2018-02-16

NSF-ITP-92-11

TIME REVERSAL VIOLATION: Examples

from cosmology, condensed matter,
and particle physics

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* Invited talk at the A. Rich Memorial Symposium on Time Reversal, Ann Arbor, Michigan 1991, to appear in the Proceedings.

In this memorial talk, I will discuss some examples of time reversal I have encountered in my research career. These examples are taken from cosmology, condensed matter, and particle physics.

I. GENESIS OF MATTER IN THE UNIVERSE

One of the most basic observational facts about our universe is that we do not live in a universe containing equal amounts of matter and antimatter. This rather strange environmental imbalance could have come about only if three necessary conditions are fulfilled, as was first emphasized by Sakharov¹ and by Toussaint, Treiman, Wilczek, and Zee.²

(1) Baryon number B must be violated. This is obvious since otherwise the net baryon number now observed in the universe would just correspond to an initial condition. Somebody had to decide how much dirt to throw in.

(2) CP, and by the CPT theorem which follows from relativistic local field theory, time reversal T must be violated. This is also obvious since otherwise baryons and antibaryons would be on equal footings.

(3) An "arrow of time" must be provided. Indeed, the expansion of the universe provides just such an arrow. In other words, we need an extrinsic violation of time reversal, i.e., in the environment, as well as an intrinsic violation of time reversal, i.e., in the laws of physics.

The third condition is less obvious and requires some explanation. Mr. Boltzmann said "My distribution depends only on energy (and hence mass)." Thus, in thermal equilibrium at temperature T the number distribution of a particle is determined by $e^{-\sqrt{p^2 + m^2}/T}$. According to the CPT theorem, a particle and its antiparticle must have exactly the

same mass. It follows then that the number of baryons must be equal to the number of antibaryons.

It is instructive to consider a toy model. We stick an empty box into the oven and turn up the temperature. In fact, the "empty" box is full of thermally created particles: $\gamma, \nu, \bar{\nu}, e^-, e^+, \mu^-, \mu^+, \pi^-, \pi^+, K^-, K^+, \bar{K}^0, K^0$, and so on. Suppose we were told that there is CP violation and that the decay $\bar{K}^0 \rightarrow \pi^- e^+ \nu$ proceeds slightly faster than the decay $K^0 \rightarrow \pi^+ e^- \bar{\nu}$. We may thus be tempted to say, aha, there will be more e^+ than e^- in the box.

Nol thunders Boltzmann. According to his distribution, there will be exactly the same number of e^+ and e^- . We see that, in this analog problem, the first two of the three conditions listed above are satisfied (namely, electron number is violated, and CP or T is violated), but not the third.

Well, what is going on? The answer, if we remember what we learned in our statistical mechanics course, is of course that the reaction $e^+ + \pi^- \rightarrow \nu + K^0$ would also proceed slightly faster and thus works to remove any excess e^+ .

Now, if we suddenly open the box, we destroy thermal equilibrium. The excess e^+ would then stream out (and the π^- won't be able to find them).

An important outstanding question in this subject is, what about the sign? Can a physicist doing K decay experiments, or measuring the electric dipole moment (see below), without going outside his or her lab, predict that the world is made of matter and not antimatter? Our ability to answer this question provides a measure of our theoretical understanding, of whether we can relate CP and T violation in $\Delta B \neq 0$ processes to that in $\Delta B = 0$ processes. There have been some attempts in this direction.

II. ELECTRIC DIPOLE MOMENT OF THE ELECTRON

It was realized³ long ago that if a particle possesses an electric dipole moment then time reversal invariance is violated. Put the particle between the two charge plates (as in a capacitor) and the particle would precess in the electric field, just as a particle with a magnetic moment would precess in a magnetic field. Run the movie backward and we see our particle with electric dipole moment precessing in the opposite direction. Time reversal is violated. When we run the movie backward, the electric field is unchanged. In contrast, a magnetic field, being generated by currents, reverses its direction under time reversal. Thus, magnetic dipole moments do not violate time reversal, and indeed, particles with magnetic dipole moments are a dime a dozen but experimentalists have never found particles with electric dipole moments.⁴

As we heard yesterday,⁵ the upper bound to the electric dipole moment of the neutron is now $(3 \pm 5) \times 10^{-26} e \cdot \text{cm}$ and of the electron $(-2.7 \pm 8.3) \times 10^{-27} e \cdot \text{cm}$.

In the modern theory of weak interaction, time reversal violation in fact emerges naturally. Phases pop up everywhere and unless the theory is exceedingly simple, they cannot be absorbed. The problem is rather to understand why time reversal violation is small, and, as Weinberg emphasized many years ago, to understand why time reversal violation (and the violation of other symmetries) do not spill into the strong interaction. We now believe that we know the answer to both questions. There is no conceptual problem or paradox (which situation of course makes it tough on theorists).

As a first, and well-known, example that time reversal violating phases pop up readily in all but the simplest theories, consider the usual one-Higgs model. Gauge invariance implies that the potential V must be a function of $(\varphi^\dagger \varphi)$. Hermiticity of the Lagrangian then requires all the parameters in V to be real. In contrast, with two Higgs doublets φ_1 and φ_2 we can have in V the term $\varphi_1^\dagger \varphi_2 + \text{hermitean conjugate}$. The

coupling ρ is not required to be real and may very well be complex. Given the huge disparity among fermion masses, and other such considerations, it is difficult for most of us to imagine that there is only one Higgs doublet.

As another example, consider the Kobayashi-Maskawa mechanism. Let us give a brief review. The charged current of the weak interaction is given by $J_\mu = \bar{U}_L \gamma_\mu \mathcal{U} \mathcal{D}_L$ where $U_L = (u, c, t, \dots)_L$ and $D_L = (d, s, b, \dots)_L$. For n families, $\mathcal{U}$ is an $n \times n$ unitary matrix, which has $2n^2 - n^2$ real parameters = n^2 real parameters since the condition $\mathcal{U}^\dagger \mathcal{U} = I$ amounts to n^2 real conditions. In contrast, an $n \times n$ orthogonal matrix has $n(n-1)/2$ real parameters. Thus, the number of phases is $n^2 - n(n-1)/2 = n(n+1)/2$. But by rotating the $2n$ left-handed quark fields, we can absorb $(2n-1)$ phases, the reason for the (-1) being that if we rotate all the left-handed quark fields by an over-all phase, J_μ is not affected. And so, the net number of phases $n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$. For less than three families, there is no phase. For three families, as is well-known, there is one phase.

Simple reasoning, yes? It is amusing to go back and look at all those papers before Kobayashi-Maskawa that either did not count the number of phases or counted incorrectly.

As is well-known, this means that for the Kobayashi-Maskawa CP-violating phase to contribute, all six quarks must be involved. This immediately implies that the effect is going to be tiny. In particular, Khriplovich and Pospelov⁶ showed that the electron's elastic dipole moment is still zero to three-loop order.

A couple of years ago, Weinberg⁷ shamed the theoretical community by pointing that a hitherto neglected three-loop graph may lead to a large electric dipole moment of the neutron. When I heard him talk about this in Santa Barbara, I naturally asked him about the electric dipole moment of the electron. Barr and I showed⁸ that in fact there is a two-loop graph that contributes to the electric dipole moment of the electron (Fig. 1a). We have pulled the Higgs field "out of the top quark loop". The point is that

since a factor of the electron mass m_e is required by chirality in any case, we may as well let an explicit Higgs boson coupling flip the chirality. It is easy to give an order of magnitude estimate of this graph in the limit of the top quark mass m_t much bigger than the Higgs mass m_H . We then have (for energies $\lesssim m_H$) an effective interaction $e^2 v^{-1} \phi F_{\mu\nu} F^{\mu\nu}$ (and $e^2 v^{-1} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$). The precise value was of course calculated long ago by the famous theoretical physicist J. Steinberger. By dimension counting, we find the coefficient of the interaction to be proportional to the Yukawa coupling of the top quark divided by its mass $f_t/m_t \sim v^{-1}$. In the effective theory we then have (Fig. 1b)

$$\left(\frac{e^2}{v}\right)\left(\frac{m_e}{v}\right) \left[\int d^4 p \frac{1}{p^2} \frac{1}{p^2} \frac{1}{p^2 - m_H^2} p \right] (\text{Im}) \sim \left[\left(\frac{e^2}{v}\right) \left(\frac{m_e}{v}\right) \log(m_t^2/m_H^2) \right] (\text{Im})$$

The factor of (m_e/v) represents the Higgs coupling to the electron, the factor of p in the numerator of the Feynman integrand comes from one factor of $F_{\mu\nu}$ in the Higgs coupling to two photons, and finally the factor (Im) measures the T violation carried by the Higgs field. We cut off the logarithmically divergent integral at the top mass m_t ; after all, a Nobel prize has been given attesting to the fact that the theory is renormalizable and so the divergence must be cut off at some scale.

Recall that there is another factor of $F_{\mu\nu}$ and so the above is the coefficient of $\bar{u} \sigma_{\mu\nu} \gamma_5 u F^{\mu\nu} \sim u^\dagger \sigma u \vec{E}$ in the non-relativistic limit), namely the electric dipole moment. A detailed calculation shows that the electric dipole moment comes out to be $3 \times 10^{-27} e - cm$ times a factor containing such unknown parameters as the ratio m_t^2/m_H^2 and the strength of CP violation, a factor that may range from zero to infinity, and which we may imagine to be of order 1.

There are also a number of other graphs similar to the one we have shown. They were subsequently computed⁹ to be of more or less the same order of the magnitude. A supersymmetric version of this discussion has also been given.¹⁰

We remark in passing that a similar two loop graph contributing to the electric

down to 10^{-12} .

Of course, theorists can always work up models¹³ with a large electric dipole moment of the electron. They are even paid to do so. The philosophy I like is that in the breaking of the "true physics" at some higher energy scale, (grand unified scale, whatever) down to low energy, unless we have some good reasons to believe in the contrary, lots of scalar fields would surely be lying around in the debris. Scalar fields can do all kinds of interesting things¹⁴ for us, in particular, generate a big electric dipole moment of the electron. In a moment of frustration, I wrote in the preprint an editorial passage as follows. (I was surprised that the editors of the Physical Review Letters actually let it stand.)

"Experimental particle physics has been somewhat dull lately. An optimist might dream of an exciting scenario in which, within the next few years, the electron will be found to have an electric dipole moment and $\mu \rightarrow 3e$ will be detected with a large triple-product correlation. And later, with higher-energy machines, doubly charged scalars may be produced with exotic decay signatures into channels such as $\tau^- e^-$ and $\mu^- e^-$."

Well, it didn't happen in the six years since that letter was published, but we can always dream on.

III. T VIOLATION IN SUPERCONDUCTORS

In condensed matter systems, time reversal is often spontaneously broken. It is instructive to compare the antiferromagnet and the ferromagnet, defined on a square lattice say. In an antiferromagnet the spins point alternately up and down. Reverse time and each spin reverses its direction. The antiferromagnetic state is not time reversal invariant. However, if we translate the time reversed state by one lattice spacing we recover the state we started with. The state is invariant under the operation (time reversal) $\times$

Fig. 1. (a) A graph contributing to the electron's electric dipole moment. CP violation indicated by the cross is assumed to exist in the propagators of the neutral Higgs bosons. (b) The same graph where the top-quark loop has been shrunk to a point.

dipole moment of the neutron may compete and even dominate the graphs considered by Weinberg.⁸

Higgs exchange may also generate T -violating electron-nucleon interactions that may give a large contribution to the electric dipole moment of atoms.¹¹

We now have had more than a quarter century of time reversal violation, but mysteriously confined to the quark sector. It would be strange if it doesn't also surface in the lepton sector. There was a lost opportunity some years ago. Remember the rumour in 1976-77 that the decay $\mu^- \rightarrow e^- \gamma$ may have been observed? With muon number conservation violated the decay $\mu^- \rightarrow e^- e^+ e^-$ would also go. This process would have provided a golden opportunity to measure the triple correlation¹² $(\vec{\sigma} \cdot (\vec{p}_e^- \times \vec{p}_{e^+}))$ with the muon naturally providing a polarization direction. Alas, the upper limit on this decay is now

(translation by one unit). In the macroscopic limit, the translation by one lattice spacing is insignificant. Thus, the low energy physics of the antiferromagnet should be effectively time reversal invariant. In contrast, upon time reversal, the ferromagnetic state goes into another state related to the original state by a rotation, which is always macroscopically significant.

Shortly after the discovery of high temperature superconductivity the possibility that the material may dynamically violate T and P was suggested.¹⁵ The new state of matter was dubbed the chiral spin liquid. The order parameter is a spin triple cross product: clearly if $\vec{S}_i$ denotes the spin operator on site i in a lattice model of the material, then the non-vanishing of the expectation value $\langle \vec{S}_i(\vec{S}_j \times \vec{S}_k) \rangle$ would signal a violation of T and P . Here i, j , and k denote three neighboring lattice sites. I won't go into further theoretical details here, as reviews of the subject are available elsewhere.¹⁶ I will just mention briefly the experimental tests.

To look for T and P violation, we can study the polarization of light reflected from a high- T_c material when the incident light is linearly polarized.^{17,18} It is fairly clear that the polarization would be rotated when T and P are violated. Indeed let the incident and reflected beam be represented by

$$E_j = b_j e^{ikz} + c_j e^{-ikz}$$

and the transmitted beam by

$$E_j = a_j e^{-\kappa_j z} + a_j e^{-\kappa_j z}$$

where for simplicity the surface of material is taken to be perpendicular to the direction of propagation, denoted by the z -axis. Let $c_{\pm} = c_1 \pm ic_2$, etc., and define the reflection coefficients $R_{\pm} = R_{-}$ by

$$c_{\pm} = R_{\pm} b_{\pm}$$

Then parity implies

$$R_{+} = R_{-}$$

and time reversal

$$R_{\pm} = (R_{\mp}^*)^{-1}$$

The (possibly complex) angle of rotation ϕ is given by $e^{2i\phi} = R_{+}/R_{-}$. Thus, we see that there is a possibility of turning a circular polarization into an elliptical polarization.

Thus far, three experiments¹⁹ have been done with two showing a rotation of the polarization axis and one showing no effect. Of course there are numerous solid state effects to worry about. Also, it is important to see both T and P violation with TP conserved. A rotation of the polarization axis shows only P violation, but not T violation. Another worry is that the statistics parameter θ may alternate in sign as we go from layer to layer, in which case the effect would be washed out.²⁰ An alternative experiment is to look at the precession of muons injected into the material²¹. In contrast to the polarized light experiment, the muon is a local probe. The experiment has been done and fails to show a signal at the expected level. Thus, we have now two negative experiments and two quasi-positive experiments.

Since many of you have devoted a good part of your lives to looking for T violation and know all the tricks and pitfalls in this business I would like to urge you to take time out to see if you can find materials that spontaneously violate T and P . Quite aside from possible relevance to high temperature superconductivity the discovery of such materials is of intrinsic interest. In particular, if such materials are also two-dimensional, we would see, on rather general grounds, the interesting phenomenon of fractional statistics.²²

In summary, I have talked about time reversal in three different fields, with rather difficult time scales for experiments. In cosmology, the experiment will never be done (or perhaps rather that it has already been done.) In particle physics, the experiment is

being done. In condensed matter physics, the experiment was done within months of the theoretical suggestion.

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