Field Strength Correlations in the QCD Vacuum*

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Abstract

We determine by numerical simulation on a lattice the correlation function of the gauge field strengths in the QCD vacuum.

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1. Introduction

The SVZ sum rules demonstrate the existence of gluon condensation in the vacuum of QCD [1]. The same indication comes from numerical simulations of the theory on the lattice [2].

A direct detection of gluon condensation could be provided by its effects on the spectrum of heavy $Q\bar{Q}$ bound states [3]. There, the main effect is Stark splitting of the levels by the chromoelectric field of the vacuum condensate. A static field strength, with infinite correlation distance and time, would be incompatible with the observed spectra of charmonium and bottomonium [4]: A finite correlation length is required. The gauge-invariant correlator

$$\mathcal{D}_{\mu\rho,\nu\sigma}(x) = \langle 0 | \text{Tr} \{ G_{\mu\rho}(x) S(x,0) G_{\nu\sigma}(0) S^\dagger(x,0) \} | 0 \rangle \quad (1.1)$$

must fall exponentially at large distances. In Eq. (1.1) $G_{\mu\rho} = gT^a G_{\mu\rho}^a$; $S(x,0) = \text{P exp} \left(i \int_0^1 dt x^\mu A_\mu(xt) \right)$; $A_\mu = gT^a A_\mu^a$; S is the Schwinger phase factor required for gauge invariance; T^a are the generators of the gauge group in the fundamental representation.

Lattice simulations with gauge group $SU(2)$ showed that $\mathcal{D}_{\mu\rho,\nu\sigma}$ does indeed fall off exponentially at large distances [5]:

$$\mathcal{D}_{\mu\rho,\nu\sigma}(x) \underset{|x| \rightarrow \infty}{\simeq} \exp(-|x|/\lambda) \quad (1.2)$$

and gave for the correlation distance λ

$$\lambda \simeq 0.13 \text{ fm.}$$

With such a correlation length the effect of the condensate on the levels of heavy $Q\bar{Q}$ bound states is drastically reduced [6], and becomes compatible with measurements.

A systematic study on non-perturbative effects in QCD, in terms of the correlation functions of the gluon field strengths in the vacuum, has been developed in Ref. [7]. A picture emerges in which the correlator of Eq. (1.1) plays a basic role in describing confinement of colour; its importance in the description of high-energy hadron scattering is illustrated in Ref. [8].

Stimulated by these models we have made a new lattice determination of the large-distance behaviour of $\mathcal{D}_{\mu\rho,\nu\sigma}$. In Ref. [5] the gauge group was $SU(2)$, and only the correlation of the longitudinal components of the chromoelectric field was studied. In this paper the gauge group is $SU(3)$, and we give a full determination of the correlation. Also, in Ref. [5] the ultraviolet cut off of the lattice was removed by computing the subtractions in perturbation theory; here we perform the subtraction by cooling the field configurations. This technique eliminates short-range fluctuations [9], leaving the long-range non-perturbative effects intact [10].

2. Computations and Results

The most general form of the correlator compatible with the invariances of the system is [7]:

$$\begin{aligned} \mathcal{D}_{\mu\rho,\nu\sigma}(x) = & (g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\rho\nu}) (\mathcal{D}(x^2) + \mathcal{D}_1(x^2)) \\ & + (x_\mu x_\nu g_{\rho\sigma} - x_\mu x_\sigma g_{\rho\nu} + x_\rho x_\sigma g_{\mu\nu} - x_\rho x_\nu g_{\mu\sigma}) \frac{\partial \mathcal{D}_1(x^2)}{\partial x^2} \end{aligned} \quad (2.1)$$

$\mathcal{D}$ and $\mathcal{D}_1$ are invariant functions of x^2 . We work in the Euclidean region.

We can define a $\mathcal{D}_\parallel(x^2)$ and a $\mathcal{D}_\perp(x^2)$ as follows: We go to a reference frame in which x^μ is parallel to one of the coordinate axes, say $\mu = 0$. Then

$$\begin{aligned} \mathcal{D}_\parallel &\equiv \frac{1}{3} \sum_{i=1}^3 \mathcal{D}_{0i,0i}(x) = \mathcal{D} + \mathcal{D}_1 + x^2 \frac{\partial \mathcal{D}_1}{\partial x^2} \\ \mathcal{D}_\perp &\equiv \frac{1}{3} \sum_{i<j=1}^3 \mathcal{D}_{ij,ij}(x) = \mathcal{D} + \mathcal{D}_1 \end{aligned} \quad (2.2)$$

On the lattice we can define a lattice operator $\mathcal{D}_{\mu\rho,\nu\sigma}^L$, which is proportional to $\mathcal{D}_{\mu\rho,\nu\sigma}$ in the continuum limit, i.e. when the lattice spacing $a \rightarrow 0$. Since the lattice analogue of the field strength is the plaquette $\Pi_{\mu\rho}(n)$ (the parallel transport along an elementary square of the lattice, lying on the $\mu\rho$ -plane), $\mathcal{D}_{\mu\rho,\nu\sigma}^L$ will be defined as:

$$\mathcal{D}_{\mu\rho,\nu\sigma}^L(\hat{d}a) = \frac{1}{2} \left\langle \text{Tr} \left\{ \Pi_{\mu\rho}(n + \hat{d}a) S(n + \hat{d}a, n) [\Pi_{\nu\sigma}^\dagger(n) - \Pi_{\nu\sigma}(n)] S^\dagger(n + \hat{d}a, n) \right\} \right\rangle. \quad (2.3)$$

The operator is shown pictorially in Fig. 1. There, $\hat{d}a$ is a line parallel to one of the coordinate axes, with integer length d in units of the lattice spacing a , joining the two sites to be correlated; S is the parallel transport along this line. The term subtracted in the parentheses on the right-hand side of Eq. (2.3) ensures that the disconnected part of the correlator is left out.

In the naïve continuum limit $a \rightarrow 0$:

$$\mathcal{D}_{\mu\rho,\nu\sigma}^L(\hat{d}a) \underset{a \rightarrow 0}{\simeq} a^4 \mathcal{D}_{\mu\rho,\nu\sigma}(\hat{d}a) + \mathcal{O}(a^6). \quad (2.4)$$

Making use of the definition (2.1) we can also write in the same naïve limit:

$$\begin{aligned} \mathcal{D}_{\parallel}^L(\hat{d}a) &\underset{a \rightarrow 0}{\simeq} a^4 \mathcal{D}_{\parallel}(d^2 a^2) + \mathcal{O}(a^6) \\ \mathcal{D}_{\perp}^L(\hat{d}a) &\underset{a \rightarrow 0}{\simeq} a^4 \mathcal{D}_{\perp}(d^2 a^2) + \mathcal{O}(a^6) \end{aligned} \quad (2.5)$$

Equations (2.4) and (2.5) are called naïve because they come from a formal expansion of the operator, and are expected to be modified, when the expectation value is computed, by lattice artefacts, i.e. by effects due to the ultraviolet cut off. These effects can be estimated in perturbation theory and subtracted. Instead we remove them by freezing the quantum fluctuations at the scale of the lattice spacing; we will comment below on this procedure and on its limitations. After cooling, $\mathcal{D}_{\parallel}^L$ and $\mathcal{D}_{\perp}^L$ are expected to obey Eq. (2.5).

We have measured the correlations on a 16^4 lattice at distances ranging from 1 to 8 lattice spacings and at $\beta = 5.8, 5.9, 6.0$. From renormalization group arguments,

$$a = \frac{1}{\Lambda_L} f(\beta). \quad (2.6)$$

As $\beta \rightarrow \infty$, $f(\beta)$ is given by

$$f(\beta) = \left(\frac{8}{33} \pi^2 \beta \right)^{\frac{51}{121}} \exp \left(-\frac{4}{33} \pi^2 \beta \right) (1 + \mathcal{O}(1/\beta)), \quad (2.7)$$

for gauge group $SU(3)$ and in the absence of quarks. At sufficiently large β one expects:

$$\begin{aligned} \mathcal{D}_{\parallel}^L f(\beta)^{-4} &= \frac{1}{\Lambda^4} \mathcal{D}_{\parallel} \left(\frac{d^2}{\Lambda^2} f^2(\beta) \right) \\ \mathcal{D}_{\perp}^L f(\beta)^{-4} &= \frac{1}{\Lambda^4} \mathcal{D}_{\perp} \left(\frac{d^2}{\Lambda^2} f^2(\beta) \right), \end{aligned} \quad (2.8)$$

where $f(\beta)$ is given by Eq. (2.7) and terms of higher order in a are negligible.

In Fig. 2 we plot $\mathcal{D}_{\parallel}^L f(\beta)^{-4}$ and $\mathcal{D}_{\perp}^L f(\beta)^{-4}$ versus $d_{\text{phys}} = (d/\Lambda)f(\beta)$. A best fit to the data with the functions

$$\begin{aligned}\mathcal{D}(x^2) &= A \exp\left(-\frac{|x|}{\lambda}\right) \\ \mathcal{D}_1(x^2) &= A_1 \exp\left(-\frac{|x|}{\lambda_1}\right)\end{aligned}\tag{2.9}$$

gives $\lambda = \lambda_1$ to a relative accuracy of 10^{-3} , and:

$$\lambda = \frac{1}{\Lambda} \frac{1}{183}, \quad \frac{A}{\Lambda^4} = 3.6 \times 10^8, \quad \frac{A_1}{\Lambda^4} = 1.25 \times 10^8\tag{2.10}$$

with $\chi^2/N_{\text{dof}} \simeq 1.37$. With the value of Λ determined from the string tension [11] we obtain:

$$\lambda = 0.22 \text{ fm}\tag{2.11}$$

We have also tried different fits, including perturbative-like terms, such as

$$\begin{aligned}\mathcal{D}(x^2) &= A \exp\left(-\frac{|x|}{\lambda}\right) + \frac{a}{|x|^4}, \\ \mathcal{D}_1(x^2) &= A_1 \exp\left(-\frac{|x|}{\lambda_1}\right) + \frac{a_1}{|x|^4}.\end{aligned}\tag{2.12}$$

The value of χ^2/N_{dof} is again acceptable; the slope of the exponential is unchanged within a few per cent, and the terms proportional to $1/|x|^4$ are small compared with the exponentials at the distances we consider. We conclude that at large distances the form (2.9) is correct, with the values (2.10) for the parameters.

We shall omit the details of our cooling procedure, which have been described in several papers (see Ref. [10] and references therein). We only remark that it is a local procedure, which affects correlations at distances that grow with the number of cooling steps as in a diffusion process. We then expect that, if the distance at which we observe the correlation is sufficiently large, lattice artefacts are frozen by cooling long before the correlation is affected: this produces a plateau in the correlation versus cooling step. Our data are the values of the correlation at the plateau; the error is the typical statistical error at the plateau, plus a systematic error which is estimated as the

difference between neighbouring points at the plateau (whenever the plateau is not long enough, looking more like an extremum). A typical behaviour is shown in Fig. 3.

As a final comment we notice that terms proportional to $1/|x|^4$ in the correlation could be observed at larger values of β , where the distance between two points (far enough in lattice units so that the correlation is not modified by cooling before lattice artefacts are eliminated) is small compared with 1 fm in physical units. However, larger lattices would then be necessary to avoid infrared artefacts.

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Figure captions

Fig. 1. The operator $\mathcal{D}_{\mu\nu,\rho\sigma}^L(\hat{d}a)$.

Fig. 2. The functions $\mathcal{D}_{\perp}^L f(\beta)^{-4}$ (upper set of data) and $\mathcal{D}_{\parallel}^L f(\beta)^{-4}$ (lower set) versus physical distance. Crosses correspond to $\beta = 5.8$, diamonds to $\beta = 5.9$, squares to $\beta = 6.0$. The lines are the best fit of Eqs. (2.9) and (2.10).

Fig. 3. A typical behaviour of $\mathcal{D}_{\parallel}^L$ (crosses) and $\mathcal{D}_{\perp}^L$ during cooling ($d = 6, \beta = 5.8$).

Fig. 1

Fig. 2

Fig. 3