**Second-Order QCD Corrections to Heavy-Quark
Forward-Backward Asymmetries**

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Abstract

We evaluate the second-order QCD corrections to the forward-backward asymmetries of b and c quarks at LEP energies, where the quark mass can be neglected. This calculation is necessary for the reduction of the theoretical error associated with the scale dependence of the leading QCD correction, given the envisaged experimental precision of a few per cent to be achieved at the end of the LEP1 phase. The dependence on the experimental cuts of the second-order QCD corrections to asymmetries and to partial widths for b and especially c quarks is also discussed.

1. INTRODUCTION

The measurements of the various asymmetries at the Z peak are a very important part of the LEP1 programme of precision tests of the electroweak theory [1]. The forward-backward asymmetries can be experimentally studied for charged leptons and heavy quarks together with the τ -polarization asymmetry, while for polarized electrons other asymmetries become accessible, the most important one being the left-right asymmetry. In particular the forward-backward asymmetry for the b quark, A_{FB}^b , can provide a very stringent constraint, since it can be measured with precision and the resulting error on the effective $\sin^2\theta_W$ (the only variable on which the asymmetries depend) is a rather small fraction of the absolute error on A_{FB}^b (for $\sin^2\theta_W \approx 0.23$ one finds [2] $\delta\sin^2\theta_W \approx -0.18 \delta A_{FB}^b$). In the present paper we evaluate the next-to-leading-order QCD correction to A_{FB}^b (or to any similar heavy-quark asymmetry) in the limit of negligible quark mass. As is well known, the first-order QCD correction, computed in refs.[2,3] for non-zero quark masses, is given by a rather complicated expression that can be simplified by expanding in $\mu = 2m_b/m_Z$. The result is [2, 3]:

$$A_{FB}^b = (A_{FB}^b)_0 F_{QCD} = (A_{FB}^b)_0 \left(1 - c_1 \frac{\alpha_s}{\pi} + c_2 \left(\frac{\alpha_s}{\pi} \right)^2 + o\left(\left(\frac{\alpha_s}{\pi}\right)^3\right) \right) \quad (1)$$

with $\langle v \rangle$ and a are the vector and axial-vector couplings of the b-quark, respectively)

$$c_1 = \left\{ 1 - \frac{2\pi\mu}{3} - \frac{2\mu^2}{3} \frac{9}{5} + \frac{\pi^2}{8} + \frac{1}{2} \left(\log \frac{\mu}{2} \right)^2 - 3 \log 2 - \frac{5}{2} \log 2 + 3\mu^2 \frac{v^2 + a^2 \log^4}{v^2 + a^2} \right\}. \quad (2)$$

Numerically, for m_b in the range 4.5 - 5 GeV, one obtains $c_1 = 0.81 - 0.79$. Note that the term linear in μ already provides a very good approximation: $c_1 \approx 0.79 - 0.77$ (we have recomputed this term and checked the result). If the second-order correction is not known, the scale ambiguity forbids to fix the size of the effective α_s to be used in eq. (1) to better than, say, $\alpha_s = 0.10 - 0.14$; as a result, the first-order QCD correction is between -2.4% and -3.4% . At present the experimental value for A_{FB}^b , corrected for B-B mixing, combined over the four LEP experiments, is given by $A_{FB}^b(\sqrt{s} = m_Z) = 0.094 \pm 0.014$ [4]. Thus a 1% ambiguity on the QCD correction is quite small when compared with the present error. However, the precision goal at LEP 1 with $10^7 Z$ is a relative error of about 5% on A_{FB}^b [2]. At this level of precision it becomes important to evaluate the second-order QCD correction. Once the coefficient c_2 of $(\alpha_s/\pi)^2$ in eq. (1) is fixed (for a specified scale choice in the leading term) the ambiguity on F_{QCD} goes down to the few per mille level. Actually the result for c_2 in the limit of $\mu = 0$ is sufficient for an adequate precision, the variation with μ being a small correction to the correction. A peculiar source of possible logarithmic dependence on μ will be discussed in the following. We also neglect imaginary parts and other similar contributions of order Γ_Z/m_Z in the $O(\alpha_s^2)$ correction, which is consistent with neglecting terms of order m_b/m_Z .

2. METHOD OF CALCULATION

We first specify a suitable definition for A_{FB}^b , which is both theoretically treatable and experimentally accessible. We define A_{FB}^b starting from the angular distribution for inclusive b-quark production $do(e^+e^- \rightarrow b + X)/d\cos\theta$, with θ being the angle between the b quark and the incoming electron directions. The forward (backward) b-quark cross-section σ_{bF} (σ_{bB}) is then obtained by integrating the inclusive b-quark angular distribution in $\cos\theta$ from 0 to 1 (-1 to 0). The asymmetry A_{FB}^b is given by:

$$A_{FB}^b = \frac{\sigma_{bF} - \sigma_{bB}}{\sigma_{bF} + \sigma_{bB}} \quad (3)$$

Experimentally one does not directly measure the b-quark direction: typically, it is the direction of a charged lepton l^- ($l = e, \mu$) from b decay that is measured, with some convenient energy and p_T cuts in order to reject leptons from charm decay and other background processes (where T stands for transverse with respect to the accompanying hadronic jet direction). If the direction of the lepton l^- is identified with the direction of the b quark and if one measures the quantity

$$(A_{FB}^b)_l = \frac{\sigma_{lF} - \sigma_{lB}}{\sigma_{lF} + \sigma_{lB}}, \quad (4)$$

which is obtained from the number of forward and backward tagging leptons, one clearly makes an error ΔA given by:

$$\Delta A = A_{FB}^b - (A_{FB}^b)_l = \frac{2(\sigma_{bF} - \sigma_{bB})(\sigma_{lF} - \sigma_{lB})}{\sigma_{bF} + \sigma_{bB}}, \quad (5)$$

where σ_{bF} is the cross-section for events where the b quark is forward but the tagging lepton l^- is backward, and vice versa for σ_{bB} . We assume that one can correct for this effect together with the effects of the E and p_T cuts on the tagging lepton and of B-B mixing. We thus restrict our consideration to the QCD corrections to the asymmetry as defined in eq. (3).

We now describe the asymmetry in lowest order, then we rederive the order- α_s QCD correction in the massless theory, and finally we set the framework for the second-order calculation.

In general the relevant matrix element squared is a product of a leptonic tensor $L_{\mu\nu}$ and of a hadronic tensor $H_{\mu\nu}$, each with a symmetric and an antisymmetric parts. The numerator of the asymmetry is determined by the antisymmetric part, the denominator by the symmetric part. When mass terms and imaginary parts are neglected, the antisymmetric part is determined by the VA+V interference terms while the symmetric part is made up by the VV+AA terms. After integration, at fixed four-momentum p_1 of the b quark, over the whole remaining phase space, the antisymmetric part of the hadron tensor reduces to the form

$$[dPS_{2,4} H_{\mu\nu} = i \epsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta H_0(x_1) . \quad (6)$$

Applied to this form the tensor $i \epsilon_{\mu\nu\alpha\beta} p_+^\alpha p_-^\beta$ arising from the leptonic term $L_{\mu\nu}$, p_+ (p_-) being the e^+ (e^-) four-momentum, produces the same result as the saturation with the tensor $-i \epsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta \cos\theta_{x_1}$, where q and x_1 are the Z four-momentum and the b energy fraction, respectively. We will make use of this trick often in the following, since contraction with $\epsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta$ is much simpler than with $\epsilon_{\mu\nu\alpha\beta} p_+^\alpha p_-^\beta$, because it only involves the final-state momenta.

At tree level we have the well-known simple formulae [2, 5]:

$$\frac{d\sigma(e^+e^- \rightarrow q\bar{q})}{d\cos\theta} = \frac{3}{8} \sigma_0 \{R(1+\cos^2\theta) + J \cos\theta\} \quad (7)$$

where, for massless quarks,

$$\begin{aligned} R &= N_c [Q^2 - 2Q v_e v \text{Re}\beta + (v_e^2 + a_e^2)(v^2 + a^2) |\beta|^2] \\ J &= N_c [-4Q a_e a \text{Re}\beta + 8 v_e v a_e a |\beta|^2] \end{aligned} \quad (8)$$

$$\text{and } \sigma_0 = \frac{4\pi\alpha^2}{3s}, v = -1/4Q\sin^2\theta w, a = -1, Q = \frac{1}{-3} \text{ for } q=b, \beta = \frac{g_{2S}^2}{s-m_Z^2 + im_Z\Gamma_Z}$$

$g = \frac{G_F}{8\sqrt{2}\pi\alpha}$. For the asymmetry we simply have:

$$(A_{FB}^b)_0 = \frac{3J}{8R}. \quad (9)$$

At the peak, $\sqrt{s} = m_Z$, the previous expression reduces to

$$(A_{FB}^b)_0(\sqrt{s} = m_Z) = \frac{3 v_e v a_e a}{(v_e^2 + a_e^2)(v^2 + a^2)}. \quad (10)$$

Mass corrections in the Born term are well known [2, 5] and can easily be taken into account, but we skip them as the corresponding correction is of the order of 10^{-4} .

We now consider the corrections of order α_s in the massless theory. In general we have virtual and real corrections. At order α_s in the massless theory, the virtual corrections are the same for vector and axial-vector couplings of the Z in all regularizations that preserve chiral symmetry. Therefore virtual corrections cancel between numerator and denominator in the asymmetry, as has been verified by the existing explicit calculations for QCD [3] and for final-state radiation in QED [6]. This implies that the contribution to the asymmetry from the real diagrams is actually finite and corresponds to the complete correction. Thus one only needs to consider the three-parton process:

$$e^+(p^+)e^-(p^-) \rightarrow q(p_1)\bar{q}(p_2)g(p_3).$$

We define, as usual, $y_{ij} = 2p_i p_j / s$, $x_1 = 1 - y_{23}$, $x_2 = 1 - y_{13}$, $x_3 = 1 - y_{12}$. After the azimuthal integration one has:

$$\frac{d\sigma(e^+e^- \rightarrow q\bar{q}g)}{dx_1 dx_2 d\cos\theta} = \frac{3}{8}(1+\cos^2\theta) \frac{d\sigma_T}{dx_1 dx_2} + \frac{3}{4} \sin^2\theta \frac{d\sigma_L}{dx_1 dx_2} + \frac{3}{4} \cos\theta \frac{d\sigma_A}{dx_1 dx_2}, \quad (11)$$

where θ is the angle between p_1 and p_- . From an explicit calculation [3] one obtains:

$$\frac{d\sigma_S}{dx_1 dx_2} = \frac{d(\sigma_T + \sigma_L)}{dx_1 dx_2} = R \sigma_0 \frac{\alpha_s}{2\pi} C_F B_S \quad (12)$$

$$\frac{d\sigma_A}{dx_1 dx_2} = \frac{1}{2} i \sigma_0 \frac{\alpha_s}{2\pi} C_F B_A \quad (13)$$

with $C_F = \frac{4}{3}$ and

$$B_S = \frac{x_1^2 + x_2^2}{y_{13} y_{23}} \theta(1 - y_{13} - y_{23}); \quad B_A = \frac{x_1^2 - x_2^2 \cos\theta_{12}}{y_{13} y_{23}} \theta(1 - y_{13} - y_{23}) \quad (14)$$

with

$$\cos\theta_{12} = 1 + \frac{2}{x_1 x_2} - \frac{2}{x_1} - \frac{2}{x_2}. \quad (15)$$

The result for the asymmetry corrected at first order in α_s can finally be written down in the form:

$$A_{FB}^b = (A_{FB}^b)_0 \left(1 + \frac{\alpha_s}{2\pi} C_F \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 (B_A - B_S) \right), \quad (16)$$

with

$$B_A - B_S = -2 \frac{1 - y_{13}}{1 - y_{23}} = -2 \frac{x_2}{x_1}, \quad (17)$$

which leads to the result

$$A_{FB}^b = (A_{FB}^b)_0 \left(1 - \frac{3}{2} C_F \frac{\alpha_s}{2\pi} \right) = (A_{FB}^b)_0 \left(1 - \frac{\alpha_s}{\pi} \right) \quad (18)$$

It is immediately seen that eq. (18) coincides with the results in eqs. (1) and (2) in the massless limit $\mu = 0$. It is also evident that the entire correction arises from the denominator in the asymmetry, i.e. the total cross-section for b -quark inclusive production, and that the correction to the numerator, i.e. to the antisymmetric coefficient function of $\cos\theta$, vanishes. In fact, from a separate well-known calculation [7], one obtains that the

hadronic cross-sections are corrected by a factor of $\{1 + \frac{3}{2} C_F \frac{\alpha_s}{2\pi}\}$. There is no simple physical reason for the vanishing of the first-order correction to the numerator in the massless limit.

The above derivation can be rephrased in a different form, which is also instructive. One can introduce a cut y to distinguish between "2-jet" and "3-jet" contributions. We call "3-jet" the contribution of phase space where $y_{ij} > y$ for all i, j pairs and "2-jet" the complementary region. In general, we can then write

$$A_{\text{FB}}^b = (A_{\text{FB}}^b)_0 \frac{1 + \frac{\alpha_s}{\pi} [A_2 + A_3]}{1 + \frac{\alpha_s}{\pi} [S_2 + S_3]} \approx (A_{\text{FB}}^b)_0 \left\{ 1 + \frac{\alpha_s}{\pi} [(A-S)_2 + (A-S)_3] \right\}. \quad (19)$$

Here $A_{2,3}$ and $S_{2,3}$ are the 2,3 jet contributions to the antisymmetric (in $\cos\theta \rightarrow -\cos\theta$) numerator and the symmetric denominator, respectively. The 2-jet contributions arise from virtual diagrams plus the soft part of real diagrams with $y_{ij} < y$ for some i, j . The hard part of real diagrams, obtained for $y_{ij} > y$, contributes to the 3-jet components. At fixed non-vanishing values of y , $A_{2,3}$ and $S_{2,3}$ are all separately finite. The difference $(A-S)_2$ stays finite when $y \rightarrow 0$. It actually vanishes in that limit because of the fact already mentioned, that the virtual diagrams contribute the same correction to numerator and denominator of the asymmetry as a consequence of chiral invariance of the massless theory at this order in the perturbative expansion. The result in eq. (18) also implies that $A_3 = -A_2$, i.e. the 2-jet and 3-jet terms in the numerator cancel, so that no corrective term survives in the numerator at order α_s in the massless theory. An explicit calculation leads to

$$A_3 = -A_2 = \frac{C_F}{2} (2 \log^2 y + 3 \log y + 1 - \frac{\pi^2}{3}) + o(y) \quad (20)$$

$$S_3 = \frac{C_F}{2} (2 \log^2 y + 3 \log y + \frac{5}{2} \frac{\pi^2}{3}) + o(y) \quad (21)$$

and thus the result in eq.(18) follows from the difference $(A-S)_3$ which was computed neglecting terms of order y (note that in the limit $y \rightarrow 0$ the directions of the b quark and of the jet axis coincide). This same method can be generalized to the massive case. For example, one can easily compute the linear term in μ appearing in eq.(2). In fact, it is evident that the matrix elements squared are only modified at order μ^2 . Therefore the whole μ term arises from $(A-S)_3$ when the massless matrix element squared is integrated over the exact 3-body phase space (with $x_{1,2} > \mu$ etc.). We have explicitly checked that the linear μ term of eq.(2) is correctly reproduced.

The present calculation of the correction of order α_s^2 is based on a generalization of the above procedure. We start from the formula, analogous to eq.(19), which is valid in the massless theory:

$$A_{\text{FB}}^b = (A_{\text{FB}}^b)_0 \frac{1 + (\frac{\alpha_s}{\pi})^2 [A_2 + A_3 + A_4]}{1 + \frac{\alpha_s}{\pi} + (\frac{\alpha_s}{\pi})^2 [S_2 + S_3 + S_4]} \approx (A_{\text{FB}}^b)_0 \left\{ 1 - \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} [1 + (A-S)_2 + (A-S)_3 + (A-S)_4] \right\}. \quad (22)$$

Here A_i and S_i refer to the 2-, 3- and 4-jet contributions at order α_s^2 (the reader should not confuse these quantities with the analogous ones in eq.(19)) defined in terms of the y cut (two partons are considered to stay within one single jet if $y_{ij} < y$). A collection of typical diagrams contributing at order α_s^2 is presented in fig. 1. As in the case of the $O(\alpha_s)$ calculation, the differences $(A-S)_i$ are less singular and simpler to compute than the individual A_i and S_i . In the following we perform a detailed analysis of the separate 2-, 3-, 4-jet contributions.

3. CALCULATION OF THE SECOND-ORDER CORRECTIONS

3.1 Two-Jet Contribution

The singular part of the 2-jet contribution is the same for the vector and the axial-vector coupling of the Z . As a consequence $(A-S)_2$ is finite and can be computed directly in the limit of $y \rightarrow 0$. In this limit, only genuine 2-parton processes contribute to $(A-S)_2$. The only non vanishing contribution to $(A-S)_2$ arises from the 2-loop diagram with a heavy-quark triangle (the last diagram of fig. 1a). All other diagrams lead to the same correction for the vector and axial-vector coupling of the Z , so that their contribution to $(A-S)_2$ vanishes. For the triangle diagram there is no contribution to the vector coupling, by the Furry theorem, while the axial-vector coupling is modified because, owing to the large t -b splitting the top-quark loop does not completely cancel the bottom-quark loop. The net result of the triangle diagrams is a shift in the effective axial-vector coupling, which was derived in refs.[8, 9]:

$$\frac{\delta a}{a} = -\frac{1}{6} I_2(z) \left(\frac{\alpha_s}{\pi} \right)^2. \quad (23)$$

where a is the axial-vector $Z \rightarrow b\bar{b}$ effective coupling, the function $I_2(z)$ is given in eq. (3) of ref. [8] and $z = (m_Z/2m_t)^2$. A good numerical interpolation for $I_2(z)$, also given in ref.[8], eq. (6) is $I_2 \approx 7.210 - 1.481z - 1.363 z^2 - 6 \log(2\sqrt{z})$. Numerically $I_2(z) = 7.4$, 10.0, 11.8 for $m_t = 100, 150, 200$ GeV respectively. Using this result in eq. (10), we have, given eq. (22), the following result for the correction to the asymmetry:

$$F_{\text{QCD}}^{2\text{-jet}} = \left\{ 1 + \left(\frac{\alpha_s}{\pi} \right)^2 (A-S)_2 \right\} = 1 - \frac{1}{6} \left(\frac{\alpha_s}{\pi} \right)^2 I_2(z) \frac{v^2 - a^2}{v^2 + a^2} \approx 1 + \left(\frac{\alpha_s}{\pi} \right)^2 0.058 I_2(z), \quad (24)$$

where the last numerical result is obtained for $\sin^2\theta_W \approx 0.23$.

3.2 Three-Jet Contribution

The 3-jet contribution is obtained from the diagrams regarded the process $e^+e^- \rightarrow b\bar{b}g$, shown in fig. 1b, combined with the terms extracted from the 4-parton

processes $e^+e^- \rightarrow b\bar{b}gg, e^+e^- \rightarrow b\bar{b}q\bar{q}$; these take into account the configurations where two out of the four partons, say i and j , form a single jet in the sense that $y_{ij} < y$. Among the diagrams for the 3-parton process $e^+e^- \rightarrow b\bar{b}g$, there is a triangle diagram with t and b quarks in the loop (the last diagram in fig. 1b), analogous to the triangle diagram for the 2-jet contribution, discussed in the previous section. This triangle diagram only enters by interference with the lowest-order diagrams for $e^+e^- \rightarrow b\bar{b}g$; it leads to a finite, small, in principle m_t -dependent, correction that we will discuss separately at the end of this section.

We can write down a generalized form of eqs. (11)-(13):

$$\frac{d\sigma(e^+e^- \rightarrow j_1 j_2 j_3)}{dx_1 dx_2 d\cos\theta} = \frac{3}{8}(1+\cos^2\theta) \frac{d\sigma_T}{dx_1 dx_2} + \frac{3}{4} \sin^2\theta \frac{d\sigma_L}{dx_1 dx_2} + \frac{3}{4} \cos\theta \frac{d\sigma_A}{dx_1 dx_2} \quad (25)$$

$$\frac{d\sigma_S}{dx_1 dx_2} = \frac{d(\sigma_T + \sigma_L)}{dx_1 dx_2} = R\sigma_0 \frac{\alpha_s}{2\pi} C_F B'_S \quad (26)$$

$$\frac{d\sigma_A}{dx_1 dx_2} = \frac{1}{2} J\sigma_0 \frac{\alpha_s}{2\pi} C_F B'_A \quad (27)$$

where by $j_1 (j_2)$ we now denote the jet that contains the b quark ($\bar{b}$ antiquark), and x_1 refers to the j_1 energy; θ is the angle between the j_1 and the e^- directions. In the small- y limit the jet and the b directions coincide. The quantities B'_A and B'_S also include the effect of second-order QCD corrections. The relation between $(A-S)_3$ and $B'_A - B'_S$ is specified by

$$(A-S)_3 = \text{coefficient of } \frac{\alpha_s}{\pi} \text{ in } \left[\frac{C_F}{2} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 (B'_A - B'_S) \right]. \quad (28)$$

The expression of B'_S is obtained from the original calculation of Ellis, Ross and Terrano [10]. Similarly B'_A is obtained from the comprehensive study of parity-odd structure functions in e^+e^- annihilation by Körner et al. [11]. After some long algebraic work the final result, when terms of $O(y)$ are omitted, is given by:

$$B'_A - B'_S = (B_A - B_S) \left[1 + \frac{\alpha_s}{2\pi} (C_F Q_C + N_C Q_N + \frac{n_f}{2} Q_T) \right] - \frac{\alpha_s}{2\pi} F_D, \quad (29)$$

where $(B_A - B_S)$ is given by eq.(17) and

$$Q_T = \frac{2}{3} \log y - \frac{10}{9} \quad (30)$$

$$Q_C = -2 \log^2 \frac{y}{y_{12}} - 3 \log y - 1 + \frac{\pi^2}{3} \quad (31)$$

$$Q_N = \log^2 \frac{y}{y_{12}} - \log^2 \frac{y}{y_{13}} - \log^2 \frac{y}{y_{23}} - \frac{11}{6} \log y + \frac{\pi^2}{6} + \frac{67}{18} \quad (32)$$

$$\begin{aligned} F_D = & -4(C_F \frac{N_C}{2}) \left(\frac{1-y_{13}}{1-y_{23}} R(y_{12}, y_{13}) + (2 - \frac{y_{13}}{1-y_{23}} - \frac{1-y_{23}}{y_{13}}) R(y_{12}, y_{23}) + \right. \\ & + \frac{y_{23}}{(1-y_{23})(1-y_{12})} + \frac{\log y_{12}}{(1-y_{12})^2} \left[(1-y_{12})^2 - \frac{y_{13}}{1-y_{23}} \right] - 2N_C \frac{1-y_{13}}{1-y_{23}} R(y_{13}, y_{23}) + \\ & + 2 \frac{\log y_{23}}{(1-y_{23})^2} (N_C y_{23} (1-y_{23}) + C_F [2(1-y_{23})^2 - 2y_{13} + 3y_{13} y_{23}]) + \\ & \left. + 2 C_F \frac{1-y_{13}}{1-y_{23}} \right) \end{aligned} \quad (33)$$

with the following definition:

$$R(x, y) = \log x \log y - \log x \log(1-x) - \log y \log(1-y) + \frac{\pi^2}{6} - Li_2(x) - Li_2(y) \quad (34)$$

and $Li_2(x) = - \int_0^x \frac{\log(1-z)}{z} dz$. It is important to observe that $R(y_{12}, y_{23})$ is $O(y_{13})$ for $y_{13} \rightarrow 0$ (recall that $y_{12} + y_{23} + y_{13} = 1$). As a consequence, as seen from the above explicit expression, the difference $B'_A - B'_S$ has only integrable singularities for $x_{1,2} \rightarrow 1$ so that the integration appearing in eq. (28) can be performed numerically at fixed $y \neq 0$. In fact power singularities at $x_{1,2} \rightarrow 1$ are present separately in B'_A and B'_S , but cancel when the difference is taken just as in the first-order calculation. However, the result of the integral over x_1 and x_2 depends on y and diverges at $y = 0$. The corresponding y singularities of $(A-S)_3$ are only removed when the 4-jet contribution $(A-S)_4$ is added. The results of the numerical calculations are presented in sections 3.3 and 4.

We now consider the triangle diagram term in the 3-jet contribution. The amplitude of the triangle diagram is computed in ref. [9] eq. (2.14), in terms of two functions called A and B . The interference of this amplitude with the Born diagrams for the process $e^+e^- \rightarrow b\bar{b}g$ has to be taken. The Born term with an axial coupling for the Z contributes to the denominator, while the vector-coupling is relevant for the numerator. The B function term does not contribute to the interference. For the symmetric part, which belongs to the denominator of the asymmetry, the reason is explained in ref. [9], just after eq. (2.14). For the numerator, it is due to the fact that the B term is proportional to q_1 and thus it vanishes when saturated with $i\epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta$, according to the procedure outlined after eq. (6) of the present paper. The result can be expressed as follows:

$$\frac{2a_b v_b}{v_b^2 + a_b^2} \rightarrow \frac{2a_b v_b [1 + (\frac{\alpha_s}{6})^2 \frac{1}{6} (I_3^A(t) - I_3^A(b))]}{\pi} \quad (35)$$

or, equivalently, one has for the correction factor to the asymmetry:

$$F_{\text{QCD}}^{\Delta_3, \text{jet}} = 1 + \frac{\alpha_s}{\pi} 2 \left[\frac{1}{6} (I_3^A(0) - I_3^A(b)) - \frac{a_b^2}{v_b^2 + a_b^2} \frac{1}{3} (I_3^S(0) - I_3^S(b)) \right]. \quad (35)$$

I_3^A and I_3^S are functions of the mass m of the quark in the triangle loop. Their expressions are

$$I_3^S = \int_0^1 dy_{12} \int_0^{1-y_{12}} dy_{13} (y_{12}^2 + 2y_{12}y_{13} + y_{12} + 2y_{13}^2 - y_{13}) \frac{F_A(y_{12})}{y_{12} + y_{13}} \quad (36)$$

with $F_A(y_{12}) = m_Z^2 A(y_{12})$ given in eq. (C1) of ref. [9] by:

$$F_A(y) = -\frac{1}{2(1-y)} \left\{ 1 + \frac{1}{1-y} \left[a(y) \log \frac{a(y)+1}{a(y)-1} - a(1) \log \frac{a(1)+1}{a(1)-1} \right] - \frac{1}{4r} \left(\log^2 \frac{a(y)+1}{a(y)-1} - \log^2 \frac{a(1)+1}{a(1)-1} \right) \right\} \quad (37)$$

where $a(y) = \sqrt{1 - \frac{1}{ry}}$ and $r = \frac{m_Z^2}{4m^2}$.

The above expressions, with $r \rightarrow \infty$, are directly adequate for the b quark. In this limit one obtains $F_A(y) = -\frac{1}{2(1-y)} \left(1 + \frac{\log(y)}{1-y} \right)$. Numerically it then follows from eqs. (36) that

$$I_3^S(b) = 1.58; \quad I_3^A(b) = 0.85. \quad (38)$$

Note that the value of $I_3^S(b)$ coincides with the result of ref. [9], eq. (3.3).

For the t quark $r < 1$ and $a(y_{12})$ can become imaginary. In the interval $y_{12} < 1/r$ one starts from the analytic continuation:

$$F_A(y) = -\frac{1}{2(1-y)} \left\{ 1 + \frac{1}{1-y} [-2\alpha(y)\tau(y) + 2\alpha(1)\tau(1) + \frac{1}{r} (\tau^2(y) - \tau^2(1))] \right\}, \quad (39)$$

with $\alpha(y) = \sqrt{\frac{1}{ry} - 1}$ and $\tau(y) = \arctan \alpha(y) - \frac{\pi}{2}$. By numerical integration of eqs. (36) one obtains:

$$\begin{array}{lll} I_3^S(0) = -0.103; & I_3^A(0) = -0.070 & \text{for } m_t = 100 \text{ GeV} \\ = -0.043; & = -0.029 & = 150 \text{ GeV} \\ = -0.024; & = -0.016 & = 200 \text{ GeV} \end{array}$$

In the final result from eq. (35) the t -quark contribution is quite small and one finds:

$$F_{\text{QCD}}^{\Delta_3, \text{jet}} = 1 + \left(\frac{\alpha_s}{\pi} \right) 2 \cdot 0.23 \quad (40)$$

3.3 Four-Jet Contribution

The 4-jet contribution (defined as $y_{ij} > y$ for all possible i, j pairs) is completely determined by the diagrams with four partons in the final state, which correspond to the processes $e^+e^- \rightarrow b\bar{b}g\bar{g}$ (fig. 1c), $e^+e^- \rightarrow b\bar{b}q\bar{q}$ ($q=u, d, c, s$; fig. 1d) and $e^+e^- \rightarrow b\bar{b}b\bar{b}$ (fig. 1e). All these processes have rates that start at order α_s^2 . By itself $(A-S)_4$ is finite only for $y > 0$, but we know (and we will check it by an explicit calculation) that the sum $(A-S)_3 + (A-S)_4$ is finite in the limit $y \rightarrow 0$. When computing the matrix elements squared for the 4-quark processes, one encounters three types of terms called D, E and F in the literature [12, 13]. We call "leading" the quark-antiquark pair from the Z vertex and "secondary" the pair produced by the virtual gluon. Then the D terms are those appearing when leading quarks and antiquarks are contracted among themselves in squaring (and consequently secondaries are also contracted among themselves) as depicted in fig. 2a. Terms of type E are those appearing when one of the leading particles is contracted with a leading one while the second leading particle is contracted with a secondary one (and, of course, in addition there is also one secondary-secondary contraction). This matching, shown in fig. 2b, can only apply to the process $e^+e^- \rightarrow b\bar{b}b\bar{b}$ with identical leading and secondary pairs. Finally in the terms of type F a leading pair is contracted with a secondary pair, giving rise to the double triangle structure shown in fig. 2c. It turns out that the dominant contribution to the asymmetry arises from the $b\bar{b}g\bar{g}$ mode plus the D terms from the $b\bar{b}q\bar{q}$ and $b\bar{b}b\bar{b}$ channels. We start by considering these terms.

Among the D terms associated with the process $e^+e^- \rightarrow b\bar{b}q\bar{q}$, with $q \neq b$ those in fig. 3 deserve a separate discussion. They correspond to the squared modulus of the sum of the two diagrams with a secondary $b\bar{b}$ pair (in the above-mentioned sense that the $q\bar{q}$ pair is created by the Z while the $b\bar{b}$ pair is radiated from the virtual gluon). We denote these terms as D_B terms, where B stands for Branco et al. [14] who studied their effect on the cross-section in detail. These terms do not contribute to the numerator of the asymmetry (as is clear from the fact that the $b\bar{b}$ pair produced by this mechanism is, after colour sum, in a state of definite charge conjugation) while they affect the inclusive $b\bar{b}$ cross-section at the denominator in a peculiar way. In fact the D_B terms are not proportional to $v_b^2 + a_b^2$ but to the light-quark couplings. Moreover their contribution is singular in the limit of massless b quarks (its most singular term behaves as $\log^3 4m_b^2/m_Z^2$). From a purely technical point of view the D_B terms are to be considered as a radiative correction to the inclusive production rate of light quarks. Note that the singularity at $m_b = 0$ of the D_B terms is indeed cancelled

by the diagrams in fig. 4 (when their interference with the lowest-order diagram is taken) which belong to the light-quark rate and have no b quarks in the final state. However the D_B terms do have b quarks in the final state and thus contribute to the measured inclusive b-quark cross-section. Note that the $b\bar{b}$ pair from the D_B terms is much softer than for the ordinary production mechanism so that a considerably smaller fraction of the events from the D_B mechanism passes the cuts in comparison with directly produced $b\bar{b}$ pairs. As a consequence we propose to include the D_B contribution in the following way. This contribution to the measured b-quark inclusive cross-section (with no cuts) can be written down as a corrective factor given, for finite m_b , by:

$$1 + \frac{\alpha_s^2}{\pi} \sum_{q=1}^4 \frac{(v_q^2 + a_q^2)}{v_b^2 + a_b^2} G(m_b, m_q), \quad (41)$$

We have repeated the calculation Branco et al. [14] and reproduced their results for the function $G(m_b, m_q)$. By numerical integration we obtain:

$$G(4.5 \text{ GeV}, 0) = 0.98; G(5 \text{ GeV}, 0) = 0.80. \quad (42)$$

Also, for charm, we find

$$G(4.5 \text{ GeV}, 1.5 \text{ GeV}) = 0.96. \quad (43)$$

On the basis of these results we derive for the asymmetry the correction factor

$$F_{\text{QCD}}^{\text{Branco}} = 1 - \frac{\alpha_s^2}{\pi} \sum_{q=1}^4 \frac{(v_q^2 + a_q^2)}{v_b^2 + a_b^2} G(m_b, m_q) f_{\text{cuts}} = 1 - \frac{\alpha_s^2}{\pi} (3.5 \pm 0.7) f_{\text{cuts}}, \quad (44)$$

where f_{cuts} is the ratio of the reduction factors due to the cuts for the D_B terms and the ordinary contributions, and the error quoted is from the ambiguity on the quark masses. Typical values of f_{cuts} will be discussed in section 4. This concludes our discussion of the D_B terms.

The channel $e^+e^- \rightarrow b\bar{b}g$ contributes to the final result terms proportional to the colour factors C_F^2 and $C_F N_C$ while the D terms from $e^+e^- \rightarrow b\bar{b}q\bar{q}$ and $e^+e^- \rightarrow b\bar{b}b\bar{b}$ combine to produce a term proportional to $C_F m^2/2$ with $n_f=5$. This last result is obtained by carefully taking into account the existence of two b-quarks in the $b\bar{b}b\bar{b}$ final state, the increased number of possible contractions and the statistical factor of 1/4 due to the presence of two pairs of identical particles. The symmetric and antisymmetric parts of the hadronic tensor are computed in refs. [12] and [13], respectively. As mentioned earlier, for the numerator of the asymmetry, we integrate over the phase space of particles other than the b quark whose four-momentum p_1 is kept fixed and contract with $-i \epsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta \frac{\cos\theta}{x_1}$ [recall eq. (6) and the related discussion]. We can compute the contribution to $(A-S)/4$ at each fixed y . Finally we add $(A-S)_3$, taken at the same values of y . We write the results for the contributions to the QCD correction factor of the processes $b\bar{b}g$ and $b\bar{b}gq$, plus the D-terms from $e^+e^- \rightarrow b\bar{b}q\bar{q}$ and $e^+e^- \rightarrow b\bar{b}b\bar{b}$, in the form:

$$F_{\text{QCD}}^D = \left\{ 1 - \frac{3}{4} C_F \frac{\alpha_s(m_Z)}{\pi} + \frac{1}{4} C_F \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{9}{4} C_F + C C_F + N N_C + T T_R \right] \right\}, \quad (45)$$

where $T_R = n_f/2 = 5/2$. The separate results on $C C_F$, $N N_C$ and $T T_R$ for 3 and 4 jets at smaller and smaller values of y are presented in table 1. Taking the differences of 3 and 4 jets for each term we obtain

$$C C_F = 5.8 \pm 0.7; \quad N N_C = -31.0 \pm 0.8; \quad T T_R = 14.2 \pm 0.1, \quad (46)$$

and consequently

$$F_{\text{QCD}}^D \approx 1 - \frac{\alpha_s(m_Z)}{\pi} - (2.6 \pm 0.4) \left(\frac{\alpha_s}{\pi} \right)^2. \quad (47)$$

Note that we have specified the appropriate scale for the first-order term which corresponds to the given second-order correction.

The E terms (fig. 2b), only present in the $b\bar{b}b\bar{b}$ channel, are proportional to one single colour factor given by $C_F(C_F - N_C/2)$. The relevant contribution to the hadronic tensor can be found in refs. [12,13]. By numerical integration we find that the net contribution of the E terms to the asymmetry vanishes (within the accuracy of the calculation) because of a cancellation between the numerator and the denominator. Thus:

$$F_{\text{QCD}}^E \approx 1. \quad (48)$$

Finally we consider the F-terms which are present for the $b\bar{b}b\bar{b}$ and the $b\bar{b}q\bar{q}$ final states. The resulting correction can be cast into the form:

$$\frac{2a_b v_b}{v_b^2 + a_b^2} \rightarrow \frac{2a_b v_b \left[1 + \left(\frac{\alpha_s}{\pi} \right)^2 \frac{1}{3} I_4^A \left(1 + \frac{1}{2} \sum_{q=1}^4 \frac{v_q}{v_b} \right) \right]}{v_b^2 + a_b^2 \left[1 + \left(\frac{\alpha_s}{\pi} \right)^2 \frac{1}{3} I_4^S \right]} \quad (49)$$

Equivalently, for the correction factor to the asymmetry, one has :

$$F_{\text{QCD}}^F \approx 1 + \frac{\alpha_s^2}{\pi} \left(\frac{1}{3} I_4^A \left(1 + \frac{1}{2} \sum_{q=1}^4 \frac{v_q}{v_b} \right) - \frac{a_b^2}{v_b^2 + a_b^2} \frac{1}{3} I_4^S \right).$$

The quantities I_4^A and I_4^S are determined by the AV+VA and AA components of the

hadronic tensor from the F terms of the $b\bar{b}b\bar{b}$ channel, respectively. The factor $(1+$

$\frac{1}{2} \sum_{q=1}^4 \frac{v_q}{v_b}$ is the change introduced by the $b\bar{b}q\bar{q}$ final state, which does not contribute to the denominator because $\sum_{q=1}^4 a_q = 0$ (this is also the origin of the $1/2$ in front of the sum: while for the $b\bar{b}b\bar{b}$ channel both $V_b A_b$ and $A_b V_b$ contribute, in the $b\bar{b}q\bar{q}$ case only $A_b V_q$ is non vanishing after summing over the light quarks). When the b -quark mass is neglected, the loop factors I_4^A and I_4^S are the same for all five flavours of quarks. The quantity I_4^S was computed in ref. [9] [see their eq. (2.13)] with the result:

$$I_4^S \approx -0.460. \quad (50)$$

Similarly, for the antisymmetric part, we start from the corresponding hadronic tensor, obtained from ref. [15], we contract with the tensor $-i \epsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta \frac{\cos\theta}{x_1}$, and integrate numerically over the phase space. As a result we obtain:

$$I_4^A \approx -0.48. \quad (51)$$

This finally implies:

$$F_{\text{QCD}}^F \approx 1 - \left(\frac{\alpha_s}{\pi}\right)^2 0.13. \quad (52)$$

4. FINAL RESULT ON THE B-QUARK ASYMMETRY

The final result for the QCD correction to A_{FB}^b is obtained by collecting the factors given in eqs. (24), (40), (44), (47), (48) and (52). This leads to:

$$\begin{aligned} F_{\text{QCD}} &\approx 1 - c_1 \frac{\alpha_s(m_Z)}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 [-(2.5 \pm 0.4) - (3.5 \pm 0.7) f_{\text{cusp}} + 0.058 I_2(z)] \approx \\ &\approx 1 - c_1 \frac{\alpha_s(m_Z)}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 [-(1.9 \pm 0.4) - (3.5 \pm 0.7) f_{\text{cusp}}], \end{aligned} \quad (53)$$

where in the last step we used the value $I_2(z) = 10$ obtained in section 3.1 for $m_t = 150$ GeV (the m_t variation from 100 to 200 GeV would lead to an additional 0.1 error). The coefficient c_1 , defined in eq. (2), was reintroduced to take mass effects into account in the lowest-order correction. We recall that the quantity f_{cusp} was introduced by eq. (44) in connection with the Branco et al. [14] contribution to the b -inclusive cross-section. This quantity should be estimated by each experiment. As an indication, we find that $f_{\text{cusp}} \approx 1/2.5$ for b quarks identified via $b \rightarrow l\nu X$ with an energy cut $E(l) > 4.5$ GeV and a transverse (with respect to the jet axis) momentum cut $p_T(l) > 1$ GeV.

We see from eq. (53) that the second-order correction to A_{FB}^b is in all cases quite small. We also note that in second order it is no longer true that, in the massless theory, the whole correction arises from the denominator (for this to be true, -1.9 should be -0.411 in the limit of negligible m_t -loop contribution).

5. THE CASE OF CHARM

Up to now we have considered the case of the b quark. We now briefly consider what is to be changed in the case of the charm quark forward-backward asymmetry A_{FB}^c . We will discuss the various contributions in the same order as in the previous case.

The 2-jet contribution can be written down in a similar form as in eqs. (23), (24). The difference is only in the overall sign of the correction and in the v and a couplings, which are now those of the c quark. Numerically, for $\sin^2\theta_W \approx 0.23$ one finds:

$$F_{\text{QCD}}^{2\text{-jet}} = 1 + \frac{1}{6} \left(\frac{\alpha_s}{\pi}\right)^2 I_2(z) \frac{v^2 - a^2}{v^2 + a^2} \approx 1 - \left(\frac{\alpha_s}{\pi}\right)^2 0.123 I_2(z) \quad (54)$$

For the 3-jet contribution the only modification has to do with the triangle diagrams. Equation (35) keeps the same form, with the same coefficients given by eqs. (36)-(39) (the b quark was taken as massless in the loop), but with the opposite sign and the v and a couplings of charm. One obtains:

$$\begin{aligned} F_{\text{QCD}}^{A3\text{-jet}} &= 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{6} (I_3^0 - I_3^A(b)) - \frac{a_c^2}{v_c^2 + a_c^2} \frac{1}{3} (I_3^S(t) - I_3^S(b)) \right] \approx \\ &\approx 1 - \left(\frac{\alpha_s}{\pi}\right)^2 0.39 \end{aligned} \quad (55)$$

Among the 4-jet contributions, what changes is the Branco et al. [14] contribution, eqs. (41)-(44), and the triangle diagram term eq. (49), while eqs. (47) and (48) for the D and E contributions, respectively, are unaltered. Equation (41) is modified into:

$$F_{\text{QCD}}^{\text{Branco}} = 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \sum_{q=1}^4 \frac{(v_q^2 + a_q^2)}{v_c^2 + a_c^2} G(m_c, m_q) f_{\text{cusp}} \quad (56)$$

where the sum runs over u, d, s and b quarks and f_{cusp}^b is the analogue of f_{cusp} for charm. For m_c in the range 1.1 - 1.7 GeV and $m_b \approx 4.5$ GeV one finds:

$$G(m_c, 0) \approx 5.5 \pm 1.3; \quad G(m_c, m_b) \approx 4.8 \pm 1.1. \quad (57)$$

We then obtain:

$$\begin{aligned}
F_{\text{QCD}}^{\text{Branco}} &= 1 - \frac{\alpha_s}{\pi} 2 \left[1.29 G(m_c, m_b) + 3.58 G(m_c, 0) \right] \hat{f}_{\text{cuts}}^c = \\
&\approx 1 - \frac{\alpha_s}{\pi} 2 (26 \pm 6) \hat{f}_{\text{cuts}}^c.
\end{aligned} \tag{58}$$

Note that this correction to A_{FB}^c , which is the inverse of that relevant for the c-quark inclusive cross-section, is quite large ($\approx 4\%$ for $\hat{f}_{\text{cuts}}^c = 1$) if the experimental cuts are not chosen in a way to minimize $\hat{f}_{\text{cuts}}^c$.

The F terms which are present for the $b\bar{b}b\bar{b}$ and the $b\bar{b}q\bar{q}$ final states are also to be modified because the couplings are different. The resulting correction can be cast into the form:

$$F_{\text{QCD}}^F = 1 - \frac{\alpha_s}{\pi} 2 \left[\frac{1}{3} \frac{1}{4} \sum_{q=1}^4 \frac{a_c v_q + v_c a_q}{v_c a_c} - \frac{a_c^2}{v_c^2 - a_c^2} \frac{1}{3} \frac{1}{4} \sum_{q=1}^4 (1 + \frac{a_q}{a_c}) \right], \tag{59}$$

where the sum runs over $q = u, d, s, b$. A numerical evaluation, using eqs. (50), (51), leads to the result (for $\sin^2\theta_W \approx 0.23$):

$$F_{\text{QCD}}^F \approx 1 - \frac{\alpha_s}{\pi} 2 \cdot 0.22. \tag{60}$$

By collecting the above results one obtains for charm:

$$\begin{aligned}
F_{\text{QCD}} &= 1 - c_1 \frac{\alpha_s(m_Z)}{\pi} + \frac{\alpha_s}{\pi} 2 \left[-(3.2 \pm 0.4) - (26 \pm 6) \hat{f}_{\text{cuts}}^c - 0.123 I_2(z) \right] \\
&\approx 1 - c_1 \frac{\alpha_s(m_Z)}{\pi} + \frac{\alpha_s}{\pi} 2 \left[-(4.4 \pm 0.4) - (26 \pm 6) \hat{f}_{\text{cuts}}^c \right],
\end{aligned} \tag{61}$$

where the value of c_1 in the case of charm is given by $c_1 \approx 0.95 - 0.93$ for $m_c = 1.1 - 1.7$ GeV. We see that the charm correction is very small provided that the experimental cuts can be chosen in a way to strongly suppress $\hat{f}_{\text{cuts}}^c$.

6. CONCLUSION

We have computed the next-to-leading QCD corrections to the forward-backward asymmetries of b and c quarks. The final results are given by eq. (53) (b quark) and (61) (c quark). Once the second-order corrections are known, the first-order term is completely well defined. In particular the numerical results given by eqs. (53) and (61) refer to a choice of scale in the leading correction given by the centre-of-mass energy m_Z . With this choice, the second-order corrections are small, at least for $\hat{f}_{\text{cuts}}$ and $\hat{f}_{\text{cuts}}^c$ small. We find it

interesting that the Branco et al. [14] terms can be as large as 0.5% for b quarks and 4% for c quarks. These contributions are usually not included in the QCD corrections to the partial inclusive widths into charm or bottom pairs because, in a technical but not physical sense, they can be considered as radiative corrections to the light-quark widths. This implies that for a precise definition of partial widths and asymmetries for b and c quarks a specification of the experimental cuts must be added in order to give a value for the suppression factors $\hat{f}_{\text{cuts}}$ and $\hat{f}_{\text{cuts}}^c$ and make the QCD corrections explicit.

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Table 1

y	TTR			CCF			NNC		
	3-jet	4-jet	Sum	3-jet	4-jet	Sum	3-jet	4-jet	Sum
0.01	15.7	-2.17	13.53	28.7	-17.9	10.8	-21.7	-17.67	-39.4
0.001	21.4	-7.24	14.16	83.4	-77.0	6.4	55.0	-86.5	-31.5
0.0001	27.13	-12.9	14.23	181.2	-175.2	6	179.0	-210.5	-31.5
0.00001	33.0	-18.8	14.20	321.9	-316.1	5.8	350.2	-381.2	-31.0

Fig. 2

Fig. 1

Fig. 3

Fig. 4