mu^2 a^2 + D_L + D_L^I) \right] (O_L^I)^{\text{LATT}} \\ & + \frac{g^2}{16\pi^2} (D_N + D_N^I) O_N^{\text{LATT}} + \frac{g^2}{16\pi^2} (D_R + D_R^I) O_R^{\text{LATT}}. \end{aligned} \quad (67)$$

r	1.00	0.75	0.50	0.25	0
D_L	-65.50	-63.80	-62.00	-59.80	-58.10
D_N	-14.40	-14.50	-14.00	-11.40	0
D_R	-1.61	-1.77	-1.92	-1.59	0
D_L^I	16.25	13.73	10.18	4.91	0
D_N^I	0.44	2.22	3.48	3.36	0
D_R^I	-2.58	-2.27	-1.72	-0.82	0

Table 2: Numerical values of the finite renormalization constants $D_{L,R,N}$ and $D_{L,R,N}^I$ for several values of the Wilson parameter r .

Now, adding the results of the two steps, we obtain the lattice effective operator corresponding to O_L in the continuum full theory; by choosing $\mu = a^{-1}$ we get

$$\begin{aligned}
O_L = & \left[1 + \frac{\alpha_S^{\text{CONT}}(a^{-1})}{4\pi} \left(-6 \log \frac{\mu^2}{m^2} + C_L \right) \right. \\
& + \frac{\alpha_S^{\text{LATT}}(a^{-1})}{4\pi} (D_L + D_L^I) \left. \right] (O_L^I)^{\text{LATT}} + \frac{\alpha_S^{\text{LATT}}(a^{-1})}{4\pi} C_S O_S^{\text{LATT}} \quad (68) \\
& + \frac{\alpha_S^{\text{LATT}}(a^{-1})}{4\pi} (D_N + D_N^I) O_N^{\text{LATT}} + \frac{\alpha_S^{\text{LATT}}(a^{-1})}{4\pi} (D_R + D_R^I) O_R^{\text{LATT}}.
\end{aligned}$$

Numerical values of D_L , D_N , D_R and of D_L^I , D_N^I , D_R^I , as a function of the Wilson parameter r , can be found in table 2. For their analytical form, and for the discussion of the various diagrams, see appendix B.

We use the reduced value $e^{(R)} = 4.53$ - see eq.(52) and discussion thereof - for the heavy quark self-energy and we choose $\beta = 6.0$, i.e. $\alpha_S^{\text{LATT}} = 0.080$,

$$\mu = a^{-1} = 2 \text{ GeV}, \quad m_b = 5 \text{ GeV}, \quad r = 1, \quad (69)$$

using the one-loop continuum value of the strong coupling constant $\alpha_S^{\text{CONT}} = 0.33$, corresponding to $\Lambda_{QCD} = 200 \text{ MeV}$ for four active quarks. In such a way, as anticipated in the introduction, we find that the coefficient of $(O_L^I)^{\text{LATT}}$ is (0.78). Neglecting the contribution of operators with different chirality, this same factor relates the lattice value of $f_B^2 B_B$ to the physical one.

To evaluate the correction factor for the B_B parameter alone, we subtract the correction to f_B^2 , and obtain (0.98).

7 Acknowledgements

We warmly thank G. Martinelli and G. C. Rossi for many useful suggestions and for help during all this work. We also thank M. Testa for discussions. One of us (A.B.) wishes to thank the TH-Division of Cern for kind hospitality.

8 Appendix A

In this appendix we show explicitly, at one-loop, that using the improved lattice prescriptions described at the end of sect.4, on-shell matrix elements of bilinear heavy-light operators (38) do not have terms of order a or $g^2 a \log a$.

The on-shell matrix elements of a generic heavy-light operator O_Γ is obtained by adding all the diagrams contributing to the irreducible vertex correction Λ_Γ and multiplying it by

$$\sqrt{Z_2^q Z_2^b} \Lambda_\Gamma(p'_0, p), \quad (70)$$

where Z_2^q is the residue of the pole of the propagator of the improved light quark, and Z_2^b that of the heavy quark.

The on-shell conditions that we use in calculating (70) are

$$p'_0 = 0 \quad (71)$$

for the heavy quark and

$$i \not{p} = -m_R \quad (72)$$

for the light quark; here the renormalized light quark mass is explicitly kept different from zero. For dimensional reasons, only graphs that involve light quarks in the loop give rise on-shell to terms of order $a \log a$. The relevant calculations for light quarks in the improved theory have been done in ref.[8]. We are not interested here in all the terms of order $(g^2 \text{const.})$ and $(ag^2 \text{const.})$, thus keeping only the logarithmically divergent terms, one gets

$$Z_2^q = (1 - g^2 C_F L)(1 - a r m_0) \quad (73)$$

where m_0 is the “bare” mass of the light quark⁴ and L is a logarithmically divergent term for $a \rightarrow 0$

$$L = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2(q^2 + \lambda^2 a^2)} = -\frac{1}{16\pi^2} \log \lambda^2 a^2 + \text{const.}, \quad (74)$$

where λ is the infra-red regulator. For the heavy quark we find

$$Z_2^b = 1 + 2g^2 C_F L, \quad (75)$$

As for the contributions of vertex diagrams, using the on-shell conditions (71,72), we get

$$\text{fig.}(1.a) \longrightarrow g^2 C_F \Gamma \left[1 - a \frac{r}{2} (i \not{p} + m_0) \right] L \quad (76)$$

$$\text{fig.}(2.a) \longrightarrow \Gamma \left[-a \frac{r}{2} i \not{p} + g^2 C_F a \frac{r}{2} m_0 L \right] \quad (77)$$

$$\text{fig.}(2.b) \longrightarrow g^2 C_F \Gamma \left[-a \frac{r}{2} i \not{p} - 2a r m_0 \right] L \quad (78)$$

On-shell, noting that, at order g^2 , the relation $g^2 m_0 = g^2 m_R$ hold, we have

$$\Lambda_\Gamma = \Gamma \left[1 + a \frac{r}{2} m_R + g^2 C_F L + g^2 C_F (-a r m_R) L \right]. \quad (79)$$

Finally, using the relation [8]

$$m_R = m \left(1 + 3g^2 C_F L \right) \quad (80)$$

we obtain for eq.(70)

$$\sqrt{Z_2^q Z_2^b} \Lambda_\Gamma = \Gamma \left(1 + \frac{3}{2} g^2 C_F L \right) \quad (81)$$

where the leading logs are the same as in the continuum effective theory, and we see that there are no terms of order $g^2 a \log a$.

⁴We neglect here the g^2/a divergent term coming from the lattice light self-energy, since it does not give rise to additional logarithmically divergent contributions.

9 Appendix B

In this appendix we will discuss with more detail the new graphs we have computed, and give the analytical expressions of their amplitudes and their numerical values as a function of the Wilson parameter r . For convenience, we will define the following quantities, which have been introduced in ref.[7]

$$\begin{aligned}
\Delta_1 &= \sum_{\mu} \sin^2 \frac{q_{\mu}}{2}, \\
\Delta_2 &= \sum_{\mu} \sin^2 q_{\mu} + 4r^2 (\Delta_1)^2 \\
\Delta_4 &= \sum_{\mu} \sin^2 q_{\mu} \\
\Delta_5 &= \sum_{\mu} \sin^2 q_{\mu} \sin^2 \frac{q_{\mu}}{2}. \\
\Delta_6 &= \sum_{\mu} \cos^4 \frac{q_{\mu}}{2} \\
\Delta_7 &= \sum_{\mu} \sin^2 \frac{q_{\mu}}{2} \cos^6 \frac{q_{\mu}}{2}.
\end{aligned} \tag{82}$$

Similar quantities, which are summed on $j = 1, 2, 3$, instead of $\mu = 0, 1, 2, 3$, will be indicated with superscript (3). Let us consider the lattice renormalization of the improved two-fermion heavy-light operators O_{Γ}^I , eq.(38). The contributions of the diagrams of fig.1 to the lattice renormalization constant are

$$\text{fig.(1.a)} \longrightarrow \frac{g^2}{16\pi^2} C_F \left[-\log \lambda^2 a^2 + d_1 + (d_2 - d^I)G \right] \Gamma, \tag{83}$$

(for the definition of $G\Gamma$ see eq.(43) of the text)

$$\text{fig.(1.b)} \longrightarrow \frac{g^2}{16\pi^2} C_F \left[-\log \lambda^2 a^2 + \frac{1}{2}e \right] \Gamma, \tag{84}$$

$$\text{fig.(1.c)} \longrightarrow \frac{g^2}{16\pi^2} C_F \left[\frac{1}{2} \log \lambda^2 a^2 + \frac{1}{2}(f + f^I) \right] \Gamma. \tag{85}$$

In eqs.(83,84,85) the quantities d_1, d_2, e and f are the same of ref.[4]; for their analytical expressions see also the appendix of ref.[7]. The new quantity d^I

is defined as

$$d^I = (-16\pi^2) \frac{r}{4} \int \frac{d^3 q}{(2\pi)^3} \frac{\Delta_4^{(3)}}{4\Delta_1^{(3)}\Delta_2^{(3)}}. \quad (86)$$

The quantity f^I , coming from the improved light quark self-energy (see also ref.[17]), is defined as

$$\begin{aligned} f^I = & (16\pi^2) \int \frac{d^4 q}{(2\pi)^4} \left\{ \frac{1}{4\Delta_1\Delta_2} \left[\frac{r^2}{2} (\Delta_1(4-\Delta_1)^2 \right. \right. \\ & - \Delta_1\Delta_6 + \frac{1}{4}\Delta_1\Delta_4 - \Delta_1^2(4-\Delta_1) \\ & - \Delta_4(4-\Delta_1) + (\Delta_4-\Delta_5) + \frac{1}{4}\Delta_1\Delta_4(4-\Delta_1) \\ & - \frac{1}{4}\Delta_1(\Delta_4-\Delta_5) + \frac{1}{4}(\Delta_4-\Delta_5)(4-\Delta_1) \\ & - \frac{1}{4}\Delta_4(4-\Delta_1)^2 + \frac{1}{4}\Delta_4\Delta_6 - \Delta_7) \\ & \left. \left. + \frac{r^2}{2\Delta_1} \left(\frac{1}{16}\Delta_4^2(4-\Delta_1) - \frac{1}{16}\Delta_4(\Delta_4-\Delta_5) \right) \right] \right\} \quad (87) \end{aligned}$$

The numerical values of d^I and f^I as functions of r are given in the first and second row of table 3.

From the diagrams of fig.2 we get

$$\text{fig.(2.a)} \longrightarrow \frac{g^2}{16\pi^2} C_F \left[-(d^I + q)G + n \right] \Gamma, \quad (88)$$

$$\text{fig.(2.b)} \longrightarrow \frac{g^2}{16\pi^2} C_F \left[-(l+m) \right] \Gamma, \quad (89)$$

$$\text{fig.(2.c)} \longrightarrow \frac{g^2}{16\pi^2} C_F (hG) \Gamma. \quad (90)$$

The analytical expressions of these new quantities are:

$$h = (-4\pi^2)r \int \frac{d^3 q}{(2\pi)^3} \frac{1}{4\Delta_1^{(3)}},$$

$$\begin{aligned}
l &= (-16\pi^2) \frac{r^2}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{3\Delta_4 - \Delta_1 \Delta_4 + \Delta_5}{4\Delta_1 \Delta_2}, \\
m &= (16\pi^2) \frac{r^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{-\frac{1}{2}\Delta_4 + 8\Delta_1 - 2\Delta_1^2}{4\Delta_1 \Delta_2}, \\
n &= (16\pi^2) \frac{r^2}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{-\Delta_4 + 4\Delta_1 - \Delta_1^2}{4\Delta_1 \Delta_2}, \\
q &= (-16\pi^2) \frac{r^3}{4} \int \frac{d^3 q}{(2\pi)^3} \frac{\Delta_4^{(3)}}{4\Delta_2^{(3)}},
\end{aligned} \tag{91}$$

while their numerical values are given in table 4.

Thus for A_Γ and A_Γ^I , eqs.(44,46), we get

$$\begin{aligned}
A_\Gamma &= d_1 + d_2 G + \frac{1}{2}(e + f), \\
A_\Gamma^I &= -(2d^I + q - h)G + \frac{1}{2}f^I - l - m + n.
\end{aligned} \tag{93}$$

In the case $\Gamma = \gamma_0 \gamma_5$, cfr. tab.1, we have

$$\begin{aligned}
A_{\gamma_0 \gamma_5} &= d_1 - d_2 + \frac{1}{2}(e + f), \\
A_{\gamma_0 \gamma_5}^I &= 2d^I + \frac{1}{2}f^I - h - l - m + n + q.
\end{aligned} \tag{94}$$

Finally, we give the analytical expression of $i\Sigma_0$ defined in eq.(50)

$$i\Sigma_0 = (16\pi^2) \int \frac{d^3 q}{(2\pi)^3} \frac{1}{8\Delta_1^{(3)}}; \tag{95}$$

for its numerical value, see eq.(51). We now discuss the diagrams referring to the four-fermion operator O_L^I , eq.(39).

The graphs in fig.(3.a), computed with the improved QCD vertex, give:

$$\begin{aligned}
\frac{g^2}{16\pi^2} \left[\frac{20}{3} (-\log \lambda^2 a^2) + \frac{10}{3} d_1 + \frac{4}{3} e + \frac{1}{3} c \right] (O_L^I)^{latt} + \\
+ \frac{g^2}{16\pi^2} [2(-d_2 + d^I)] O_N^{latt}.
\end{aligned} \tag{96}$$

The graphs in fig.(3.b), which involve only light quarks in the loops, give (see also ref.[17])

$$\begin{aligned} \frac{g^2}{16\pi^2} \left[\frac{8}{3} \log \lambda^2 a^2 + \frac{4}{3} (f + f^I) + \frac{1}{3} (v + v^I) \right] (O_L^I)^{latt} + \\ + \frac{g^2}{16\pi^2} \frac{4}{3} (w + w^I) O_R^{latt}, \end{aligned} \quad (97)$$

where f, f^I come from the improved light quark self-energy (see the previous discussion of two-fermion operators). The quantities v and w are the same of ref.[7] while the analytical expressions of the quantities v^I and w^I are

$$\begin{aligned} v^I &= (-16\pi^2) \int \frac{d^4 q}{(2\pi)^4} \frac{r^2}{8\Delta_1\Delta_2} \left[-4\Delta_1(\Delta_4 - \Delta_5) \right. \\ &\quad + 4\Delta_1\Delta_4(4 - \Delta_1) + r^2(-2(\Delta_4 - \Delta_5)\Delta_1^2 \\ &\quad \left. + 2\Delta_4\Delta_1^2(4 - \Delta_1)) \right] \\ w^I &= (16\pi^2) \int \frac{d^4 q}{(2\pi)^4} \frac{r^2}{32\Delta_1\Delta_2^2} \left[-4\Delta_1(\Delta_4 - \Delta_5) \right. \\ &\quad + 4\Delta_1\Delta_4(4 - \Delta_1) - \frac{1}{2}\Delta_4^2(4 - \Delta_1) \\ &\quad \left. + \frac{1}{2}\Delta_4(\Delta_4 - \Delta_5) \right] \end{aligned} \quad (98)$$

We give the numerical values of v^I and w^I in the third and fourth row of table 3.

Now we compute the new graphs of fig.4 and 5, corresponding to the rotated part of the operator, eq.(39). The graphs in fig.(4.a) and (5.a) give

$$\frac{g^2}{16\pi^2} \left(\frac{10}{3} n \right) (O_L^I)^{LATT} + \frac{g^2}{16\pi^2} 2 (-h + q + d^I) O_N^{LATT}. \quad (99)$$

Fig.(4.b) and (5.b) contain only light fermions in the loop - see also ref.[17] - and they give respectively

$$\frac{g^2}{16\pi^2} \left(-\frac{1}{3} s \right) (O_L^I)^{LATT} + \frac{g^2}{16\pi^2} \left(-\frac{1}{3} s \right) O_R^{LATT} \quad (100)$$

and

$$\frac{g^2}{16\pi^2} \left[-\frac{8}{3} (l + m) \right] + \frac{g^2}{16\pi^2} \left[\frac{2}{3} (l + m) \right] O_R^{LATT}. \quad (101)$$

r	1.00	0.75	0.50	0.25	0
d^I	-4.04	-3.66	-3.07	-2.02	0
f^I	-3.63	-2.84	-1.92	-0.84	0
v^I	-6.69	-5.73	-4.44	-2.34	0
w^I	0.82	0.80	0.72	0.45	0

Table 3: *Numerical values of the finite renormalization constants d^I , f^I , v^I and w^I for several values of the Wilson parameter r .*

The analytical expression for s is

$$\begin{aligned}
s = & (16\pi^2) \int \frac{d^4}{(2\pi)^4} \frac{2\tau^2}{\Delta_2^2} \left[\frac{1}{4} \Delta_4 (4 - \Delta_1) \right. \\
& + \frac{\tau^2}{4} ((\Delta_4 - \Delta_5) - \Delta_4 (4 - \Delta_1)) \left(\frac{\Delta_4}{4} - \Delta_1 \right) \\
& \left. - \frac{\Delta_4}{16\Delta_1} (\Delta_4 (4 - \Delta_1) + \Delta_4 - (\Delta_4 - \Delta_5)) \right]; \quad (102)
\end{aligned}$$

we give numerical values for s in the last row of table 4. For comparison with ref.[17], we get

$$s = 8\pi^2 I_6, \quad (103)$$

and

$$l + m = \pi^2 I_7 \quad (104)$$

where I_6 and I_7 are defined in [17]. The contributions of the graphs in fig.(4.c) are zero.

Finally we find, cfr. eq.(67)

$$\begin{aligned}
D_L^I &= -\frac{10}{3}n - \frac{4}{3}f^I + \frac{8}{3}(m + l) - \frac{1}{3}v^I + \frac{1}{3}s \\
D_N^I &= 2(h - 2d^I - q) \\
D_R^I &= \frac{1}{3} [s - 4w^I - 2(l + m)]. \quad (105)
\end{aligned}$$

r	1.00	0.75	0.50	0.25	0
h	-9.88	-7.41	-4.94	-2.47	0
l	-3.42	-2.52	-1.54	-0.55	0
m	7.35	5.92	4.08	1.74	0
n	0.73	0.63	0.47	0.22	0
q	-2.02	-1.20	-0.54	-0.11	0
s	3.41	3.20	2.79	1.72	0

Table 4: *Numerical values of the finite renormalization constants h , l , m , n , q and s for several values of the Wilson parameter r .*

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Fig.1 - One-loop vertex and self-energy diagrams of local two-fermion operator.

Fig.2 - One-loop diagrams for the vertex correction due to the rotated part of the improved two-fermion operator.

Fig.3 - One-loop vertex and self-energy diagrams of local four-fermion operator.

Fig.4 - One-loop diagrams for the vertex correction due to the rotated part of the improved four-fermion operator. Fermion-fermion vertex contribution.

Fig.5 - One-loop diagrams for the vertex correction due to the rotated part of the improved four-fermion operator. Fermion-gluon vertex contribution.