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**ELECTROMAGNETIC FORM FACTORS IN THE
LIGHT FRONT DYNAMICS**

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Abstract

It is shown that electromagnetic vertex of a nucleus (and of any bound system), expressed through the wave function in the light front dynamics at relativistic values of momentum transfer, contains a contribution of non-physical form factors increasing the total number of invariant form factors (for the deuteron - from 3 up to 11). This fact explains an ambiguities in the form factors calculated previously. The physical and unphysical form factors are covariantly separated. Explicit expressions for physical form factors of systems with spin 0, $1/2$ and 1 through the vertex functions are obtained.

1. Introduction

At present an experimental investigations of the cross sections for electron elastic scattering from nuclei are advanced in region of relativistic momentum transfer of the order of a few GeV/c (ref.¹).

A theoretical approach to relativistic composite systems (in particular, to nuclei) and to their electromagnetic form factors, standing out against a background of other approaches, is the light front dynamics. In this dynamics the state vectors and the wave functions (WF's) are defined not at fixed moment of time (hypersurface $t = \text{const}$), but on a hypersurface, which is given by the equation of a light front (for example, in one of the versions of the approach this is the equation $t+z = 0$). Advantage of the light front dynamics is determined, in particular, by the fact that the light front WF's have the same physical sense (the probability amplitudes) as a nonrelativistic ones and the theory is simplified due to suppression of the vacuum fluctuations. More detailed reasoning is given in the review²). The deuteron electromagnetic form factors in the light front dynamics were calculated in refs.³⁻⁸). Comparison of the model calculations of an electromagnetic form factors for a relativistic system in different approaches (including the light front dynamics) is given in ref.⁹).

There are a few versions of the light front dynamics which differ from each other by definition of the hypersurface: the light front surface $t+z = 0$; the light front surface defined by the invariant equation $\omega x = 0$, where $\omega = (\omega_0, \omega)$, $\omega^2 = 0$ - see below; surface $t = 0$ in the infinite momentum frame. In all these versions we meet one and the same serious problem, without solution of which any reliable calculations are impossible. We explain this problem in example of the deuteron electromagnetic form factors defined according to refs.¹⁰) as follows:

$$\begin{aligned} \langle \lambda' | J_p | \lambda \rangle &= \dot{e}_\mu^{\lambda'}(p') \{ (p + p')_\rho [F_1(q^2) g_{\mu\nu} + F_2(q^2) q_\mu q_\nu] + \\ &+ G_1(q^2) [g_{\mu\rho} q_\nu - g_{\nu\rho} q_\mu] \} e_\nu^\lambda(p) \equiv \dot{e}_\mu^{\lambda'}(p') T_{\mu\nu}^\rho e_\nu^\lambda(p) \end{aligned} \quad (1.1)$$

where $e_v^\lambda(p)$ is the deuteron polarization vector, p and p' are the initial and final deuteron momenta respectively, $q = p' - p$. The form factors are expressed through the light front WF by means of time-ordered (non-covariant) perturbation theory in one or another "convenient" frame of reference (for example, in the reference frame with Z-axis directed along $p + p'$, $z|q = p' - p$, ref.6))

In order to find by eq.(1.1) the form factors, we should express any triple of matrix elements $\langle \lambda' | J_p | \lambda \rangle$ coming in eq.(1.1) through WF. For example, in ref.4) the following matrix elements of the current $J_+ = J_0 + J_3$ are calculated: $\langle 1 | J_+ | 1 \rangle$, $\langle -1 | J_+ | 1 \rangle$ and $\langle 1 | J_+ | 0 \rangle$. According to terminology of ref.6) they give so called solution "B" for the form factors. If decomposition (1.1) with three form factors is really valid for electromagnetic vertex expressed through WF in the light front dynamics, then all the rest of the matrix elements would be determined by the first three ones and would result in the same form factors.

In ref.6) it was found that calculation of form factors by the matrix elements $\langle 1 | J_+ | 1 \rangle$, $\langle -1 | J_+ | 1 \rangle$ and by $\langle 0 | J_+ | 0 \rangle$ instead of $\langle 1 | J_+ | 0 \rangle$ (solution "A") gives the result which considerably differs at $q^2 \sim m^2$ from the solution "B" (by ~ 30 per cent for the function $A(q^2)$ in the cross section and by ~ 1 order for $B(q^2)$ in the region of its minimum). Besides that, calculation in other system of reference (with other direction of Z-axis obtained, for example, by rotation around q), which is less convenient but equivalent in principle to the first one, would give other result. So, the problem concerned consists in ambiguities of the form factors found in the light front dynamics.

In the framework of the relativistic quantum mechanical approach with fixed number of particles this problem was investigated in refs.11,12). In these papers an attempt was undertaken to construct a covariant operator of current $\hat{J}_\mu(0)$ which is transformed by means of the same Poincare group generators (depending on interaction) which act on WF. For such a covariant current operator the decomposition (1.1) determined by three form factors would be valid. In ref.12) a set of rather complicated constraints for $\hat{J}_\mu(0)$ was obtained. They were studied in ref.13) in the lowest order of the expansion of the current operator in powers of the momentum transfer, but closed solution for $\hat{J}_\mu(0)$ taking into account an interaction, was not found.

In the present paper we give solution of the problem of electromagnetic form factors in the light front dynamics. This solution is obtained by means of explicitly covariant approach developed in ref.¹⁴⁾ (see also review²⁾). In this approach WF is defined on the light front surface given by invariant equation $\omega x = 0$, where ω is four-vector: $\omega = (\omega_0, \vec{\omega})$, $\omega^2 = 0$. Detailed group-theoretical analysis of this approach is given in refs.¹⁵⁾. Particular case $\omega = (1, 0, 0, -1)$ corresponds to the light front equation $t + z = 0$ mentioned above. The problem considered manifests itself in our case in the fact, that owing to appearance of four-vector ω the number of the spin structures and, hence, the number of corresponding invariant functions (form factors) in decomposition of electromagnetic vertex of a system in invariant amplitudes exceeds the number of physical form factors. For example, as it will be shown below (sect.6) for the case of deuteron instead of 3 form factors we have 11 ones, the spin structures at new 8 form factors depending on ω . Increase of the number of spin structures just corresponds to increase of the number of independent matrix elements $\langle \lambda' | J_+ | \lambda \rangle$ expressed through WF. Due to explicit relativistic covariance of the approach¹⁴⁾ the form factors do not depend on a system of reference and on direction of z -axis.

In the theory at $t + z = 0$ the contribution of these structures is not separated from contribution of physical form factors. In our approach the physical form factors and other contributions are separated from each other by indication of their independence or, on the contrary, the dependence on ω . This allows to obtain the physical form factors in terms of initial electromagnetic vertex in explicit form (for a system with $J = 1$ the form factors are given by eqs.(6.9-11) in sect.6, for a system with $J=1/2$ - by eqs.(A.5), (A.6) in appendix).

Plan of the paper is the following. In sect.2 the properties of WF's and the graph technique on the light front $\omega x = 0$ is briefly described. The problem concerned is explained in the simplest example of spinless system. Electromagnetic vertex of such a system is determined now by two form factors instead of one. In sect.3, assuming that the spinless system in its turn consists of two spinless particles, we express both (physical and unphysical) form factors of this system through WF. In sect.4 it is shown that graphs not incorporated previously and not expressed through WF (in particular, so called instantaneous exchanges) must cancel completely the contribution depending on ω in electromagnetic

vertex, but do not change the contribution independent of ω . By this way, the latter gives the physical form factors. In sect.5 in the framework of the model calculation we check that independent of ω contribution to form factor of spinless system does coincide with electromagnetic form factor calculated independently by means of the Bethe-Salpeter function. In sect.6 the decomposition of the light front electromagnetic vertex of arbitrary system with $J = 1$ (in particular, for deuteron) in invariant amplitudes is obtained. It contains 11 terms. Explicit formulas for 3 physical form factors are given (eqs.(6.9-11)). In sect.7 we express these form factors through WF, assuming that the system with $J = 1$ consists of two spinless particles. In sect.8 the check analogous to one carried out in sect.5 is fulfilled for the form factors of system with $J = 1$. Sect.9 contains concluding remarks. In appendix the decomposition of the light front electromagnetic vertex of a system with $J = 1/2$ in invariant amplitudes is obtained and corresponding explicit expressions for form factors are given.

2. Electromagnetic vertex of spinless particle

We begin with the simplest case of electromagnetic form factor of spinless system (e.g. π -meson). The form factor $F(q^2)$ of this system is defined by the equality:

$$\langle p' | J_p(0) | p \rangle = (p + p')_p F(q^2) \quad (2.1)$$

Our aim is to express the form factor $F(q^2)$ through the light front WF. We will work in the covariant approach mentioned above with the light front defined by the invariant equation $\omega x = 0$. As it was shown in ref.^{14a)} (see ref.²⁾), two-body WF on the light front $\omega x = 0$ is parametrized in the momentum space as follows:

$$\psi = \psi(k_1, k_2, p, \omega\tau) \quad (2.2)$$

where $k_1 + k_2 = p + \omega\tau$; k_1 and k_2 are the constituent momenta, p is the total momentum of the system, τ is a scalar parameter. All the four-momenta are on the respective mass shells: $k_1^2 = k_2^2 = m^2$, $p^2 = M^2$, $(\omega\tau)^2 = 0$. Four-momentum $\omega\tau$ in eq.(2.2) means that WF is always off-energy-shell. On the mass shell $k_1^2 = k_2^2 = m^2$, $p^2 = M^2$ and on the energy shell (i.e., under the condition $k_1 + k_2 = p$) WF would turn into a constant. We call non-equal to zero (and proportional to ω) difference $k_1 + k_2 - p = \omega\tau (\neq 0)$ as the four-momentum of a

spurion.

From these four-vectors one can construct the pair of variables $\mathbf{R}_\perp$, x analogous to well known variables in the infinite momentum frame^{2,14)}:

$$\mathbf{R} = \mathbf{k}_1 - x\mathbf{p}, \quad \mathbf{R} = (\mathbf{R}_\perp, \mathbf{R}_\parallel, R_0), \quad x = \omega \mathbf{k}_1 / \omega \mathbf{p} \quad (2.3)$$

where $\mathbf{R}_\parallel \parallel \vec{\omega}$, $\mathbf{R}_\perp \cdot \vec{\omega} = 0$. Since $R\omega = 0$, it follows that $\mathbf{R}_\parallel = R_0$ and $R^2 = -\mathbf{R}_\perp^2 = \text{inv. WF}$ depends on $\mathbf{R}_\perp$ and x : $\psi = \psi(\mathbf{R}_\perp, x)$.

Another pair of variables consists of relativistic relative momentum $\mathbf{q}$ (coinciding with $\mathbf{k}_1$ in the reference system, where $\mathbf{k}_1 + \mathbf{k}_2 = 0$) and vector $\mathbf{n}$ (direction of $\vec{\omega}$ in this system): $\psi = \psi(\mathbf{q}, \mathbf{n})$. This pair of variables is convenient for smooth matching with nonrelativistic WF. Explicit expressions for $\mathbf{q}$, $\mathbf{n}$ in terms of initial four-momenta are given in refs.^{2,14)}.

Calculating machinery is given by special graph technique developed by Kadyshevsky¹⁶⁾ and adjusted to the light front case in ref.^{14a)}. Kadyshevsky's technique¹⁶⁾ is invariant representation of time-ordered perturbation theory. By replacement of all the variables according to eq.(2.3) its light front version^{14a)} can be transformed^{14b)} to the form of the time-ordered perturbation theory in the infinite momentum frame (given by the Weinberg rules¹⁷⁾).

The graphs in this graph technique are obtained from the given Feynman graph by numbering its vertices by all the possible ways and by consecutive passing through them the spurion (dashed) line. Four-momenta of all the particles, including intermediate ones, are on the respective mass shells. Corresponding propagator is $\theta(\omega p) \delta(p^2 - m^2)$ (multiplied by $(\hat{p} + m)$ for the case of spin 1/2). Spurion has four-momentum $\omega \tau$ (its propagator is $1/[2\pi(\tau - i0)]$) and ensures conservation of four-momenta in the vertices, like the case of WF. One should integrate over all the internal particle momenta and over the scalar parameters τ in infinite limits taking into account conservation of four-momenta in the vertices, including spurion four-momenta.

We emphasize that in any given order of perturbation theory any given Feynman on-mass-shell amplitude and the sum of the on-energy-shell light-front amplitudes, obtained from this Feynman amplitude, coincide with each other. Every light front amplitude in this sum depends on ω , but this dependence disappears in the total sum. There exists one-to-one correspondence (ref.^{14b)}) between graphs and amplitudes of this light front graph technique and the Weinberg

technique (ref.17)).

We shall come to expression of form factors through WF by calculating the amplitude corresponding to graph fig.1a. For simplicity we consider a two-body system, but all the results are evidently applied to a many-body systems. First of all we pay attention to the fact that this amplitude, generally speaking, is not factorized in a product of two vertices of the photon interaction with colliding particles and the photon propagator. This problem was discussed in ref.14b).

Indeed, let us write down the part of expression for amplitude of graph fig.1a corresponding to the photon propagator and adjoined spurion lines:

$$M \sim \int (...) \theta(\omega(p' - p)) \delta[(p' - p + \omega\tau_2 - \omega\tau_3)^2] \frac{d\tau_2}{\tau_2 - i0} \frac{d\tau_3}{\tau_3 - i0} \quad (2.4)$$

The dots designate all the rest part of amplitude. For definiteness we suppose that $\omega(p' - p) > 0$. Then the amplitude of the graph, which differs from fig.1a by the direction of the photon line, contains $\theta(\omega(p - p'))$ and therefore equals zero. Integrating eq.(2.4) over $d\tau_2$ by means of δ -function, we get:

$$M \sim \int (...) \frac{d\tau_3}{(2\omega q\tau_3 - q^2 - i0)(\tau_3 - i0)} \quad (2.5)$$

From eq.(2.5) it is seen that this amplitude does not contain explicitly the factor $1/q^2$, that does not allow to extract a form factor. Absence of factorization is connected with the fact that, as we have already emphasized, only the sum of all the light-front graphs coincides with the Feynman amplitude, containing $1/q^2$ and independent of ω .

However, as it is seen from eq.(2.5), the factorization can be obtained under the following condition 14b):

$$\omega(p' - p) = \omega q = 0 \quad (2.6)$$

implied as limit $\omega q \rightarrow +0$. Under this condition the electromagnetic vertex can be represented by the graph fig.1b with the photon four-momentum $q^2 \neq 0$. Since the vertex function F_p depends on ω besides p and p' , then its structure has the form:

$$F_p = (p + p')_p F(q^2) + \omega_p \frac{(p + p')^2}{2(\omega p)} g_1(q^2) \quad (2.7)$$

The factor $(p + p')^2/2(\omega p)$ at g_1 is separated for convenience.

The invariant functions F and g_1 could depend on the scalar ωq ,

but under the condition (2.6) this dependence is absent.

In the case of spinless system the problem considered in the present paper consists, firstly, in appearance in eq.(2.7) of the extra item proportional to ω_p (besides the form factor $F(q^2)$) and, secondly, in existence of infinite number of graphs not expressed through WF but having the same order relative to electromagnetic interaction as the graph fig.1a has. Some of these graphs are shown in fig.2. The problem is in the following: how to deal with the item in eq.(2.7) proportional to ω_p and how to take into account this infinite series of graphs not reduced to the graph fig. 1a.

3. Spinless constituents in the state with $J = 0$

Let us calculate for example the functions F and g_1 for the case of spinless constituents. Amplitude of graph fig.1b has the form:

$$F_p = \int [(p + \omega\tau_2 - k)_p + (p' + \omega\tau_1 - k)_p] \theta(\omega(p - k)) \delta((p + \omega\tau_2 - k)^2 - m^2) \times \theta(\omega(p' - k)) \delta((p' + \omega\tau_1 - k)^2 - m^2) \theta(\omega k) \delta(k^2 - m^2) \Gamma_1 \Gamma_2 \times \frac{d\tau_1}{2\pi(\tau_1 - i0)} \frac{d\tau_2}{2\pi(\tau_2 - i0)} d^4k \quad (3.1)$$

Three factors $\theta(\dots)\delta(\dots)$ in eq.(3.1) are the particle propagators, the factors $1/\tau_j$ are the spurion propagators, Γ 's are the bound state vertices. Integrating by means of δ -functions over $d\tau_1 d\tau_2 dk_0$ we get:

$$F_p = \frac{1}{(2\pi)^2} \int \frac{(p + p' + \omega\tau_1 + \omega\tau_2 - 2k)_p \theta(\omega(p - k)) \theta(\omega(p' - k))}{(m^2 - (p - k)^2 - i0)(m^2 - (p' - k)^2 - i0)} \Gamma_1 \Gamma_2 \frac{d^3k}{2\varepsilon(k)} \quad (3.2)$$

where $\varepsilon(k) = (k^2 + m^2)^{1/2}$.

The vertex Γ is connected with WF ψ by the relation^{14a)} :

$$\psi(k_1, k_2, p, \omega\tau) = \sqrt{\frac{2\pi}{m}} \frac{\Gamma}{s_1 - M^2} \quad (3.3)$$

where $s_1 = (k_1 + k_2)^2 = (p + \omega\tau)^2$. Taking into account that $m^2 - (p - k)^2 = (1 - \omega k/\omega p) \cdot (s_1 - M^2)$ and analogously for $m^2 - (p' -$

$k)^2$, we obtain:

$$F_p = \frac{m}{(2\pi)^3} \int \frac{(\underline{p} + \underline{p}' + \omega\tau_1 + \omega\tau_2 - 2k)_p \theta(\omega(\underline{p} - k)) \psi_1 \psi_2 \frac{d^3 k}{2\varepsilon(k)}}{(1 - \omega k/\omega p)^2} \quad (3.4)$$

where ψ_1 and ψ_2 depend on the momenta in the vertices 1 and 2 respectively. Equating right-hand-sides of eqs.(3.4) and (2.7) with each other and multiplying them in turn by ω_p and $(\underline{p} + \underline{p}')_p$, we get:

$$F(q^2) = \frac{m}{(2\pi)^3} \int \frac{\theta(\omega(\underline{p} - k))}{(1 - x)} \psi_1 \psi_2 \frac{d^3 k}{2\varepsilon(k)} \quad (3.5)$$

$$g_1(q^2) = \frac{m}{(2\pi)^3} \int \frac{\theta(\omega(\underline{p} - k))}{(1 - x)^2} \left\{ x + 2 \frac{x[k(\underline{p} + \underline{p}')] - M^2}{(\underline{p} + \underline{p}')^2 (1 - x)} \right\} \psi_1 \psi_2 \frac{d^3 k}{2\varepsilon(k)} \quad (3.6)$$

where $x = \omega k/\omega p$. It is convenient to introduce the variables defined by the equality (2.3). Analogously to eq.(2.3) we represent the four-momentum transfer: $q = (\vec{\Delta}, \mathbf{q}_{||}, q_0)$, where $\vec{\Delta}\vec{\omega} = 0$, $\mathbf{q}_{||}||\vec{\omega}$, $q^2 = -\vec{\Delta}^2$. In terms of variables $\mathbf{R}_\perp, x$ the expressions for F, g_1 obtains the form:

$$F(q^2) = \frac{m}{(2\pi)^3} \int \psi((\mathbf{R}_\perp^2, x) \psi((\mathbf{R}_\perp - x\vec{\Delta})^2, x) \frac{d^2 \mathbf{R}_\perp dx}{2x(1-x)} \quad (3.7)$$

$$g_1(q^2) = \frac{2m}{(2\pi)^3} \int \frac{(\mathbf{R}_\perp^2 + m^2 - M^2(1-x)^2 - x\mathbf{R}_\perp\vec{\Delta} + x\vec{\Delta}^2/2)}{4M^2 + \vec{\Delta}^2} \times \psi((\mathbf{R}_\perp^2, x) \psi((\mathbf{R}_\perp - x\vec{\Delta})^2, x) \frac{d^2 \mathbf{R}_\perp dx}{2x(1-x)^3} \quad (3.8)$$

where $0 < |\mathbf{R}_\perp| < \infty$, $0 < x < 1$

The formula (3.7) has been found in refs.¹⁸⁾.

For the system with small binding energy $|\varepsilon| \ll m$ WF is

concentrated in the main at $x \sim 1/2$, $R_{\perp}^2 \sim m|\epsilon|$. For such a system the function g_1 at $|q^2| \ll m^2$ has smallness $\sim |\epsilon|/m$ in comparison with F , that agrees with its relativistic origin. As it will be shown in sect.5, at $|q^2| \lesssim m^2$ the function g_1 becomes comparable with F . Therefore the problem connected with appearance in eq.(2.7) of the item containing g_1 obtains particular acuteness.

4. Graphs not expressed through WF.

In order to solve this problem we analyse the structure of graphs not expressed through WF. We show that all corresponding them non-zero amplitudes, like the item with g_1 in eq.(2.7), are proportional to ω_p (or depends on ω through a tensor structures in the case of non-zero spin).

At first we consider the graph fig.2a and show that in the case when all the particles including photon are scalar ones, the corresponding amplitude tends to zero at $\omega(p' - p) \rightarrow 0$. This amplitude is determined by the integral over the product of four pairs of θ - and δ -functions corresponding to the particle propagators and of three factors $1/\tau_j$ (the spurion propagators) :

$$M = \frac{g^2}{2\pi} \int \theta(\omega(p' - k)) \delta[(p' + \omega\tau_1 - k)^2 - m^2] \theta(\omega(p - k)) \\ \times \delta[(p - k + \omega\tau_2 - \omega\tau_3)^2 - m^2] \theta(\omega k) \delta(k^2 - m^2) \theta(\omega(p' - p)) \\ \times \delta[(p' - p + \omega\tau_3)^2] \Gamma_1 \Gamma_3 \frac{d\tau_1}{2\pi(\tau_1 - i0)} \frac{d\tau_2}{2\pi(\tau_2 - i0)} \frac{d\tau_3}{2\pi(\tau_3 - i0)} d^4k \quad (4.1)$$

Here g is the interaction constant of a scalar photon; Γ_1 and Γ_3 are the vertex functions for the vertices 1 and 3 respectively.

From the arguments of the δ -functions we find :

$$\tau_1 = (m^2 - (p' - k)^2)/2\omega(p' - k) \quad (4.2a)$$

$$\tau_2 = \tau_3 + (m^2 - (p - k)^2)/2\omega(p - k) \quad (4.2b)$$

$$\tau_3 = -(p' - p)^2/2\omega(p' - p) \quad (4.2c)$$

Integrating over $d\tau_j$ ($j = 1, 2, 3$) and over dk_0 , we get :

$$M = - \frac{g^2}{(2\pi)^4} \frac{1}{q^2} \times \int \frac{\theta(\omega(p' - p))\theta(\omega(p - k))\theta(\omega(p' - k))}{[m^2 - (p' - k)^2 - i0][m^2 - (p - k)^2 - \frac{\omega(p - k)}{\omega(p' - p)}(p' - p)^2 - i0]} \times \Gamma_1 \Gamma_3 \frac{d^3 k}{2\varepsilon(k)} \quad (4.3)$$

At $\omega(p' - p) \rightarrow 0$ the factor $\omega(p - k)/\omega(p' - p)$ in the denominator in eq.(4.3) tends to infinity and the amplitude M tends to zero.

As it is seen from eqs.(4.2a-c) the factor $1/\omega(p' - p)$ in the denominators $1/\tau_j$ comes twice: in $1/\tau_3$ (here it is canceled by derivative over τ_3 from the argument of the δ -function $\delta((p' - p)^2 + 2\omega(p' - p)\tau_3)$) and in $1/\tau_2$. In its turn this follows from the conservation of four-momenta in the vertices 3 and 4 provided the position of the vertex 4 is left shifted relative to the vertex 3. Therefore it is evident that the amplitude of any graph (for example, with any number of exchanges between constituents and with their arbitrary order in time), provided the emission of photon precedes in time to decomposition of the bound state, tends to zero at $\omega(p' - p) \rightarrow 0$.

Analogously one can prove that the amplitude of graph fig.3, which is expressed through WF but contains an interaction (vertex 3) between the time moments of the photon emission and absorption, tends to zero at $\omega(p' - p) \rightarrow 0$ as well.

The amplitude of the graph fig.2b contains the product of θ -functions $\theta(\omega(k - p))\theta(\omega(p' - k)) = \theta(\omega(k - p))\theta(\omega q - \omega(k - p))$ corresponding to the propagators of the lines 3-2 and 3-1. Therefore at $\omega q \rightarrow 0$ the corresponding integration region tends to zero and, hence, the amplitude becomes proportional to ωq and also tends to zero.

Physical sense of nullifying of the amplitudes concerned becomes transparent in the framework of the equal-time description in the infinite momentum frame with the infinite momentum directed along $\mathbf{p} + \mathbf{p}' \parallel \mathbf{z}$. At this point of view the condition $\omega(p' - p) = \omega q = 0$ means that $\mathbf{q}$ is purely transverse: $\mathbf{q} = (\mathbf{q}_\perp, q_z = 0, q_0 = 0)$. Since $q_0 = 0$, such an exchange does not contain retardation. In this sense

the vertices 4 and 2 in fig.2a and fig.3 are simultaneous. Hence, there is no interaction in the time interval between these vertices and the graphs figs.2a and 3 containing the vertex 3 in this zero time interval do not contribute. The graph fig.2b is absent since transverse exchange with $q_0 = 0$ is unable to create a pair with nonzero energy.

Note that the amplitude corresponding to the graph fig.4a (and any graph of this type), in which an exchange particle (meson) propagates simultaneously with the intermediate photon and have no any interaction during the time interval between emission and absorption of the photon, does not equal zero even if $\omega q = 0$. Such a graph is a particular case of that one shown in fig.4b, which corresponds to the presence of many-body sectors in the state vector. Form factor is determined by the sum of contributions of all the essential Fock components. If an intermediate meson has a charge, one should take into account its interaction with the photon (meson exchange currents).

In the case of the vector photon the amplitude of the graph shown in fig.2b does not tend to zero at $\omega q \rightarrow 0$, but becomes proportional to ω . Indeed, the following factor corresponds to the vertex 3 in fig.2b:

$$(p_1 + p_2)_\rho = (p' - p + \omega \tau_2)_\rho \quad (4.4)$$

where p_1 and p_2 are the momenta corresponding to the lines 3-1 and 3-2,

$$\tau_2 = \frac{m^2 - (p - k)^2}{2\omega(k - p)} + \frac{m^2 - (p' - k)^2}{2\omega(p' - k)} \quad (4.5)$$

The θ -functions lead to the equalities $\omega(p' - p) > 0$, $\omega(k - p) > 0$, $\omega(p' - k) > 0$ and, hence, $\omega p < \omega k < \omega p'$. Therefore, if $\omega q \rightarrow 0$, $\omega p' \rightarrow \omega p$, the denominators in eq.(4.5) tend to zero and τ_2 increases up to infinity. This leads to the compensation of small factor ωq mentioned above. As a result, the amplitude of the graph in fig.2b has the finite limit proportional to ω . Since the parameter ω appears in numerator only in the combination $\omega \tau$, it is clear that any graph not expressed through WF and surviving under the condition $\omega q = 0$ will contain ω . In the case of intermediate particle with spin 1/2 the factor ω occurs from the four-momentum $\hat{p}$ in the propagator $(\hat{p} + m)\theta(\omega p)\delta(p^2 - m^2)$.

Note that the graphs shown in figs.2a, 3 tends to zero at $\omega q \rightarrow 0$ even for the vector photon, since the terms $\omega \tau_j$ in numerators of corresponding amplitudes come in such combinations that the items

containing the factor $1/\omega(\mathbf{p}' - \mathbf{p})$ cancel each other.

Thus, the final expression for electromagnetic vertex of the system considered obtains the following form:

$$F_p = (\mathbf{p} + \mathbf{p}')_p F(q^2) + \omega_p \frac{(\mathbf{p} + \mathbf{p}')^2}{2\omega p} g_1(q^2) + \omega_p \frac{(\mathbf{p} + \mathbf{p}')^2}{2\omega p} g_2(q^2) \quad (4.6)$$

where $F(q^2)$ and $g_1(q^2)$ are given by eqs. (3.7) and (3.8) respectively, $g_2(q^2)$ is the total contribution of the graphs not expressed through WF.

As it was noted above, the representation of a Feynman graph as a sum of covariant time-ordered graphs results in the dependence of corresponding amplitudes on ω ; this dependence vanishing after the summation of all these graphs. In the framework of perturbation theory it was explicitly shown in ref.12). Similarly, an amplitudes of the time-ordered perturbation theory are not covariant but their sum is covariant. The vertex function defined by eq. (4.6) also can not depend on an intermediate auxiliary construction - the light front hypersurface - which is used for correct definition of relativistic WF (depending on this hypersurface). On the light front $t+z=0$ (four-vector ω is fixed) the dependence on ω turns into the dependence on frame of reference. The latter is impossible in any Lorentz-invariant theory. Therefore the following equality must be valid:

$$g_1(q^2) + g_2(q^2) = 0 \quad (4.7)$$

So, the problem of ω -dependence of electromagnetic vertex is resolved by the following way. It is necessary to confine ω by the condition $\omega q = 0$ and decompose the vertex function expressed through WF in invariant amplitudes keeping the structures which depend on ω . Under the condition $\omega q = 0$ the contribution of all the graphs not expressed through WF depends on ω and, being taken into account, it completely cancels the ω -dependent part of the vertex function, but does not change its ω -independent part. In practice it is hardly possible to calculate this contribution directly, but as it was shown, there is no need to do this.

Thus, the practical recipe consists in decomposition of the vertex function shown in fig.1b (or of corresponding vertex function for many-body system) in invariant amplitudes and in calculation of the coefficients at the ω -independent structures. These coefficients are just the physical observable form factors. In such a form this recipe is valid for arbitrary spins of the bound system and of its constituents. In the simplest case of scalar particles, as it is

seen from eq.(2.7), in order to find the physical form factor $F(q^2)$ one should multiply the expression (3.4) by ω_p . We emphasize that in spite of the fact that in practice the ω -dependent terms in decomposition of the vertex function are omitted, their extraction is necessary for correct calculation of the physical form factors.

5. System with $J=0$. The model

Mutual cancellation of the terms containing g_1 and g_2 in eq.(4.6), following from rather general principles of relativistic invariance, is a very delicate property of the theory since the functions g_1 and g_2 have quite different origin: g_1 is expressed through WF according to eq.(3.8), while g_2 is not expressed through WF and, hence, hardly can be calculated in explicit form. To be sure that our statement is correct and only the first term in the right-hand-side of eq.(4.6) survives, we will calculate the electromagnetic form factor in the Wick-Cutkosky model^{19,20} (two scalar particles interacting by exchange of scalar massless particle) by two following methods:

(i) Independently from the approach discussed above, namely, through the Bethe-Salpeter function which is known in this model;

(ii) By means of the formula (3.7).

The coincidence of results of these two methods, where the calculation (i) plays role of a standard, confirms that the terms with g_1 and g_2 in eq.(4.6) indeed cancel each other and the correct result for the form factor is given by the single term with F . Here for the completeness of our exposition we repeat some results of the paper^{14d}).

The matrix element of the electromagnetic current operator corresponding to the Feynman graph shown in fig.5 is given by the following analytical expression:

$$\begin{aligned}
 (p + p')_\rho F(t) &= i \int (p + p' - 2k_1)_\rho \Phi\left(\frac{p}{2} - k_1, p\right) \Phi\left(\frac{p'}{2} - k_1, p'\right) (m^2 - k_1^2) \frac{d^4 k_1}{(2\pi)^4} \\
 &= - \frac{ic^2}{4\pi} \int (p + p' - 2k)_\rho \Psi(p, p', k) \frac{d^4 k}{(2\pi)^4} \quad (5.1)
 \end{aligned}$$

where Φ is the Bethe-Salpeter function, $t = q^2$. The solution for this

function found in refs.¹⁹⁾ (see the reviews²⁰⁾) can be represented in the form:

$$\Phi(k, p) = \frac{-ic}{\sqrt{4\pi}} \left[(m^2 - \frac{M^2}{4} - k^2 - i0)(m^2 - (\frac{p}{2} + k)^2 - i0)(m^2 - (\frac{p}{2} - k)^2 - i0) \right]^{-1} \quad (5.2)$$

where $c = 2^6 \pi m^{1/2} \kappa^{5/2}$, $\kappa = (m|\epsilon|)^{1/2} = m\alpha/2$, $|\epsilon| = 2m - M = m\alpha^2/4$, ϵ is the binding energy, $\alpha = g^2/(16\pi m^2)$, g is the coupling constant in Hamiltonian $H^{int} = -g\phi^2\chi$, ϕ is the scalar field with mass m , χ is the massless field. The function Ψ is given by the formula:

$$\Psi(k, p, p') = \left[(m^2 - \frac{M^2}{4} - (\frac{p}{2} - k)^2 - i0)(m^2 - \frac{M^2}{4} - (\frac{p'}{2} - k)^2 - i0) \right. \\ \left. \times (m^2 - (p - k)^2 - i0)(m^2 - (p' - k)^2 - i0)(m^2 - k^2 - i0) \right]^{-1} \quad (5.3)$$

This solution is valid for the case of $|\epsilon| \ll m$ ($\alpha \ll 1$).

Let us calculate the form factor in the most interesting relativistic region of the momentum transfer $|t| \gg m^2$. Neglecting the difference $m^2 - k^2$ in comparison with $pk, p'k$, we can approximately write:

$$m^2 - \frac{M^2}{4} - (\frac{p}{2} - k)^2 \approx pk = \frac{m^2 - (p - k)^2}{2}$$

(and similarly for p'). By means of the Feynman parametrization the function Ψ can be represented as follows:

$$\Psi = 4 \cdot 4! \int_0^1 dx \int_0^1 dy \frac{xy}{[(k - d)^2 - \beta^2 + i0]^5} \quad (5.4)$$

where $d = py + p'x$, $\beta^2 = |t|xy + m^2 - M^2(x + y)(1 - x - y)$. Multiplying both sides of eq.(5.1) by $(p + p')_\rho$, we find after integration over d^4k :

$$F(t) \approx - \frac{2^3 m^6 \alpha^5}{\pi} \frac{d}{d|t|} J(|t|) \quad (5.5)$$

where

$$J(|t|) = \int_0^1 dx \int_0^{1-x} dy \frac{1 - x - y}{[|t|xy + m^2 - M^2(x + y)(1 - x - y)]^2} \quad (5.6)$$

Introducing new variables $u = xy, w = x + y$, we can rewrite eq.(5.6) in the following form:

$$J(|t|) = \frac{2}{m^4} \int_0^{1/4} du \int_{2\sqrt{u}}^1 dw \frac{1}{\sqrt{w^2 - 4u}} \frac{1-w}{\left[\frac{|t|}{m^2} u + 4 \left(\frac{1}{2} - w \right)^2 + \alpha^2 w(1-w) \right]^2} \quad (5.7)$$

If $|t|/m^2 \gg 1$, $\alpha \ll 1$, then the following two regions give the main contribution to eq. (5.7):

- (1) $u \ll 1, w \ll 1$
- (2) $u \ll 1, |1/2 - w| \ll 1$

For the contribution $J^{(1)}$ from the region (1) we find:

$$J^{(1)}(|t|) = \frac{2}{m^4} \int_0^{1/4} du \frac{1}{\left(\frac{|t|}{m^2} u + 1 \right)^2} \int_{2\sqrt{u}}^1 dw \frac{1}{\sqrt{w^2 - 4u}} \approx \frac{1}{m^2 |t|} \left\{ \ln \left(\frac{|t|}{m^2} \right) + \dots \right\} \quad (5.8)$$

where the dots denote the term of the order of 1. The contribution $J^{(2)}$ from the region (2) has the form:

$$J^{(2)}(|t|) = \frac{2}{m^4} \int_0^{1/4} du \int_0^1 dw \frac{1}{\left[\frac{|t|}{m^2} u + 4 \left(\frac{1}{2} - w \right)^2 + \frac{\alpha^2}{4} \right]^2} \approx \frac{2\pi}{\alpha m^2 |t|} \quad (5.9)$$

The substitution of $J = J^{(1)} + J^{(2)}$ in eq. (5.5) gives:

$$F(t) = \frac{2^3 \alpha^5 m^4}{\pi |t|^2} \left\{ \ln \left(\frac{|t|}{m^2} \right) + \frac{2\pi}{\alpha} \right\} \quad (5.10)$$

Logarithmic part of eq. (5.10) was found in ref.²¹). In all "reasonable" region of t under the condition $\alpha \ll 1$ the second term in the right-hand-side of eq. (5.10) dominates in comparison with the first one.

Now we will calculate the form factor in terms of the light front WF. For the system considered this WF was found in ref.^{14d}). It has the following form:

$$\psi(q, n) = \frac{2^3 \pi^{1/2} \kappa^{5/2}}{(q^2 + \kappa^2)^2 (1 + |nq|/\epsilon(q))} \quad (5.11)$$

WF (5.11) differs from the nonrelativistic Coulomb WF of the ground state by the factor depending on n and also by the construction of relativistic relative momentum q . In terms of variables $R_\perp$ and x (eq. (2.3)) this WF is rewritten as^{14d})

$$\psi(R_{\perp}, x) = \frac{2^7 \pi^{1/2} \kappa^{5/2}}{\left[\frac{R_{\perp}^2 + m^2}{x(1-x)} - M^2 \right]^2 (1 + |1 - 2x|)} \quad (5.12)$$

Substituting eq. (5.12) in eq. (3.7), integrating over $d\phi$ in $d^2 R_{\perp} = R_{\perp} dR_{\perp} d\phi$ and introducing the variable $y = R_{\perp}/m$, we find:

$$F(t) = \frac{2^6 \alpha^5}{\pi} \int_0^1 dx x^3 (1-x)^3 \int_0^{\infty} dy y \frac{y^2 + \left(\frac{x\Delta}{m}\right)^2 + a^2}{(1 + |1 - 2x|)^2 (y^2 + a^2)^2 D^{3/2}} \quad (5.13)$$

where

$$a^2 = (1 - 2x)^2 + \alpha^2 x(1 - x) \quad (5.14)$$

$$D = [(y - x\Delta/m)^2 + a^2][(y + x\Delta/m)^2 + a^2] \quad (5.15)$$

If $|t|/m^2 \gg 1$, $\alpha \ll 1$, the integral (5.13) is determined by contributions from the following two regions:

(1) $m/\Delta \ll x \ll 1$; (a) $y \sim 1$, (b) $y \sim x\Delta/m \gg 1$

(2) $|x - 1/2| \ll 1$, (a) $y \sim a$, (b) $y \sim x\Delta/m \sim \Delta/2m$ we denote these contributions as $I^{(1)}$ and $I^{(2)}$ respectively. The functions $I^{(1)}$ and $I^{(2)}$ are given by the following expression:

$$\begin{aligned} I^{(1)} &= \frac{2^4 \alpha^5 m^2}{\pi \Delta^2} \int_0^1 \frac{x dx}{(x\Delta/m)^2 + 1} \int_0^{\infty} \frac{y dy}{(y^2 + 1)^2} \\ &+ \frac{2^2 \alpha^5 m^2}{\pi \Delta^2} \int_0^1 \frac{x dx}{(x\Delta/m)^2 + 1} \int_{-\infty}^{\infty} \frac{dy}{(y^2 + 1)^{3/2}} = \frac{2^3 m^4 \alpha^5}{\pi |t|^2} \ln \frac{|t|}{m^2} \\ I^{(2)} &= \frac{2^4 \alpha^5 m^4}{\pi \Delta^4} \int_0^1 dx \int_0^{\infty} \frac{y dy}{[y^2 + (2x - 1)^2 + \alpha^2/4]^2} \end{aligned} \quad (5.16)$$

$$+ \frac{2^2 \alpha^5 m^4}{\pi \Delta^4} \int_0^1 dx \int_{-\infty}^{\infty} \frac{dy}{[y^2 + (2x-1)^2 + \alpha^2/4]^{3/2}} \approx \frac{2^3 m^4 \alpha^5}{\pi |t|^2} \frac{2\pi}{\alpha} \quad (5.17)$$

Two items in the right-hand-side of eqs. (5.16) and (5.17) correspond to contributions from the regions (a) and (b) respectively. The sum $F(t) = I^{(1)} + I^{(2)}$ coincides with the formula (5.10).

The function g_1 is obtained by substituting WF (5.12) in eq. (3.8). We adduce here its asymptotics at $|t| \gg m^2$:

$$g_1 \approx 2^5 \alpha^4 (m^2/|t|)^2 \quad (5.18)$$

Numerical calculation (see table 1) shows that this coincidence takes place not only in the asymptotic region $|t| \gg m^2$ but for any values of momentum transfer. Expected accuracy of the coincidence has the order of α , since from a Feynman graph we obtain, in particular, time-ordered graphs corresponding to the meson exchange between the vertices. Such graphs are taken into account automatically by calculation with the Bethe-Salpeter function, but they do not contribute when only two-body Fock component is kept in the state vector. In the case of $\alpha \ll 1$ these graphs, containing intermediate many-body states, have a relative order of α and are negligible.

Thus, we have demonstrated that the form factor is in fact determined by ω -independent part of eq. (4.6) and, hence, the terms with g_1 and g_2 cancel each other. This cancellation is rather nontrivial, since the functions g_1 and F in the region $|t| \gg m^2$ (see fig. 6) become comparable with each other (at large $|t|$ the function g_1 is even greater than F).

If the momentum transfer is small in comparison with the particle mass, the form factor is expressed through nonrelativistic WF:

$$F(t) = \int \psi^2(r) \exp\left(\frac{i}{2} \vec{\Delta} \cdot \vec{r}\right) d^3 r \quad (5.19)$$

Substituting the Coulomb WF $\psi(r) = \kappa^{3/2} e^{-\kappa r} / \pi^{1/2}$ in eq.(5.19) we find:

$$F(t) = \frac{2 \alpha^4 m^4}{(|t| + 4 \alpha m)^2} \quad (5.20)$$

Note that nonrelativistic formula (5.20) practically coincides with the results of calculations of the form factor using the Bethe-Salpeter function (eqs.(5.1), (5.2)) and the light front WF (eqs.(3.7), (5.12)) not only in nonrelativistic limit $|t| \ll m^2$, but also in the region $|t| \gtrsim m^2$ where logarithmic term in eq.(5.10) is still negligible. This is seen from the table 1. Asymptotic form of eq.(5.20) is identical to the second term in the right-hand-side of eq.(5.10).

6. Formfactors of a system with $J = 1$

The analysis carried out below is applicable to arbitrary system with spin 1, in particular, to the important case of deuteron.

The calculation of the graph shown in fig.1b leads to dependence of the results on ω and to appearance of eight additional spin structures in the corresponding vertex function:

$$I_{\lambda\lambda}^{\rho} = e_{\mu}^{\lambda'}(p') \{ T_{\mu\nu}^{\rho} + B_{\mu\nu}^{\rho} e_{\nu}^{\lambda}(p) \} \quad (6.1)$$

where $T_{\mu\nu}^{\rho}$ represented by eq.(1.1) and is determined by the physical form factors F_1 , F_2 and G_1 .

$$B_{\mu\nu}^{\rho} = \omega_p [B_1 g_{\mu\nu} + B_2 q_{\mu} q_{\nu} + B_3 \omega_{\mu} \omega_{\nu} + B_4 (q_{\mu} \omega_{\nu} - q_{\nu} \omega_{\mu})] + B_5 p_{\rho} \omega_{\mu} \omega_{\nu} + B_6 p_{\rho} (q_{\mu} \omega_{\nu} - q_{\nu} \omega_{\mu}) + B_7 (g_{\mu\rho} \omega_{\nu} + g_{\nu\rho} \omega_{\mu}) + B_8 q_{\rho} (q_{\mu} \omega_{\nu} + q_{\nu} \omega_{\mu}) \quad (6.2)$$

Here $P = p + p'$, $B_1, \dots, B_8$ are invariant functions. Tensor $B_{\mu\nu}^{\rho}$ as well as $T_{\mu\nu}^{\rho}$ is symmetric with respect to the simultaneous permutations $p \leftrightarrow p'$ ($q \rightarrow -q$) and $\mu \leftrightarrow \nu$. Note that the structures at B_7 and B_8 in eq.(6.2) (and, hence, all the expression (6.1)) do not satisfy the current conservation, that is $q_{\rho} I_{\lambda\lambda}^{\rho} \neq 0$.

The total number of independent spin structures can be calculated without their explicit construction. Owing to ω -dependence, the helicity indices in the vertex function is not

restricted by the equality $\lambda = \lambda_1 + \lambda_2$. (λ is the photon helicity, λ_1, λ_2 are the hadron helicities) Therefore we have $3 \times 3 \times 3 = 27$ initial structures. P-invariance reduces this number to 14. T-invariance (or C-invariance in annihilation channel) eliminates 4 structures in addition and finally 10 independent structures remain. This calculation automatically takes into account the condition of transversality of electromagnetic current which would lead to a relation between B_7 and B_8 ; in this case the number of independent invariant amplitudes would reduce from 11 to 10. However ω -dependent terms in eq.(6.1) not obligatory satisfy this condition. Therefore the functions B_7 and B_8 are independent and we obtain 11 structures (or invariant amplitudes): there are 3 structures in $T_{\mu\nu}^P$ and 8 ones in $B_{\mu\nu}^P$.

Incorporation of the graphs not expressed through WF leads to independence of results on ω , i.e. to the cancellation of the ω -dependent structures in eq.(6.1). Hence, the total vertex function is determined by three form factors F_1, F_2 and G_1 , according to eq.(1.1). These form factors in principle could be found (i.e., expressed through the vertex function $I_{\lambda\lambda}^P$) directly from the formula (6.1). However this derivation is rather cumbersome. Therefore we use less cumbersome method. At first we separate a polarization vector in WF: $\psi^\lambda(p) = \epsilon_\nu^\lambda \psi_\nu(p)$, after that we identify WF of the bound system with $\psi_\nu(p)$. Then the graph in fig.1b corresponds to the tensor $J_{\mu\nu}^P$, which has the following structure determined by 20 invariant amplitudes:

$$J_{\mu\nu}^P = A_{\mu\nu}^P + \tilde{B}_{\mu\nu}^P + C_{\mu\nu}^P \quad (6.3)$$

where

$$A_{\mu\nu}^P = P_\rho [A_1 g_{\mu\nu} + A_2 q_\mu q_\nu + A_3 P_\mu P_\nu + A_4 (q_\mu P_\nu - q_\nu P_\mu)] + \\ + A_5 (g_{\mu\rho} q_\nu - g_{\nu\rho} q_\mu) + A_6 (g_{\mu\rho} P_\nu + g_{\nu\rho} P_\mu) + A_7 q_\rho (q_\mu P_\nu + q_\nu P_\mu) \quad (6.4)$$

Tensor $\tilde{B}_{\mu\nu}^P$ has exactly the same structure as $B_{\mu\nu}^P$ (with change of all B_j to $\tilde{B}_j$) and $C_{\mu\nu}^P$ has the following form:

$$C_{\mu\nu}^P = \omega_\rho [C_1 P_\mu P_\nu + C_2 (P_\mu \omega_\nu + P_\nu \omega_\mu) + C_3 (q_\mu P_\nu - q_\nu P_\mu)] + \\ + C_4 P_\rho (P_\mu \omega_\nu + P_\nu \omega_\mu) + C_5 q_\rho (P_\mu \omega_\nu - P_\nu \omega_\mu) \quad (6.5)$$

Because of the identities $\tilde{e}_\mu^{\lambda}(p')P_\mu = -\tilde{e}_\mu^{\lambda}(p')q_\mu$, $P_\nu e_\nu^\lambda(p) = q_\nu e_\nu^\lambda(p)$, the tensor $J_{\mu\nu}^\rho$ being contracted with $\tilde{e}_\mu^{\lambda}(p')$ and $e_\nu^\lambda(p)$ reduces to eq. (6.1):

$$\tilde{e}_\mu^{\lambda}(p') J_{\mu\nu}^\rho e_\nu^\lambda(p) = I_{\lambda\lambda}^\rho \quad (6.6)$$

with

$$F_1 = A_1, \quad F_2/2M^2 = A_2 - A_3 + 2A_4, \quad G_1 = A_5 + A_6 \quad (6.7)$$

$$B_2 = \tilde{B}_2 - C_1 + 2C_3, \quad B_4 = \tilde{B}_4 - C_2, \quad B_6 = \tilde{B}_6 - C_4, \quad B_8 = \tilde{B}_8 - C_5;$$

$$\tilde{B}_j = B_j \text{ if } j = 1, 3, 5, 7. \quad (6.8)$$

Substituting the tensors A , $\tilde{B}$, C from eqs. (6.4), (6.2) and (6.5) in eq. (6.3) and contracting this equality by turns with 20 independent tensor structures, coming in the formulas (6.2), (6.4), (6.5), we obtain the system of 20 equations for A_j , $\tilde{B}_j$ and C_j , the amplitudes of interest ($A_1, \dots, A_6$) being found from the subsystem containing 11 rather simple equations. The result is as follows:

$$F_1 = J_{\mu\nu}^\rho \frac{\omega_\rho}{2(\omega p)} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} - \frac{P_\mu \omega_\nu}{(\omega p)} + P^2 \frac{\omega_\mu \omega_\nu}{4(\omega p)^2} \right] \quad (6.9)$$

$$\frac{F_2}{2M^2} = -J_{\mu\nu}^\rho \frac{\omega_\rho}{2(\omega p)q^2} \left[g_{\mu\nu} - 2 \frac{q_\mu q_\nu}{q^2} - \frac{P_\mu \omega_\nu}{(\omega p)} + M^2 \frac{\omega_\mu \omega_\nu}{(\omega p)^2} - \frac{q_\mu \omega_\nu}{(\omega p)} \right] \quad (6.10)$$

$$G_1 = J_{\mu\nu}^\rho \left\{ g_{\mu\rho} \left[\frac{q_\nu}{q^2} + \frac{\omega_\nu}{2(\omega p)} \right] + \frac{\omega_\rho}{2(\omega p)} \left[-P^2 \frac{q_\mu \omega_\nu}{2(\omega p)q^2} + \frac{q_\mu P_\nu}{q^2} + P^2 \frac{\omega_\mu \omega_\nu}{4(\omega p)^2} - \frac{P_\mu \omega_\nu}{2(\omega p)} \right] \right. \\ \left. + P_\rho \left[\frac{q_\mu \omega_\nu}{2(\omega p)q^2} - \frac{\omega_\mu \omega_\nu}{4(\omega p)^2} \right] - q_\rho \frac{q_\mu \omega_\nu}{2(\omega p)q^2} \right\} \quad (6.11)$$

The formulas (6.9-11) give the solution of the problem of finding the form factors for a bound system with spin 1. The tensor $J_{\mu\nu}^\rho$ is given by the graph in fig.1b (or by corresponding graph for many-body system) and is expressed through WF. The rest of the amplitudes ($\tilde{B}_{1-8}$ and C_{1-5}) also can be found in explicit form, but we do not adduce them here. In spite of the fact that the expressions (6.9-11) include ω explicitly, under the condition $\omega q = 0$ the form

factors F_1 , F_2 and G_1 depend only on q^2 .

7. System with spin 1 consisting of two spinless particles

Let us find the form factors F_1 , F_2 and G_1 for the system with spin $J = 1$ consisting of two spinless particles. The light front WF of this system has the following form:

$$\psi^\lambda(k_1, k_2, p, \omega\tau) = e_\nu^\lambda(p) \psi_\nu \quad (7.1)$$

$$\psi_\nu = k_{1\nu} \phi_1(s_1, t_1) + \frac{\omega_\nu}{(\omega p)} \phi_2(s_1, t_1) \quad (7.2)$$

where $s_1 = (p + \omega\tau)^2$, $t_1 = (p - k_1)^2$, ϕ_1 and ϕ_2 are invariant functions.

Note that in other representation pointed out in refs.^{14c,d} WF of the system considered acquires the form extremely close to nonrelativistic one:

$$\tilde{\psi}_m(q, n) = f_1(q^2, nq) Y_{1m}(q) + f_2(q^2, nq) Y_{1m}(n) \quad (7.3)$$

The invariant functions ϕ_{1-2} and f_{1-2} are connected by the relations:

$$\phi_1 = -(3/4\pi)^{1/2} f_1 \quad (7.4)$$

$$\phi_2 = \left(\frac{3}{4\pi}\right)^{1/2} M \left(\frac{\sqrt{s_1} - M}{2} \left[\frac{\sqrt{s_1}}{M} \left(1 - \frac{\omega k_1}{\omega p}\right) + \frac{\omega k_1}{\omega p} \right] f_1 - f_2 \right) \quad (7.5)$$

These relations can be obtained by comparing WF's (7.1) and (7.3) in the center-of-mass system, where $\mathbf{p} + \tilde{\omega}\tau = 0$. The rotation matrix, connecting WF's (7.1) and (7.3) becomes unit one in this system.

Analogously to eq.(3.4) the tensor $J_{\mu\nu}^p$ can be written as follows:

$$J_{\mu\nu}^p = \frac{m}{(2\pi)^3} \int \frac{(p + p' + \omega\tau_1 + \omega\tau_2 - 2k)_\rho}{(1 - \omega k/\omega p)^2} \theta(\omega(p - k)) \psi_\mu(p') \psi_\nu(p) \frac{d^3 k}{2\varepsilon(k)} \quad (7.6)$$

Substituting WF (7.2) in eq.(7.6) and, in turn, eq.(7.6) in eqs.(6.9-11) we find:

$$F_1 = \frac{m}{(2\pi)^3} \int \frac{d^3 k}{2\varepsilon(k)} \frac{\theta(\omega(p - k))}{1 - \omega k/\omega p} \left[m^2 - \frac{(kq)^2}{q^2} + \frac{P^2(\omega k)^2}{4(\omega p)^2} - (kP) \left(\frac{\omega k}{\omega p} \right) \right] \phi_1 \phi_1' \quad (7.7)$$

$$G_1 = - \frac{m}{(2\pi)^3} \int \frac{d^3 k}{2\varepsilon(k)} \frac{\theta(\omega(p-k))}{(1 - \omega k/\omega p)^2} \left(\frac{\omega k}{\omega p} \right) \left[m^2 - \frac{(kq)^2}{q^2} + \frac{P^2(\omega k)^2}{4(\omega p)^2} - (kp) \left(\frac{\omega k}{\omega p} \right) \right] \phi_1 \phi_1' \quad (7.8)$$

$$F_2 = 2M^2 \frac{m}{(2\pi)^3} \int \frac{d^3 k}{2\varepsilon(k)} \frac{\theta(\omega(p-k))}{1 - \omega k/\omega p} \left\{ \frac{1}{q^2} \left[\frac{2(kq)^2}{q^2} - \frac{P^2(\omega k)^2}{4(\omega p)^2} + (kp) \left(\frac{\omega k}{\omega p} \right) - m^2 \right] - \frac{1}{4} \left(\frac{\omega k}{\omega p} \right)^2 \right\} \phi_1 \phi_1' \quad (7.9)$$

In variables $\mathbf{R}_\perp$, x (see eq.(2.3)) these expressions for the form factors are rewritten as:

$$F_1 = \frac{m}{(2\pi)^3} \int \frac{d^2 \mathbf{R}_\perp dx}{2x(1-x)} \left(\frac{(\mathbf{R}_\perp \vec{\Delta})^2}{\Delta^2} - \mathbf{R}_\perp^2 \right) \phi_1 \phi_1' \quad (7.10)$$

$$G_1 = \frac{m}{(2\pi)^3} \int \frac{d^2 \mathbf{R}_\perp dx}{2(1-x)^2} \left(\mathbf{R}_\perp^2 - \frac{(\mathbf{R}_\perp \vec{\Delta})^2}{\Delta^2} \right) \phi_1 \phi_1' \quad (7.11)$$

$$F_2 = 2M^2 \frac{m}{(2\pi)^3} \int \frac{d^2 \mathbf{R}_\perp dx}{2x(1-x)} \left\{ \frac{1}{\Delta^2} \left(2 \frac{(\mathbf{R}_\perp \vec{\Delta})^2}{\Delta^2} - \mathbf{R}_\perp^2 - x(\mathbf{R}_\perp \vec{\Delta}) \right) \right\} \phi_1 \phi_1' \quad (7.12)$$

where

$$\phi_1 \equiv \phi_1(\mathbf{R}_\perp^2, x); \quad \phi_1' \equiv \phi_1((\mathbf{R}_\perp - x\vec{\Delta})^2, x). \quad (7.13)$$

One can see that only the function ϕ_1 (or f_1) contributes to the form factors but the term with ω_v in WF (7.2) (or with $Y_{1m}(\mathbf{n})$ in eq.(7.3)) drops out from the results. We believe this fact to be concerned to the particular case considered above. For example, let us consider the light front WF corresponding to a scalar system consisting of a particles with spin 1/2. This WF has the following form:

$$\psi = \phi_1 \bar{u}_1(k_1) U_c \bar{u}_2(k_2) + \phi_2 \bar{u}_1(k_1) \hat{\omega} U_c \bar{u}_2(k_2) \quad (7.14)$$

where $\bar{u}_{1,2}(k_{1,2})$ are bispinors of the particles, U_c is the charge conjugation matrix: $U_c = \gamma_2 \gamma_0$, $\hat{\omega} = \omega_\mu \gamma_\mu$. In this case the term with ϕ_2 analogous to that one in eq.(7.2) gives contribution to the form factor.

8. System with $J = 1$. The model

From the results of sect.6 one can see that the structure of the transition current of the system with $J = 1$, in contrast to the case of $J = 0$, is quite nontrivial. Here the complete cancellation of the ω -depending structures occurs in each of eight independent pairs. Therefore it is interesting to check in the Wick-Cutkosky model that the rest of eq.(6.1), independent of ω , infact yields the correct result for the form factors.

Analogously to procedure of sect.5, at first we will express the form factors through the Bethe-Salpeter function. This function has the following form:

$$\Phi_\lambda(k, p) = e_\nu^\lambda(p) k_\nu \Phi_1(k, p)$$

$$\Phi_1(k, p) = -ic_1 \left[(m^2 - \frac{M^2}{4} - k^2 - i0)^2 (m^2 - (\frac{p}{2} + k)^2 - i0) (m^2 - (\frac{p}{2} - k)^2 - i0) \right]^{-1} \quad (8.1)$$

where $c_1 = 2^7 (\pi m)^{1/2} \kappa_1^{7/2}$, $\kappa_1 = (m|\epsilon_1|) = m\alpha/4$, $|\epsilon_1| = 2m - M = m\alpha^2/16$. Mass M here differs from M for the state with $J = 0$ in sect.5. Here as well as in eq.(5.2) $|\epsilon_1| \ll m$ ($\alpha \ll 1$).

Analogously to the derivation of eq.(5.1) we can obtain the following expression for the tensor $T_{\mu\nu}^p$ in eq.(1.1):

$$T_{\mu\nu}^p = i \int (p + p' - 2k)_\rho k_\mu k_\nu \Phi_1(\frac{p}{2} - k, p) \Phi_1(\frac{p'}{2} - k, p') (m^2 - k^2) \frac{d^4 k}{(2\pi)^4} \quad (8.2)$$

Comparing eq.(8.2) with eq.(1.1) we find:

$$F_1 = -i \frac{\alpha^7 m^8}{32\pi^3} \int d^4 k \left[1 - \frac{kP}{p^2} \right] \left[k^2 - \frac{(kq)^2}{q^2} - \frac{(kP)^2}{p^2} \right] \Psi_1(k, p, p') \quad (8.3)$$

$$G_1 = i \frac{\alpha^7 m^8}{16\pi^3} \int d^4 k \frac{kP}{p^2} \left[k^2 - \frac{(kq)^2}{q^2} - \frac{(kP)^2}{p^2} \right] \Psi_1(k, p, p') \quad (8.4)$$

$$F_2 = -i \frac{\alpha^7 m^8}{32\pi^3(1+\eta)} \int d^4k \left[k^2 \left(1 - 4 \frac{kP}{P^2}\right) + 2 \left(3 \frac{kP}{P^2} - 1\right) \left(\frac{(kq)^2}{q^2} + \frac{(kP)^2}{P^2}\right) - \frac{1}{2\eta} \left(1 - 2 \frac{kP}{P^2}\right) \left(3 \frac{(kq)^2}{q^2} + \frac{(kP)^2}{P^2} - k^2\right) \right] \Psi_1(k, p, p') \quad (8.5)$$

where $\eta = -q^2/4M^2$, the function Ψ_1 is given by:

$$\Psi_1 = \left[\left(m^2 - \frac{M^2}{4} - \left(\frac{p}{2} - k\right)^2 - i0\right) \left(m^2 - \frac{M^2}{4} - \left(\frac{p'}{2} - k\right)^2 - i0\right) \right]^{-1} \Psi(k, p, p') \quad (8.6)$$

and Ψ , in turn, is determined by eq.(5.3).

Using the Feynman parametrization we rewrite eq.(8.6) for $|t| \gg m^2$ as follows:

$$\Psi_1 \approx 4 \cdot 6! \int_0^1 dx \int_0^{1-x} dy \frac{(xy)^2}{[(k-d)^2 - \beta^2 + i0]^7} \quad (8.7)$$

where all the notations were explained after the formula (5.4).

The further analysis is quite analogous to that one made in sect.5. It leads to the following asymptotics of the form factors:

$$F_1 \approx - \frac{\alpha^7}{4\pi} \left(\frac{m^2}{|t|}\right)^3 \left\{ \ln \left(\frac{|t|}{m^2}\right) + \frac{4\pi}{\alpha} \right\} \quad (8.8)$$

$$F_2 \approx - \frac{12\alpha^7}{\pi} \left(\frac{m^2}{|t|}\right)^4 \left\{ \ln \left(\frac{|t|}{m^2}\right) + \frac{4\pi}{\alpha} \right\} \quad (8.9)$$

$$G_1 \approx \alpha^6 \left(\frac{m^2}{|t|}\right)^3 \quad (8.10)$$

Light front WF in this model was found in ref.^{14d)}. The function ϕ_1 in eqs.(7.10-12) has the form:

$$\phi_1(R_\perp, x) = \frac{2^{11} \pi^{1/2} \kappa_1^{7/2}}{\left[\frac{R_\perp^2 + m^2}{x(1-x)} - M^2 \right] (1 + |1 - 2x|)^2} \quad (8.11)$$

Substituting eq.(8.11) into eqs.(7.10-12), integrating over $d\phi$ and introducing $y = R_\perp/m$ we find:

$$F_1 = -\frac{2^4}{\pi} \alpha^7 \int_0^\infty dy y^3 \int_0^1 dx \frac{x^5(1-x)^5}{\beta^4(x)(y^2 + a_1^2)^3 D^{3/2}} \quad (8.12)$$

$$G_1 = \frac{2^4}{\pi} \alpha^7 \int_0^\infty dy y^3 \int_0^1 dx \frac{x^6(1-x)^4}{\beta^4(x)(y^2 + a_1^2)^3 D^{3/2}} \quad (8.13)$$

$$F_2 = 3 \frac{2^8}{\pi} \alpha^7 \int_0^\infty dy y^3 \int_0^1 dx \frac{x^7(1-x)^5(y^2 - (\frac{x\Delta}{m})^2 - a_1^2)}{\beta^4(x)(y^2 + a_1^2)^3 D^{5/2}} \quad (8.14)$$

where $\beta(x) = 1 + |1 - 2x|$, $a_1^2 = (1 - 2x)^2 + \alpha^2 x(1 - x)/4$, D is defined by eq.(5.15) with change of a by a_1 .

Calculation, analogous to one made in sect.5 reproduces the asymptotic expressions (8.8-10). As well as in the case of $J = 0$, the coincidence of the results, obtained with use of Bethe-Salpeter function and light front WF, takes place not only in asymptotics but also in all the range of momentum transfer (an accuracy of coincidence has the order of α). It is demonstrated by numerical calculation, results of which are shown in the table 2. This coincidence confirms the correctness of finding the form factors by eqs.(6.9-11).

Let us also calculate the form factors in nonrelativistic approximation. Using the Coulomb nonrelativistic WF in coordinate representation

$$\Psi_m = \frac{u(r)}{r} Y_{1m}(r/r), \quad u(r) = (4/3)^{1/2} \kappa_1^{5/2} r^2 e^{-\kappa_1 r} \quad (8.15)$$

we find:

$$F_1 = -G_1 = - \int_0^\infty u^2(r) \left[j_0\left(\frac{|q|r}{2}\right) + j_2\left(\frac{|q|r}{2}\right) \right] dr = - \frac{\alpha^6}{\left(\alpha^2 + \frac{|t|}{m^2}\right)^3}$$

$$|t| \approx \infty - \alpha^6 \left(\frac{m^2}{|t|}\right)^3 \quad (8.16)$$

$$F_2 = - \frac{3}{2\eta} \int_0^\infty u^2(r) j_2\left(\frac{|q|r}{2}\right) dr = - \frac{48\alpha^6}{\left(\alpha^2 + \frac{|t|}{m^2}\right)^4} |t| \approx \infty - 48\alpha^6 \left(\frac{m^2}{|t|}\right)^4 \quad (8.17)$$

where, as well as before, $\eta = -q^2/4M^2 = q^2/4M^2$, j_0 and j_2 are the spherical Bessel functions.

In the range of $|t|$, where logarithmic term in eqs.(8.8) and(8.9) is negligible, the results of relativistic calculation (in both approaches) coincide with ones obtained by extrapolation of nonrelativistic formulas (8.16-17) in relativistic region (see table 2).

Charge F_{CH} , magnetic F_{MAG} and quadrupole F_Q form factors are expressed through F_1 , F_2 , and G_1 by usual formulas:

$$F_{CH} = -F_1 - \frac{2\eta}{3} [F_1 + G_1 - F_2(1 + \eta)] \quad (8.18)$$

$$F_Q = F_1 + G_1 - F_2(1 + \eta) \quad (8.19)$$

$$F_{MAG} = \frac{G_1}{\sqrt{1 + \eta}} \quad (8.20)$$

9. Conclusion

Let us give brief resume of the results obtained above.

In calculations of the vertex function for interaction of a bound system with photon (fig.1b) in the light front dynamics at $t + z = 0$ it was found that the result is noncovariant. This noncovariance leads to increase of the number of independent matrix elements in comparison with eq.(1.1) and to their dependence on direction of z -axis or, in the case of the infinite momentum frame, on the direction of its motion. In the framework of the explicitly covariant approach developed in refs.2,14) this property manifests

itself in dependence of a result on four-vector ω , defining the position of the light front $\omega x = 0$. This dependence is parametrized explicitly, through additional tensor structures constructed with help of ω (for a system with $J = 1$ there is 8 such extra tensors, see eq.(6.2)). Increase of the number of this structures corresponds to increase of the number of independent matrix elements. These structures cancel in the total (not exhausted by the graph fig.1b) expression for the vertex function. This statement is based only on the Lorentz-invariance of the theory at a whole. Remainder does not depend on ω and determines the physical form factors.

In order to find the form factors there is no need to calculate the contribution canceling the ω -dependence in the vertex function shown in fig.1b. It is enough to extract the physical form factors from this vertex function. For a system with $J = 1$ the expressions for these form factors through the vertex function are represented by the formulas (6.9-11). The model calculations confirm that this method gives the correct results.

From the analysis carried out above it follows that any fixed value of ω leads to wrong results if only graph in fig.1b is taken into account. For example, at $\vec{\omega} \parallel \mathbf{p} + \mathbf{p}'$ the spatial part (in the index p) of the tensor at the coefficient B_1 in eq.(6.2) coincides with that one at F_1 in eq.(1.1). Therefore in this case the coefficient at $(\mathbf{p} + \mathbf{p}')g_{\mu\nu}$ (which naively could be taken for the form factor F_1) is not this form factor, but is the sum of F_1 with B_1 , and with other terms. Similar to the functions F and g_1 in fig.6, in relativistic region this sum can considerably differ from F_1 . In other words, at fixed value of ω (or position of z -axis at $\omega = (1, 0, 0, -1)$ used in refs.³⁻⁸) the physical $T_{\mu\nu}^p$ and unphysical $B_{\mu\nu}^p$ contributions are not separated from each other. Here the advantages of covariant approach on the light front $\omega x = 0$ in comparison with noncovariant one at $t + z = 0$ reveal themselves very distinctly.

In practice we simply omit the structures depending on ω after their separation and finding the physical form factors. But simple ignoring of this structures from the very beginning also leads to wrong results. For example, if instead of eq.(2.7) we represent the vertex function F_p in the usual form for Feynman vertex $F_p = (\mathbf{p} + \mathbf{p}')_p \tilde{F}$, and then, in order to find $\tilde{F}$, multiply both sides of the equality by $(\mathbf{p} + \mathbf{p}')_p$, then for $\tilde{F}$ we would get the sum of the

integrals defined by eqs. (3.7) and (3.8): $\tilde{F} = (p + p')_\rho F_\rho / (p + p')^2 = F + g_1$, whereas the correct form factor is given by eq. (3.7) for F . Just the same situation takes place for the case of the system with spin $J = 1$.

In introduction we have mentioned that the papers^{4,6)} dealt with the current component $J_+ = J_0 + J_3$. For the case of spinless system in the theory with $t + z = 0$, i.e. with $\omega = (1, 0, 0, -1)$, this component is indeed distinguished, since due to equality $\omega_+ = \omega_0 + \omega_z = 0$ the second term in eq. (2.7) does not contribute. However for the deuteron case the decomposition (6.1) for the component with $p = +$, like other components, all the same contains extra contributions. As it was discussed above, this fact just corresponds to increase of the number of the current J_+ matrix elements, noticed in ref.⁶⁾. If, according to refs.^{11 12)} the covariant operator of electromagnetic current is constructed then it would directly lead to the physical form factors, without contribution of the term $B_{\mu\nu}^p$ in eq. (6.1). In other words, it would contain in itself the contribution canceling $B_{\mu\nu}^p$. Namely owing to this reason the explicit construction of the current operator turns out to be difficult. From the results of the present paper it is seen that this problem can be avoided.

The calculations of the deuteron electromagnetic form factors in the covariant light front dynamics by the method developed above will be published in future paper.

Appendix

We give here expressions for the form factors of a system with spin 1/2 (e.g., ³He or nucleon) through the electromagnetic vertex in the light front dynamics.

Electromagnetic current of such a system is determined by charge F_{CH} and magnetic F_{MAG} form factors:

$$J_\mu = \bar{u}_2 \Gamma_\mu u_1$$

$$\Gamma_\mu = 2M(F_{CH} - F_{MAG})P_\mu/P^2 + F_{MAG} \gamma_\mu \quad (A.1)$$

Here $\bar{u}_2 = \bar{u}(p')$, $u_1 = u(p)$ are bispinors, $P = p + p'$.

Appearance of four-vector ω in the problem discussed leads to increase of number of form factors. Analogously to the case of spin $J = 1$ (sect.6) this number can be calculated without their explicit

construction. As it was mentioned above (sect.6) the particle helicities not obligatory satisfy the equality $\lambda = \lambda_1 + \lambda_2$. Therefore we have $2 \times 2 \times 3 = 12$ initial amplitudes. P-parity decreases this number twice, T-parity (or C-parity in annihilation channel) gives one additional constraint. So, five independent invariant amplitudes remain. Corresponding decomposition has the form:

$$\begin{aligned} \bar{u}_2 \hat{J}_\mu u_1 = & \alpha_1 \bar{u}_2 \gamma_\mu u_1 + \alpha_2 \bar{u}_2 u_1 P_\mu + \alpha_3 \bar{u}_2 u_1 \omega_\mu + \alpha_4 \bar{u}_2 \hat{\omega} u_1 P_\mu \\ & + \alpha_5 \bar{u}_2 \hat{\omega} u_1 \omega_\mu \end{aligned} \quad (A.2)$$

Here $\hat{\omega} = \omega_\mu \gamma_\mu$, $\hat{J}_\mu$ is 4x4 matrix. Left-hand-side of eq.(A.2) is given by the graph fig.1b (or by corresponding graph for many-body system) and is expressed through WF. In contrast to the case of $J = 1$ (cf. eq.(6.1)) it is impossible to construct the structures which do not conserve the current, since it turns out that such terms don't conserve other symmetries taken into account in eq.(A.2). Similary to the cases of spins 0 and 1, the items with α_{3-5} in eq.(A.2) containing ω must be canceled in the total expression for the vertex function. Therefore the physical form factors are given by the first two terms:

$$F_{MAG} = \alpha_1 \quad (A.3)$$

$$\frac{2M}{P^2} (F_{CH} - F_{MAG}) = \alpha_2 \quad (A.4)$$

Multiplying eq.(A.2) at the left by u_2 and at the right by $\bar{u}_1$, summarizing over polarizations, then multiplying in turn by γ_μ , P_μ , etc. and calculating the traces of matrices, we get the system of equations for α_j . Solving this system, we find α_1 , α_2 , and F_{CH} , F_{MAG} . The result has the form:

$$F_{CH} = [2(\omega p)J_3 - (4M^2 + P^2)J_1/2M - J_4 + P^2J_2/(\omega p)]/4(\omega p)q^2 \quad (A.5)$$

$$F_{MAG} = [2(\omega p)J_3 - 2MJ_1 - J_4 + P^2J_2/2(\omega p)]/4(\omega p)q^2 \quad (A.6)$$

where

$$J_1 = \text{Tr}[(\hat{p}' + M)\hat{J}_\mu \omega_\mu (\hat{p} + M)] \quad (A.7)$$

$$J_2 = \text{Tr}[(\hat{p}' + M)\hat{J}_\mu \omega_\mu (\hat{p} + M)\hat{\omega}] \quad (A.8)$$

$$J_3 = \text{Tr}[(\hat{p}' + M)\hat{J}_\mu (\hat{p} + M)\gamma_\mu] \quad (A.9)$$

$$J_4 = \text{Tr}[(\hat{p}' + M)\hat{J}_\mu P_\mu (\hat{p} + M)\hat{\omega}] \quad (A.10)$$

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Table 1.

Electromagnetic form factor of spinless system in the Wick - Cutkosky model for $\alpha = 0.08$. Columns BS, LF and NR represent the results of calculations using the Bethe-Salpeter function (eqs.(5.1-2)), light front WF (eqs.(3.7), (5.12)) and nonrelativistic WF (eq.(5.20)) respectively.

$ t /m^2$	F		
	BS	LF	NR
0	1.00E+00	9.67E-01	1.00E+00
0.01	5.14E-01	4.98E-01	5.17E-01
1	6.02E-04	5.75E-04	6.23E-04
25	1.01E-06	9.76E-07	1.05E-06
100	6.40E-08	6.20E-08	6.55E-08
225	1.28E-08	1.24E-08	1.29E-08

Table 2.

Electromagnetic form factors of system with spin $J = 1$ in the Wick-Cutkosky model for $\alpha = 0.1$. Columns BS, LF and NR represent the results of calculations using the Bethe-Salpeter function (eqs.(8.3-6)), light front WF (eqs.(7.10-12), (8.11)) and nonrelativistic WF (eqs.(8.16-17)) respectively.

$ t /m^2$	$-F_1$			G_1			$-F_2$		
	BS	LF	NR	BS	LF	NR	BS	LF	NR
0	1.00E+00	9.67E-01	1.00E+00	1.00E+00	9.67E-01	1.00E+00	4.80E+03	4.66E+03	4.80E+03
0.01	1.25E-01	1.19E-01	1.25E-01	1.24E-01	1.19E-01	1.25E-01	2.99E+02	2.89E+02	3.00E+02
1	9.28E-07	8.74E-07	9.72E-07	9.26E-07	8.59E-07	9.72E-07	4.44E-05	4.17E-05	4.62E-05
25	6.18E-11	5.75E-11	6.40E-11	5.97E-11	5.49E-11	6.40E-11	1.16E-10	1.11E-10	1.23E-10
100	9.73E-13	9.09E-13	1.00E-12	9.28E-13	8.57E-13	1.00E-12	4.62E-13	4.44E-13	4.80E-13
225	8.58E-14	8.02E-14	8.79E-14	8.17E-14	7.55E-14	8.79E-14	1.77E-14	1.71E-14	1.78E-14

Figure captions

Fig. 1(a). Graph, corresponding to the expressed through WF amplitude of scattering from bound system.

(b). Electromagnetic vertex of bound system in the light front dynamics.

Fig.2. Graphs not expressed through WF.

Fig.3. Graph, corresponding to the amplitude expressed through WF and tending to zero under the condition $\omega_q = 0$.

Fig.4. Graph, corresponding to admixture of many-body states in the state vector.

Fig.5. Feynman electromagnetic vertex expressed through the Bethe-Salpeter function.

Fig.6. Physical (F) and unphysical (g_1) form factors of system with zero spin (see eq.(2.7)) calculated by eqs.(3.7) and (3.8) respectively with WF (5.12) for $\alpha = 0.08$.

Fig. 1

Fig. 2

Fig. 3

(a)

(b)

Fig. 4

Fig. 5

Fig. 6

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