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PARITY VIOLATIONS BY NEUTRAL CURRENTS IN MUONIC ATOMS

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A B S T R A C T

The 2S-1S transition in low Z muonic atoms is shown to be extremely sensitive to possible parity violation, such as may be introduced by neutral currents. The most striking effects of parity violation are found in the case of muonic Li and Be where they are estimated to be of the order of 10% on the basis of current models.

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A central issue in weak interactions is the question of existence of neutral currents. On the theoretical side their existence would indicate the possibility of unifying electromagnetic and weak interactions using gauge theories¹⁾ which have the attractive feature of being renormalizable.

The tangible evidence for neutral currents is presently controversial: a number of neutrino-like interactions²⁾ without muons or electrons as well as a few candidates³⁾ for the leptonic process $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ have been observed in high-energy neutrino experiments.

If neutral currents exist they could imply a basic modification of electromagnetic phenomena, the most striking of which is the prediction of parity violation.

In this spirit Bouchiat-Bouchiat^{4)*)} have recently drawn attention to the possibility of observing parity violations and hence neutral currents by exciting certain states in heavy electronic atoms using intense polarized photon beams.

The purpose of the present article is to show that under the right conditions the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition in muonic atoms is extremely sensitive to parity violation with the possibility of large effects in circular polarization and angular correlations.

Any hydrogenic atom is basically a system unusually sensitive to parity mixing in states of principal quantum number $n \geq 2$. The reason is that the non-relativistic Bohr atoms has exactly degenerate states of opposite parity with the same n . In this case the slightest parity violation would cause complete mixing. In actual atoms the degeneracy is lifted, primarily by the effects of self mass, vacuum polarization and the finite size of the central nucleus^{**)}. Still, the levels are only slightly perturbed for light nuclei, as compared to the spacing of states of different n , so that the parity mixing is strongly enhanced.

In most models the parity mixing parameter is proportional to the weak interaction coupling constant G , but must be a dimensionless quantity. If only processes involving the same lepton as the one forming the atom are considered and finite nuclear size is neglected, the lepton mass is the only scale available and the mixing must be proportional to Gm_ℓ^2 . Since the mass ratio $(m_\mu/m_e)^2 \approx 4 \times 10^4$ one immediately concludes that muonic atoms are much more sensitive to parity mixing than electronic atoms. This argument is invalidated by vacuum polarization

*) References to earlier work on parity violation in electronic atoms are given in this article.

***) Other effects such as nuclear polarizability, higher-order vacuum polarization, weak interaction shifts, etc., are completely negligible for our purpose with the exception of some minor influence from the hyperfine splitting.

which is largely due to virtual electron pairs both in ordinary and muonic atoms. Moreover, in heavy atoms the nuclear r.m.s. radius $\langle r^2 \rangle^{1/2}$ provides a new scale. Nevertheless in light muonic atoms the essence of the argument above remains valid as seen from the actual parity effects given in Table 1.

Of all the spontaneous transitions in a hydrogenic atom, only the $2S_{1/2} - 1S_{1/2}$ will show exceptional sensitivity to parity mixing with a reasonable branching ratio. All excited states in the atom with the exception of the $2S_{1/2}$ state decay principally via strong E1 transitions. Any parity violating effect in the amplitude for these E1 transitions is proportional to both the mixing parameter and a more forbidden transition, and will be quite small. For the $2S_{1/2}$ state, on the contrary, the E1 transition to the $2P_{1/2}$ state is either completely forbidden or has a small phase space. Its decay to the ground state is either by two-photon decay or by the M1 transition (see Fig. 1). This M1 transition is strongly hindered and vanishes in the allowed non-relativistic approximation. The matrix element comes from relativistic effects and retardation with the rate for a massive nucleus given by

$$W_{M1}^{\gamma}(2S_{1/2} - 1S_{1/2}) = 5.16 \times 10^{-4} Z^{10} \text{ sec}^{-1} \quad (1)$$

For low Z this rate is very low and the dominant decay mode is via two-photon emission

$$W^{2\gamma}(2S_{1/2} - 1S_{1/2}) = 1.70 \times 10^3 Z^6 \text{ sec}^{-1} \quad (2)$$

Furthermore, the $2P_{1/2}$ level which is nearly degenerate with the $2S_{1/2}$ level decays rapidly to the ground state by E1 transition with the rate

$$W_{E1}^{\gamma}(2P_{1/2} - 1S_{1/2}) = 1.29 \times 10^{11} Z^4 \text{ sec}^{-1} \quad (3)$$

Due to parity violation the $2P_{1/2}$ mixes with the $2S_{1/2}$ state so that the M1 transition acquires an E1 component: $M1 \rightarrow M1 + \eta E1$, where η is the mixing parameter. Therefore, the relative admixture amplitude in the matrix element is $\eta(E1/M1)$. Apart from the unusually strong admixture expected because the parity mixing states are nearly degenerate, there will therefore be an additional strong enhancement from the factor E1/M1 which for light elements is of the order 10^6 .

Up to this point the arguments given are quite general. Optimal condition for parity mixing can be obtained by choosing Z so as to make the $2P_{1/2} - 2S_{1/2}$ energy difference exceptionally favourable. In a light muonic atom the $2S_{1/2}$ is more bound than the $2P_{1/2}$ state due to the attractive vacuum polarization. With increasing nuclear size this attraction is rapidly compensated by the repulsive finite size

effect. A cross-over will occur for a particular Z for which the $2P_{\frac{1}{2}}-2S_{\frac{1}{2}}$ energy difference is anomalously small.

The vacuum polarization correction to low Z atoms was approximately calculated two decades ago by Foldy and Eriksen⁵⁾. Recent more accurate results⁶⁻⁸⁾ are available for some elements. For our purposes the agreement between these results is quite satisfactory.

The nuclear finite size correction shifts in practice only the $2S_{\frac{1}{2}}$ level. The contribution to the $2P_{\frac{1}{2}}-2S_{\frac{1}{2}}$ energy separation is given by

$$\Delta E_{size} = -\frac{1}{12} \alpha^4 \mu^3 Z^4 \langle r^2 \rangle \quad (4)$$

where μ is the reduced mass. Taking measured r.m.s. radii for ${}^4\text{He}$ ⁹⁾, ${}^6\text{Li}$, ${}^7\text{Li}$ ¹⁰⁾, ${}^9\text{Be}$ and ${}^{12}\text{C}$ ¹¹⁾, we have computed the energy separation $\Delta E = E(2P_{\frac{1}{2}}) - E(2S_{\frac{1}{2}})$:

$$\begin{aligned} \Delta E(\text{H}) &= 0.2 \text{ eV} ; \Delta E({}^4\text{He}) = 1.4 \text{ eV} ; \Delta E({}^6\text{Li}) = 1.1 \text{ eV} \\ \Delta E({}^7\text{Li}) &= 1.5 \text{ eV} ; \Delta E({}^9\text{Be}) = -2.2 \text{ eV} ; \Delta E({}^{12}\text{C}) = -33 \text{ eV}. \end{aligned}$$

Manifestly, the finite size correction is of the same size as the vacuum polarization shift in Li and Be and it has completely overwhelmed vacuum polarization already in ${}^{12}\text{C}$. It is hence to be expected that parity violation effects are particularly important in the Li, Be region. We will therefore concentrate the further discussion to the light elements although much of the following can be immediately adapted also to heavy atoms.

We consider the single-photon emission in the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition. The normal hindered M1 amplitude and the allowed E1 amplitude which has been admixed interfere with each other.

The relevant M1 amplitude, denoted by A_{M1} , is conventionally expressed by quantizing along the photon direction

$$A_{M1} = e\sqrt{2} \left[\delta_{m_i, -1/2} \delta_{m_f, 1/2} \varepsilon_- + \delta_{m_i, 1/2} \delta_{m_f, -1/2} \varepsilon_+ \right] I_{M1} \quad (5)$$

where $\varepsilon_{\pm} = \mp(1/\sqrt{2})(\varepsilon_x \pm i\varepsilon_y)$ are the photon polarization vectors; m_i and m_f refer to the component of the initial and final muon spin in the direction of the photon momentum. The quantity I_{M1} is a radial integral over the large and small components g and f of the Dirac wave function

$$I_{M1} = \int_0^{\infty} dr r^2 j_1(kr) \left[g_{1S}^*(r) f_{2S}(r) + f_{1S}^*(r) g_{2S}(r) \right]. \quad (6)$$

Similarly, the E1 amplitude A_{E1} is given by

$$A_{E1} = ie\sqrt{2} \left[\delta_{m_i, -1/2} \delta_{m_f, 1/2} \epsilon_- - \delta_{m_i, 1/2} \delta_{m_f, -1/2} \epsilon_+ \right] I_{E1} \quad (7)$$

where

$$I_{E1} = \int_0^\infty dr r^2 j_0(kr) \left[g_{1s}^*(r) f_{2p}(r) + \frac{1}{3} f_{1s}^*(r) g_{2p}(r) \right]. \quad (8)$$

To leading order for a point-like Coulomb field I_{M1} and I_{E1} are given by

$$I_{M1} = \frac{(\alpha Z)^4}{27\sqrt{2}}, \quad I_{E1} = \frac{-16}{81} \sqrt{\frac{2}{3}} (\alpha Z).$$

These lead to the M1 and E1 total transition rates (1) and (3) for muonic atoms and are in agreement with results previously quoted in the literature^{12,13)} for electronic atoms.

By parity mixing the initial state has the wave function $|2S_{1/2}\rangle + \eta|2P_{1/2}\rangle$ and the total transition amplitude is $A_{M1} + \eta A_{E1}$ (*). For unpolarized muons the circular polarization of the γ -ray is hence

$$a = \frac{W_+ - W_-}{W_+ + W_-} = \frac{2 \operatorname{Im} \eta \cdot I_{M1} I_{E1}}{I_{M1}^2 + (\operatorname{Im} \eta)^2 I_{E1}^2}. \quad (9)$$

Substituting I_{M1} and I_{E1} yields for small admixture amplitude

$$a \simeq - \frac{64}{3\sqrt{3} (\alpha Z)^3} \operatorname{Im} \eta \quad (10)$$

The experimental detection of the circular polarization seems difficult. A more convenient experiment would be the detection of the correlation of the emitted γ -ray with the muon polarization $\vec{P}$ given by

$$\frac{dW_{18}(\vec{k}_\gamma, \vec{P})}{d\Omega} = \frac{W_{18}}{4\pi} (1 \pm a \hat{P} \cdot \hat{k}_\gamma). \quad (11)$$

Here the signs refer to the case of initial state polarization (+) and final state polarization (-) of the muon with the other state unpolarized.

*) The amplitudes A_{M1} and A_{E1} are relatively imaginary. The parameter η is also imaginary if time-reversal invariance is assumed. Therefore, the relative phase of the E1 and M1 transitions is of importance.

If one chooses a case where the initial muon is unpolarized, the angular correlation of the photon and the electron from the decay of the final muon is proportional to the quantity

$$1 + \frac{1}{3} a \cos \theta_{e\gamma}. \quad (12)$$

We now calculate the mixing parameter in a model with a neutral current mediated via a heavy vector boson Z. In the external field approximation, the interaction Hamiltonian is given by

$$H_{int}(\vec{x}) = j_\lambda(\vec{x}) A^{\lambda, ext}(\vec{x})$$

where $j_\lambda(\vec{x})$ is the leptonic neutral current which contains the muonic part

$$j_\lambda^{\mu\mu}(x) = \bar{\Psi}_\mu(x) (a + b \gamma_5) \gamma_\lambda \Psi_\mu(x). \quad (13)$$

Here a and b are constants. The external field is easily found to be

$$A^{\lambda, ext}(\vec{x}) = \frac{g(Z, N)}{M_Z^2} g^{\lambda 0} \delta(\vec{x}) \quad (14)$$

where M_Z is the mass of the heavy boson and $g(Z, N)$ denotes its coupling to the nucleus.

The interaction Hamiltonian can thus be expressed in terms of the effective coupling constants

$$g_1 = \frac{a g(Z, N)}{M_Z^2}, \quad g_2 = \frac{b g(Z, N)}{M_Z^2} \quad (15)$$

in the form ^{*)}

$$H_{int}(\vec{x}) = \bar{\Psi}_\mu(\vec{x}) [g_1 \gamma_0 + g_2 \gamma_5 \gamma_0] \Psi_\mu(\vec{x}) \delta(\vec{x}) \simeq [g_1 - g_2 \frac{\vec{\sigma} \cdot \vec{p}}{m_\mu}] \delta(\vec{x}),$$

where the approximate right-hand side is the non-relativistic limit. We note that the assumption of vector boson exchange is mainly an artifice to emphasize that the interaction is short-ranged. In the form above with free effective coupling constants, the Hamiltonian has considerable generality. The effective coupling constants g_1 and g_2 should be considered as having a typical scale of AG , where A is the nucleon number.

*) The operator $\vec{\sigma} \cdot \vec{p}$ should be symmetrized to act equally on initial and final states.

The parity violating part of the interaction above connects the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ states by the term $\gamma_0\gamma_5$. To leading order in $Z\alpha$ we have

$$\langle 2P_{\frac{1}{2}}, m_P | H_{\text{int}} | 2S_{\frac{1}{2}}, m_S \rangle = -i g_2 \frac{\sqrt{3}}{32\pi} \mu^3 (\alpha Z)^4 \delta_{m_P, m_S}, \quad (16)$$

and the mixing parameter η is this quantity divided by $E_{2S_{\frac{1}{2}}} - E_{2P_{\frac{1}{2}}}$. Substitution in Eq. (10) leads to the asymmetry parameter

$$a = \frac{2g_2}{3\pi} \frac{\mu^3 (\alpha Z)}{E_{2S_{\frac{1}{2}}} - E_{2P_{\frac{1}{2}}}}. \quad (17)$$

A concrete model for neutral currents has been given by Weinberg¹⁴⁾. In this model the effective parity violating coupling g_2 is given by

$$g_2^{\text{Weinberg}} = \frac{G}{\sqrt{2}} \left[\frac{1}{2} (Z-N) - 2 \sin^2 \theta_w Z \right]$$

where θ_w is the Weinberg angle. So as to form an opinion about at which level the parity effects are expected to enter, we have tabulated the asymmetry parameter a for the Weinberg coupling and the atoms mentioned above (see Table 1). For illustration we have chosen the extreme cases $\sin^2 \theta_w = 0$ and 1 as well as the more realistic case $\sin^2 \theta_w = 0.4$.

A glance at the table reveals that expected effects are very large. We would like to emphasize that this feature does not originate from the specific model chosen, but is due to the inherent enhancement factors discussed before. Therefore, independent of the particular theory which may be wrong, even negative experiments would serve the important purpose of limiting future theories. It is also interesting to compare muonic and ordinary ions. For this reason we have calculated the ratio of the asymmetries for the two cases by using experimental results¹⁵⁾ for the Lamb shifts of the ordinary ions whenever available. One notes the smallness of the electronic effects.

Present experimental limits on the parity breaking interactions are very poor and would correspond to vastly enhanced rates of the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition. So for example, Poppe¹⁶⁾ finds an upper limit in Pb corresponding to 5×10^3 of the Weinberg coupling, which would correspond to $W^Y(2S-1S)$ in Li of about 4×10^4 times stronger than the value we have used here. A similar upper limit is obtained from the one-photon transition in muonic helium by Placchi et al.¹⁷⁾.

We now turn to the question of the backgrounds to the one-photon decay of the $2S_{\frac{1}{2}}$ state. An intrinsic background comes from the two-photon decay of this state with the configuration where one of the photons carries essentially all the available energy and the other nearly none. For low Z ions, the total rate for the

two-photon decay, $W^{2\gamma}$, is larger than that for the one-photon decay by several orders of magnitude. Using the formulae given in Ref. 18, we have estimated the two-photon fractional branching ratio, $\delta W^{2\gamma}$, where the soft photon carries at most the fraction ε of the total available energy; we find

$$\delta W^{2\gamma} \approx .95 \varepsilon^2 W^{2\gamma}.$$

To suppress this background to the level of the signal, given by the one-photon rate W^γ , requires $\varepsilon \approx \sqrt{W_{M1}^\gamma / W^{2\gamma}}$.

For comparison, we have tabulated the one- and two-photon rates [$W(\gamma)$ and $W(2\gamma)$, respectively] and the quantity $\sqrt{W_{M1}^\gamma / W^{2\gamma}}$ (see Table 1). We observe that the level of accuracy discussed above requires a resolution for Li and Be of about 1% or better in the maximum energy of the X-rays which are in the region of about 20 keV. Another possible way of eliminating this background may be by the use of an appropriate absorber.

The practical problem of an actual experiment is associated with the following points: the probability of population of the 2S state, the branching ratio into the one-photon mode and the muon polarization. (We will not consider detection of circular polarization which we believe difficult experimentally.)

The $2S_{\frac{1}{2}}$ state is expected to be populated by a fraction of the order of a few percent of all stopped muons, as has been shown directly in the case of He¹⁷⁾. In light elements the lifetime of this state is very much larger than that of the $2P_{\frac{1}{2}}$ state and in practice of the order of the muon lifetime (nuclear absorption is negligible). An important part of the physical background would be the $2P_{\frac{1}{2}}-1S_{\frac{1}{2}}$ photons which have very closely the same energy. Simple time discrimination can be used to eliminate this background.

The long lifetime of the $2S_{\frac{1}{2}}$ state exposes the atom to various molecular effects which depend on exact experimental conditions. First, the presence of electrons in the muonic atom leads to radiationless de-excitation with a rate of about $2 \times 10^9 \text{ sec}^{-1}$ ¹⁹⁾. This would reduce the one-photon branching ratio by several orders of magnitude. Normally all electrons are believed to get ejected from the light atoms during the muonic cascade. It is important to prevent re-capture of electrons which strongly militates against the use of metal targets which are rich in free electrons. Secondly, Stark mixing in collisions provides momentary parity breaking. While it will not simulate asymmetries or circular polarization, it can mix the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ levels and produce de-excitation by E1 radiation. This effect is particularly important in the gaseous state, but it is reduced in a symmetric lattice.

In carrying out parity experiments of the type discussed above, it is most likely easier to detect a correlation between the photon and the polarization rather than the circular polarization. Muons stopped are usually strongly polarized; about 15% of this polarization remains after the muon cascades to the $2S_{\frac{1}{2}}$ state and can be used for correlation measurements. An alternative is to use unpolarized muons and measure the polarization of the final muon directly by its decay electron, i.e. an electron-photon correlation.

Finally, we note that the parity breaking effects are very much larger for light muonic atoms than for heavier muonic atoms. For heavy muonic atoms the finite nuclear size dominates the $2P_{\frac{1}{2}}-2S_{\frac{1}{2}}$ energy shift according to Eq. (4). As seen from the expression for the asymmetry parameter (17) the typical asymmetry is of the order 10^{-3} to 10^{-4} . The smallness of the effect makes the question of systematic experimental errors more serious. Further, no time separation of the E1 versus M1 transition can be made in practice, though some possibilities for energy discrimination exist. The branching ratio for the M1 transition is rather poor, since the $2S_{\frac{1}{2}}-2P_{\frac{1}{2}}$ becomes physical and Auger de-excitation is a normally competing branch. Although we believe that the prospects are rather poor, this possibility should also be considered as an experimental alternative.

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Table 1

Element	$a \times 10^2$ ($\sin \Theta_w = 0$)	$a \times 10^2$ ($\sin \Theta_w = 1$)	$a \times 10^2$ ($\sin^2 \Theta_w = 0.4$)	a_{e1}/a_{mu}	W_{M1}^Y	$W^2 Y$	$\sqrt{W_{M1}^Y / W^2 Y}$
H	-2.6	+ 7.7	+1.5	-7×10^{-3}	4.6×10^{-4}	1.5×10^3	5×10^{-4}
$^4\text{He}^{(+)}$	0	+ 7.6	+3.0	-3×10^{-3}	5.1×10^{-1}	1.1×10^5	2×10^{-3}
$^6\text{Li}^{(++)}$	0	+22.2	+8.9	-0.5×10^{-3}	3.0×10	1.2×10^6	5×10^{-3}
$^7\text{Li}^{(++)}$	+1.4	+18.2	+8.1	-	3.0×10	1.2×10^6	5×10^{-3}
$^9\text{Be}^{(3+)}$	-1.3	-22.1	-9.6	-	5.3×10^2	6.9×10^6	9×10^{-3}
$^{12}\text{C}^{(5+)}$	0	- 3.1	-1.2	10^{-3}	3.1×10^4	7.9×10^7	2×10^{-2}

The one- and two-photon transitions
between the levels $n=1$ and 2

Fig. 1