SPECTRA OF HEAVY BARYONS FROM QCD SPECTRAL SUM RULES

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ABSTRACT

We study the masses and couplings of the heavy $\Sigma_{b,c}^*$ and $\Lambda_{b,c}$ baryons within QCD spectral sum rules (QSSR). The $\Sigma_{b,c}^*$ is found to be degenerate with the $\Sigma_{b,c}$ within the errors of the method, while the $\Lambda_{b,c}$ is lighter than the $\Sigma_{b,c}$.

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INTRODUCTION

In a recent paper, we have studied the masses of the $\Sigma_{b,c}$ baryons and their mass difference using QCD spectral sum rules (QSSR) [1], where we have found that the mass difference $M_{\Sigma_b} - M_{\Sigma_c}$ can be predicted with a good accuracy, and independently of the form of the sum rules, to be about 3.4 GeV. We pursue this investigation for the $\Sigma_{b,c}^*$ and $\Lambda_{b,c}$ in this paper.

1. THE $\Sigma_{b,c}^*$ BARYONS

We shall be concerned with the two-point correlator:

$$S_{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0 | T J_\mu^{\Sigma_b^*}(x) J_\nu^{\Sigma_c^*}(0) | 0 \rangle \quad (1a)$$

$$\equiv g^{\mu\nu} (\bar{q} F_1 + F_2) + \dots$$

built from the interpolating Σ_b^* operator :

$$J_\mu^{\Sigma_b^*}(x) = \frac{1}{\sqrt{3}} \epsilon^{\alpha\beta\lambda} : 2 (q_\alpha^T C \gamma^\mu q_\beta) q_\lambda + (q_\alpha^T C \gamma^\mu q_\beta) Q_\lambda : \quad (1b)$$

As usual, we perform a QCD analysis of the correlator, within an operator product expansion (OPE), which takes into account the non-perturbative condensate contributions. We evaluate the correlator to leading order of α_s and for condensates up to dimension $D = 6$. Then, we obtain :

$$\text{Im } F_1^{\Sigma_b^*}(x) = \frac{M^4}{960 \pi^3} \left\{ (1-x) \left(-29 + x + x^2 - \frac{39}{x} + \frac{6}{x^2} \right) - 60 \ln x \right\}$$

$$\text{Im } F_2^{\Sigma_b^*}(x) = \frac{M^5}{576 \pi^3} \left\{ (1-x) \left(1 + x + \frac{73}{x} + \frac{9}{x^2} \right) + 12 \left(3 + \frac{4}{x} \right) \ln x \right\}$$

$$\text{Im } F_3^{\Sigma_b^*}(x) = -\frac{\langle \alpha_s G^2 \rangle}{2^5 3^2 \pi^2} (1-x)(7 - 3x - x^2)$$

$$\text{Im } F_4^{\Sigma_b^*}(x) = -\frac{M \langle \alpha_s G^2 \rangle}{3^2 2^5 \pi^2} \left\{ \left(1 - \frac{1}{x} \right) (8-x + 11x^2) - 15 \ln x \right\}$$

$$\text{Im } F_5^{\Sigma_b^*}(x) = -\frac{M \langle \bar{\psi}\psi \rangle}{3 \pi} (1-x)^2$$

$$\text{Im } F_2^{\Sigma_c^*}(x) = -\frac{M^2 \langle \bar{\psi}\psi \rangle}{3^2 \pi} \left(-3 + x^2 + \frac{2}{x} \right)$$

$$\text{Im } F_1^{\Sigma_c^*}(x) = \frac{7}{3^2 2^2 \pi} M_0^2 \langle \bar{\psi}\psi \rangle \frac{x^2}{M}$$

$$\text{Im } F_2^{\Sigma_c^*}(x) = \frac{1}{3^2 \pi} M_0^2 \langle \bar{\psi}\psi \rangle \left(1 + \frac{3}{4} x^2 \right)$$

$$F_1^{D=6}(q^2) = \rho \frac{2^2 \langle \bar{\psi}\psi \rangle^2}{3^2 M^2 - q^2}$$

$$F_2^{D=6}(q^2) = \rho \frac{2}{3} \frac{M \langle \bar{\psi}\psi \rangle^2}{M^2 - q^2}, \quad (2)$$

where $x = M^2/t$. We test the validity of this expression from the well-known result at $M = 0$. This expression also satisfies the perturbative $E^3 [t \equiv (M + E)^2]$ threshold behaviour test for a 3-body particle with one heavy quark and two light ones.

We use the standard "one-resonance" plus "QCD continuum" parametrization of the spectral function. As in the case of Σ_c , we use the ratios $\sqrt{R_i} (i = 1, 2)$ of the Laplace transform sum rules for obtaining the Σ_c^* mass. We study the stability of the result with respect to the sum-rule variable τ and to the continuum threshold t_c . As is the case for Σ_c , the inclusion of the $D = 6$ condensate destroys the stability. The onset of τ -stability in the absence of the $D = 6$ condensate is reached for $t_c = 8 \text{ GeV}^2$ and for $\tau_0 \approx 0.2 \text{ GeV}^{-2}$. In this case, and using the input parameters of Ref. [2], we obtain $M_{\Sigma_c^*} \approx 2.15\text{--}2.40 \text{ GeV}$, including and omitting the four-quark condensate respectively. We also take a larger range of t_c values, until $t_c = 16 \text{ GeV}^2$ where the t_c -stability is reached. In this case, the τ -stability is reached at 0.6 GeV^{-2} , which corresponds to the value $M_{\Sigma_c^*} \approx 2.92 \text{ GeV}$. The most conservative prediction of the sum rule should therefore be $M_{\Sigma_c^*} \approx 2.15\text{--}2.92 \text{ GeV}$.

We make an analogous analysis for the Σ_b^* . The onset of τ -stability is reached for $t_c \approx 36 \text{ GeV}^2$ at $\tau_0 = 0.1 \text{ GeV}^{-2}$, which yields $M_{\Sigma_b^*} \approx 5.35\text{--}5.74 \text{ GeV}$.

At the t_c -stability, the corresponding value of the Σ_b^* mass is 6.2 GeV . Therefore, we deduce the conservative prediction of $5.4\text{--}6.2 \text{ GeV}$ for $M_{\Sigma_b^*}$. One should remark that the ratio of τ_0 for the Σ_b and Σ_c is about M_b/M_c , as qualitatively expected.

We test the method independence of the result by using the ratio of moments at $q^2 = 0$ for the Σ_c^0 . Stability in the number of derivatives n is reached for $t_c \approx 36 \text{ GeV}^2$ and for $n = 1$, while the t_c -stability is reached for $n = 5$ and $t_c \approx 52 \text{ GeV}^2$. The moments give the range $M_{\Sigma_c^0} \approx 5.4\text{--}6.4 \text{ GeV}$, in agreement with the results from the Laplace sum rule.

From these previous results, we estimate the mass difference between Σ_c^0 and Σ_c^+ from the values of $M_{\Sigma_c^{b,c}}$, obtained within the same conditions of t_c . The interaction between the Laplace and moments results leads to the final accurate estimate :

$$M_{\Sigma_c^0} - M_{\Sigma_c^+} \approx 3.3 \text{ GeV} , \quad (3)$$

which is independent from t_c and from the method used. This result is remarkably similar to the Σ_b and Σ_c mass difference of about 3.4 GeV [1]. Indeed, within the accuracy of the QSSR method, we find that the Σ_c^0 and Σ_c^+ are almost degenerate already for the Σ_c , which is the same conclusion as that obtained from the non-relativistic potential approaches [3], but here from a fully relativistic method near the original QCD.

We can finally estimate the residues of the resonance from the Laplace or moments sum rules or from both. Using the standard stability criteria, we obtain $|Z_{\Sigma_c^+}|^2 \approx (1.1 - 2.2) 10^{-3} \text{ GeV}^6$ and $|Z_{\Sigma_c^0}|^2 \approx (2.0 - 5.4) 10^{-3} \text{ GeV}^6$, where the normalization is the same as for the Σ_q in Ref. [1].

2. THE $\Lambda_{b,c}$ BARYONS

Here, we shall consider the correlator built from the following interpolating current :

$$J^{\Lambda_b}(x) = \epsilon^{\alpha\beta\lambda} : (q_\alpha^T C \gamma_5 q_\beta) q_\lambda + b (q_\alpha^T C q_\beta) \gamma_5 q_\lambda : , \quad (4a)$$

where the heavy quark can be static and b is a free parameter. A QCD analysis of the correlator shows that in the chiral limit where the light-quark mass is zero, the chiral condensates do not contribute :

$$\text{Im } F_1^{\text{pert}}(x) = \frac{(1+b^2)M^6}{512 \pi^3} \left\{ (1-x^2) \left(1 - \frac{8}{x} + \frac{1}{x^2} \right) - 12 \ln x \right\}$$

$$\text{Im } F_2^{\text{pert}}(x) = \frac{(1-b^2)M^6}{128 \pi^3} \left\{ (1-x) \left(1 + \frac{10}{x} + \frac{1}{x^2} \right) + 6 \left(1 + \frac{1}{x} \right) \ln x \right\}$$

$$\text{Im } F_1^q(x) = \frac{(1+b^2)}{768 \pi^2} \langle \alpha_s G^2 \rangle (1-x)(1+5x)$$

$$\text{Im } F_2^q(x) = \frac{(1-b^2)}{384 \pi^2} M \langle \alpha_s G^2 \rangle \left\{ (1-x) \left(7 + \frac{2}{x} \right) + 6 \ln x \right\}$$

$$F_1^{D=6}(q^2) = \rho \frac{\langle \bar{\psi}\psi \rangle^2 (1-b^2)}{6 M^2 - q^2}$$

$$F_2^{D=6}(q^2) = M \rho \frac{\langle \bar{\psi}\psi \rangle^2 (1+b^2)}{M^2 - q^2} . \quad (4b)$$

Using an analogous analysis, we find no τ -stability and therefore we cannot make any reliable predictions for the $\Lambda_{b,c}$ mass. However, if we compare the ratios $\sqrt{R_1}$ for the Σ_q and Λ_b baryons with the same value of t_c and at the same point τ_0 where the Σ_q sum rule has a minimum, we see that the four-quark condensate tends to make the ratios smaller for the Λ_b . This effect tends, however, to disappear for larger value of t_c . Therefore, we deduce from this analysis the practical (but not rigorous) upper bounds of 207 MeV and 163 MeV for $M_{\Sigma_c} - M_{\Lambda_c}$ and $M_{\Sigma_b} - M_{\Lambda_b}$, respectively, which indicates that Λ_b is lighter than Σ_q in accordance with potential model expectations [3]. We should also notice that the gluon condensate tends to decrease the Λ_b mass. We have tried another choice of interpolating current than the one in Eq. [4a]. Using the current which describes the Λ in the light-quark systems, we find that the choice of the current does not affect the $\Sigma_q - \Lambda_b$ mass difference, while the sum rule for the Λ_b does not yet show any stability, despite the existence of chiral quark condensate contributions in this case.

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