Final States in Heavy Quark Leptoproduction at Small x^*

G. Marchesini

Dipartimento di Fisica, Università di Parma and
INFN, Gruppo Collegato di Parma, Italy

and

B.R. Webber[†]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

The process of heavy quark production in ep collisions at very high energies is investigated in detail using a new Monte Carlo simulation of QCD which includes leading contributions in the small- x region. The simulation generates a gluon structure function which satisfies the Lipatov evolution equation, together with the associated initial-state gluon radiation, and combines this with the off-mass-shell gluon-photon fusion cross section. The resulting heavy quark and associated final-state hadron distributions are asymptotically different from those of the conventional treatment based on Altarelli-Parisi evolution and on-shell gluon-photon fusion, but the differences are not great at HERA energies.

CERN-TH.6495/92
UPRF 92-332
May 1992

*Research supported in part by the UK Science and Engineering Research Council and in part by the Italian Ministero dell'Università e della Ricerca Scientifica e Tecnologica.

[†]On leave of absence from Cavendish Laboratory, University of Cambridge, UK.

1 Introduction

Heavy flavour production is a hard process with a scale, much larger than the QCD scale Λ , set by the quark mass M and therefore it can be treated in perturbative QCD by resummation of collinear logarithms (powers of $\ln M^2/\Lambda^2$). On the other hand at very high energies this scale is much smaller than the collision c.m. energy $\sqrt{s}$, which implies that the process occurs in the so-called small- x region. In this region a second large parameter, $\ln x$, enters into the perturbative expansion, with $x \simeq M^2/s \ll 1$. Much progress has been made over the past few years in the theoretical understanding of small- x processes. In particular it is now possible to compute both the asymptotic production cross section and the exclusive distributions to leading order. The key results are as follows.

1) *Anomalous dimension* $\gamma_N(\alpha_S)$. The all-loop expression for the leading contributions to $\gamma_N(\alpha_S)$ has been known for a long time [1] but has only recently been studied and applied in the framework of hard processes [2, 3, 4]. It is given by the expansion

$$\gamma_N(\alpha_S) = \frac{\bar{\alpha}_S}{N-1} + 2\zeta_3 \left(\frac{\bar{\alpha}_S}{N-1} \right)^4 + 2\zeta_5 \left(\frac{\bar{\alpha}_S}{N-1} \right)^6 + 12\zeta_3^2 \left(\frac{\bar{\alpha}_S}{N-1} \right)^7 + \dots \quad (1)$$

where $\bar{\alpha}_S = C_A \alpha_S / \pi$ and ζ_i is the Riemann zeta function. Notice that there are no leading terms of order α_S^2 , α_S^3 , and α_S^5 . Recall that the limit $x \rightarrow 0$ corresponds to $N \rightarrow 1$, where N is the energy moment index. This expression is consistent with the 2-loop result [5] for $N \rightarrow 1$, apart from a subleading term $c \alpha_S^2 / (N-1)$. Although each term is singular only at $N = 1$, the expansion (1) develops a square root singularity at $N = 1 + (4 \ln 2) \bar{\alpha}_S$. The presence of this singularity at $N > 1$ implies that the behaviour of the structure function for $x \rightarrow 0$ is more singular than that given by any finite number of loops.

2) *Coefficient function* $C_N(\alpha_S)$. Recently a way has been found to sum all leading terms $\alpha_S^n / (N-1)^n$ in the coefficient function as well. The key development [6, 7, 8] in this study has been a generalized factorization theorem, in which one takes into account the off-shellness of the hard scattered parton. From the all-loop expression for the anomalous dimension and the coefficient function one is able to obtain the heavy flavour production cross section at high energy.

3) *Improved coherent branching*. In order to compute less inclusive quantities one needs a theoretical understanding of small- x processes which goes beyond $\gamma_N(\alpha_S)$ and $C_N(\alpha_S)$. In the case of finite x , all asymptotic exclusive quantities can be computed by means of a coherent branching algorithm [9, 10] corresponding to the resummation of all leading collinear and infrared logarithms. In order to be able to perform a similar calculation for small- x processes, one needs to extend this branching algorithm to resum also all leading logarithms for $x \rightarrow 0$. This extension has been achieved in Ref. [3, 4] and the improved coherent branching has the following characteristics.

- Correct coherence for all values of x including those near zero. The resulting structure function for x near zero satisfies the Lipatov equation and hence contains the all-loop anomalous dimension in Eq. (1).
- Since a running coupling constant is used in the branching, the solution of the Lipatov equation is generalized to include renormalization group effects.

- This branching allows a unified treatment of x finite and x near zero. The resulting structure function satisfies the unique equation [11] which reduces to the Altarelli-Parisi equation for finite x and to the Lipatov equation for small x .
- Inclusion of final-state branching according to the usual time-like coherent branching algorithm permits a full event simulation.
- The off-shell elementary hard scattering is naturally included since the parton virtuality is generated during the branching. In this way the all-loop corrections to the coefficient function are also taken into account.

The new coherent branching algorithm forms the basis of a Monte Carlo program [12] which generates deep inelastic processes for all values of x including $x \rightarrow 0$. This program has been used mainly to study the differences between the new branching algorithm and the conventional one, which corresponds to keeping only the first term in the expansion of the anomalous dimension and coefficient function (one-loop approximation).

In this paper we extend the Monte Carlo program of Ref. [12] to study heavy flavour production at HERA energy and at the energy of a hypothetical LEP $\times$ LHC ep machine with beam momenta of 80 and 8000 GeV/c, which we shall call LEHC. We include in the program the relevant off-shell hard subprocess cross section. In this way our calculation includes all-loop corrections for $N \rightarrow 1$ both in the anomalous dimension and the coefficient function. We focus our attention on the calculation of distributions of the final state partons, including the heavy quarks.

In Sect. 2, the cross section for the process is given in term of the off-shell factorization theorem. The off-shell subprocess is considered in detail, especially the question of how the relevant upper limit μ on the gluon virtuality, which determines the amount of initial-state radiation, emerges naturally from the dynamics of the subprocess. The coherent parton branching algorithm is reviewed both in the improved all-loop formulation (Sect. 3) and in the conventional one-loop approximation (Sect. 4). Sect. 5 presents the results of the Monte Carlo simulation of heavy quark electroproduction at small x . Here we compare and contrast the conventional prescription of one-loop branching plus on-shell subprocess with the improved all-loop plus off-shell formulation. Finally in Sect. 6 some conclusions are drawn.

2 Cross section

Consider the cross section for heavy quark electroproduction via photon-gluon fusion at a c.m. energy $\sqrt{s}$ much greater than the quark mass M (Fig. 1). In order to use the k_t -factorization theorem, which allows the resummation of leading infrared logarithms in the coefficient function, we have to consider the elementary subprocess $\gamma^* g^* \rightarrow Q\bar{Q}$ (see Fig. 1) in which not only the photon momentum q but also the incoming gluon momentum $k = k_n$ is off-shell. For $M^2 \ll s$ the dominant phase space region is the one in which k and q have small fractions of the proton and electron momenta respectively. We can write $k \simeq xp_p + k_t$ and $q \simeq yp_e + q_t$, where k_t and q_t are orthogonal to both p_p and p_p , so that $k^2 \simeq -k_t^2$ and

$q^2 = -Q^2 \simeq -q_t^2$. According to the results in Ref. [6, 7, 8], the heavy flavour cross section for $M^2 \ll s$ is then expressible in the form

$$\sigma(ep \rightarrow Q\bar{Q}X) \sim \frac{\alpha}{2\pi} \int \frac{dx}{x} \frac{dy}{y} \frac{dQ^2}{Q^2} d^2k_t [1 + (1-y)^2] \mathcal{F}(x, k_t) \hat{\sigma}(\hat{s}, M; k_t, q_t). \quad (2)$$

The object $\hat{\sigma}$ is a generalized subprocess cross section for the off-shell photon and gluon to produce a heavy quark-antiquark pair with c.m. energy-squared $\hat{s}$. Such a quantity can be defined in an unambiguous, gauge-invariant way in the small- x region. The explicit form of the off-shell differential cross section for photon-gluon fusion is [6, 7]

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{4} e_Q^2 \alpha_s x y s \sqrt{1 - \frac{4M^2}{\hat{s}}} \left[\frac{1}{(\hat{t} - M^2)(\hat{u} - M^2)} - \frac{1}{k_t^2 q_t^2} \left(1 + \frac{x_Q y_{\bar{Q}} s}{\hat{t} - M^2} + \frac{y_Q x_{\bar{Q}} s}{\hat{u} - M^2} \right)^2 \right] \quad (3)$$

where $d\Omega$ represents the element of solid angle in the subprocess c.m. frame, $\hat{t} = (p_Q - q)^2$ and $\hat{u} = (p_{\bar{Q}} - q)^2$ are the usual Mandelstam variables, and we have written the heavy quark and antiquark momenta as

$$p_Q = x_Q p_p + y_Q p_e + p_{tQ}, \quad p_{\bar{Q}} = x_{\bar{Q}} p_p + y_{\bar{Q}} p_e + p_{t\bar{Q}}. \quad (4)$$

If, for example, the gluon momentum k goes on-shell, $k_t^2 \rightarrow 0$, then $\hat{t} \rightarrow M^2 - xy_{\bar{Q}}s$ and $\hat{u} \rightarrow M^2 - xy_Qs$, so the $1/k_t^2$ singularity is cancelled and we recover the usual on-shell expression. On the other hand, when k_t^2 becomes much larger than M^2 and Q^2 then the cross section is suppressed.

The function $\mathcal{F}(x, k_t)$ in Eq. (2) is a generalized proton structure functions giving the probability (per unit of $\ln x$) of finding a gluon at longitudinal momentum fraction x and transverse momentum k_t . When integrated over transverse momentum up to some limit μ , the generalized structure function becomes the usual structure function giving the gluon momentum fraction distribution at scale μ^2

$$\int_0^{|k_t|=\mu} d^2k_t \mathcal{F}(x, k_t) = F(x, \mu^2). \quad (5)$$

The transverse momentum k_t arises from the emission of gluons in the course of the evolution of the structure function from the typical hadronic scale up to scale μ , as shown in Fig. 1. The kinematical bound on k_t is set by the total hadronic energy, but the important values of k_t contributing to the integral in Eq. (2) are bounded by the characteristic scale μ of the hard subprocess cross section $\hat{\sigma}$, which is of the order of M or Q .

In the conventional method of calculation, valid for finite x , one assumes that k_t is bounded by μ and puts the hard subprocess cross section $\hat{\sigma}$ on-shell ($k_t^2 = 0$). We can analyse the reliability of this procedure in the light of the generalization of the factorization theorem in Eq. (2). The dynamical cutoff on the gluon virtuality is illustrated in Fig. 2, which shows the partially integrated cross section

$$\sigma(k_t^2) \equiv \int \frac{dx}{x} d\phi_{qk} \hat{\sigma}(\hat{s}, M; k_t, q_t) (1-x)^5, \quad (6)$$

where ϕ_{qk} represents the azimuthal angle between the photon and gluon transverse momenta, the factor $(1-x)^5$ represents the typical suppression at large x for a gluon structure function,

and the heavy quark is taken to be a b quark of mass $M = 5$ GeV. The cross section is shown, normalized to its on-shell value, for fixed values of the Bjorken variable $x_B = Q^2/ys$ and various values of Q^2 , at (a) HERA and (b) LHC energies. We see that when $W^2 \gg M^2 \gg Q^2 \gg \Lambda^2$, $W^2 = ys - Q^2 \simeq Q^2/x_B$ being the hadronic c.m. energy squared, the natural cutoff is around $k_t^2 \sim 4M^2$, while for deep inelastic production with $W^2 \gg Q^2 \gg M^2$ the cutoff becomes Q^2 . Thus the dynamics of the hard subprocess provide a natural upper limit on the evolution of the initial-state gluon cascade, of the order of

$$\mu^2 = Q^2 + 4M^2, \quad (7)$$

indicated by the arrows in Fig. 2, beyond which the contribution to the cross section is suppressed. However at small Q^2 the dynamical suppression in k_t is at a significantly smaller scale. A smaller scale is found also for small $W \simeq M$.

In the conventional method of calculation, the on-shell subprocess cross section $\sigma(0)$ is multiplied by the gluon structure function $F(x, \mu^2)$. This is equivalent to approximating Fig. 2 by a step-function which drops sharply from the on-shell value to zero at $k_t^2 = \mu^2$, with μ given by Eq. (7). It can be seen that this procedure has two main effects on the value obtained for the heavy quark production cross section:

- (i) The on-shell subprocess cross section is larger in the region $k_t^2 < \mu^2$, so the cross section there is over-estimated;
- (ii) The ‘tail’ of the cross section at $k_t^2 > \mu^2$ is ignored, which leads to an under-estimate.

Asymptotically, the second effect dominates, so the true asymptotic cross section is expected [6, 7, 8] to be larger than the conventional on-shell Born approximation. However, at subasymptotic energies the first effect is important and one over-estimates the cross section. When Q^2 is small the cross section in the conventional approach is over-estimated even more.

3 All-loop coherent branching

The generalized structure function $\mathcal{F}$ and subprocess cross section $\hat{\sigma}$ provide a complete description of the fully inclusive heavy quark electroproduction cross section at small x , but for many purposes we would like to know the more exclusive properties of the final state. In particular, we note from Fig. 1 that a subprocess with characteristic scale μ involves “initial-state” emission of gluons with transverse momenta $k_t \lesssim \mu$. For a fully exclusive description of the process one should consider that the emitted gluons and heavy quark pair undergo final-state branching with some infrared cutoff Q_0 . The necessity of such a cutoff is related to parton confinement, which can be treated phenomenologically by some hadronization model which is consistent with coherence. After hadronization, this “initial-state radiation” leads to an associated hadron multiplicity and transverse energy flow that should be taken into account in studies of the final state.

The coherent branching algorithm of Refs. [3, 4], on which the program of Ref. [12] is based, provides the means for such studies. In this Section we recall its main features and describe its application to the present problem.

In an initial-state gluon cascade like that in Fig. 1, a gluon of low virtuality k_0^2 evolves towards higher spacelike virtuality and lower energy via multigluon emission $q_1, \dots, q_n$. We first introduce the kinematic variables as follows. We write the emitted gluon momenta as

$$q_i = y_i(p_p + \xi_i p_e) + q_{ti}, \quad \xi_i = q_{ti}^2 / s y_i^2, \quad (8)$$

where

$$y_i = (1 - z_i)x_{i-1}, \quad x_i = z_i x_{i-1}. \quad (9)$$

The variable ξ_i is then essentially the angle-squared of the emitted gluon with respect to the incoming proton, while x_i and y_i are the p_p -momentum fractions of the exchanged and emitted gluons, and z_i is the momentum fraction for the branching $(i-1) \rightarrow i$.

Similarly for the heavy quark pair momentum we write

$$p_Q + p_{\bar{Q}} = Y(p_p + \Xi p_e) + Q_t, \quad \Xi = \frac{\hat{s} + Q_t^2}{s Y^2}, \quad (10)$$

where $\hat{s}$ is the $\gamma^* g^* \rightarrow Q \bar{Q}$ subprocess centre-of-mass energy squared.

The important features of the branching are the following.

(i) *Coherence.* For $x \rightarrow 0$ and $x \rightarrow 1$, interference among soft gluons gives multiparton distributions for the initial-state branching which are confined to an angular-ordered region:

$$\xi_0 < \xi_1 < \dots < \xi_n < \Xi \quad (11)$$

which suggests the introduction of the scaled transverse momentum

$$q'_{ti} = x_{i-1} \sqrt{s \xi_i} = q_{ti} / (1 - z_i), \quad (12)$$

for which the ordering becomes

$$q'_{ti} > z_{i-1} q'_{ti-1}. \quad (13)$$

The distribution of the successive values of q'_{ti} is controlled by the Sudakov form factor defined below. Notice that, due to the rescaling by z_{i-1} in Eq. (13), the angular-ordered phase space (11) for $x \rightarrow 0$ gives essentially no constraint on the transverse momenta of emitted partons.

(ii) *Initial and final conditions.* The cascade evolves from ξ_0 , related to a lower scale μ_0 at which the input proton structure function is given, up to Ξ . In terms of the variable q'_{ti} , the initial emitted gluon satisfies the condition

$$q'_{t1} > x_0 \sqrt{s \xi_0} \equiv \mu_0 \quad (14)$$

where the distribution of x_0 is specified by the input structure function. The last emitted gluon in Fig. 1 must satisfy the condition

$$q'_{tn} < x_{n-1} \sqrt{s \Xi} = \frac{x_{n-1}}{Y} \sqrt{\hat{s} + Q_t^2}. \quad (15)$$

This means that the branching stops when the next gluon with momentum q_{n+1} would exceed the bound (15).

(iii) *Splitting function.* The usual gluon splitting function is modified by the presence of the non-Sudakov form factor and becomes

$$P_g(z_i, \alpha_S) = \bar{\alpha}_S \left[\frac{1}{1-z_i} + \frac{\Delta_{ns}^{(i)}}{z_i} - 2 + z_i(1-z_i) \right], \quad \bar{\alpha}_S = \frac{C_A \alpha_S}{\pi}. \quad (16)$$

The non-Sudakov form factor is generated by the resummation of virtual corrections of non-eikonal type, which are relevant only for low x , and screens the singularity of the splitting function as $z_i \rightarrow 0$. It has the form

$$\Delta_{ns}^{(i)} = \Delta_{ns}(z_i, q_{ti}^2, k_{ti}^2) = \exp \left[-\bar{\alpha}_S(k_{ti}^2) \ln \left(\frac{1}{z_i} \right) \ln \left(\frac{k_{ti}^2}{z_i q_{ti}^2} \right) \right]. \quad (17)$$

For the simulation presented in this paper, we neglect the finite terms $-2 + z_i(1-z_i)$ in (16), which are unimportant for both $x \rightarrow 0$ and $x \rightarrow 1$. For the same reason we also neglect the quark contribution to the branching process. These simplifications allow both the form factors and the distributions of q'_{ti} and z_i to be calculated analytically in terms of simple functions.

(iv) *Argument of α_S .* In general [13], the appropriate hard scale for the argument of α_S is expected to be $(1-z_i)|k_i^2|$, which tends to k_{ti}^2 as $z_i \rightarrow 0$ and to q_{ti}^2 as $z_i \rightarrow 1$. Therefore k_{ti}^2 is taken as the argument of α_S in the non-Sudakov form factor (17) and in the $1/z_i$ term of (16), while the argument is q_{ti}^2 for the soft gluon emission term $1/(1-z_i)$ and in the Sudakov form factor. Thus the final form used for the splitting function is

$$\tilde{P}_g(z_i, q_{ti}^2, k_{ti}^2) = \frac{\bar{\alpha}_S(q_{ti}^2)}{1-z_i} + \frac{\bar{\alpha}_S(k_{ti}^2)}{z_i} \Delta_{ns}(z_i, q_{ti}^2, k_{ti}^2). \quad (18)$$

(v) *Infrared cutoff and Sudakov form factor.* Although the structure function is an infrared-finite quantity, for a full description of the final state an infrared cutoff is necessary. As shown in higher-order calculations, the proper cutoff is in transverse momentum, which in terms of our variables becomes

$$z_i < 1 - Q_0/q'_{ti}. \quad (19)$$

This cutoff is imposed on the real emission and enters correspondingly into the definition of the Sudakov form factor, in such a way that inclusive quantities are infrared-finite.

The Sudakov form factor Δ_s is thus given by

$$\Delta_s(q'_{ti}, z_{i-1} q'_{ti-1}) = \exp \left[- \int_{(z_{i-1} q'_{ti-1})^2}^{q_{ti}^2} \frac{dq_t'^2}{q_t'^2} \int_0^{1-Q_0/q'_t} \frac{dz}{1-z} \bar{\alpha}_S(q_t^2) \right]. \quad (20)$$

Note that the q'_t integration region corresponds to the angular ordering constraint (13). The argument of α_S is the square of the transverse momentum $q_t = (1-z)q'_t$. As usual, this form factor is used to generate the distribution of the variable q'_{ti} in the branching process.

(vi) *Momentum sum rule.* According to the momentum sum rule, the gluon anomalous dimension γ_N should vanish for $N = 2$ in the absence of quarks. This quantity is obtained from the splitting function (18) regularized according to the form factor (20):

$$\gamma_2 = \int_0^1 dz_i \left\{ \left(\frac{\bar{\alpha}_S(q_{ti}^2)}{1-z_i} \right)_+ z_i + \bar{\alpha}_S(k_{ti}^2) \Delta_{ns}^{(i)} \right\} = \int_0^1 dz_i \left\{ -\bar{\alpha}_S(q_{ti}^2) + \bar{\alpha}_S(k_{ti}^2) \Delta_{ns}^{(i)} \right\} = \mathcal{O}(\alpha_S^2). \quad (21)$$

Within our approximation scheme this is negligible, but nevertheless it means that a violation of the momentum sum rule can build up during the cascade. In principle, the splitting function could be modified slightly by including a correction of order α_S^2 to give $\gamma_2 = 0$ at each branching. For simplicity, we prefer to apply a global correction by renormalizing all output distributions by the ratio of the input and output momentum sums.

(vii) *Transverse momentum random walk.* For finite x , the values of the transverse momenta in a QCD cascade increase towards the hard vertex. For $x \rightarrow 0$ this is no longer true. We see from Eq. (13) that for $z_{i-1} \rightarrow 0$ the value of $q_{ti} \sim q'_{ti}$ is free to be smaller than the previous value, although in any case larger than the cutoff Q_0 as required by Eq. (19). Correspondingly, the exchanged transverse momentum $k_{ti} = |\mathbf{q}_{t1} + \dots + \mathbf{q}_{ti}|$ performs a random walk, and can reach arbitrarily small values. As is well known, the Lipatov equation has no singularity at $k_t = 0$, owing to a cancellation between real and virtual (non-Sudakov) contributions. However, for running α_S the presence of $\alpha_S(k_{ti}^2)$ in Eq. (18) implies that a cutoff $k_t > Q_0$ must be introduced. Since the region $k_t < Q_0$ is an asymptotically vanishing fraction of phase space, and no pole singularity is associated with it, the effect of the k_t cutoff is asymptotically negligible.

(viii) *Convolution with hard subprocess cross section.* When the stopping condition (15) has been reached, we compute the off-shell subprocess cross section in Eq. (2) using the final exchanged gluon momentum k_n and the previously-generated virtual photon momentum q , as shown in Fig. 1. As was done in the derivation of Eq. (2), the small component of the gluon momentum along p_e , and of the photon momentum along p_p , are neglected in the calculation of the cross section.

(ix) *Final-state emission.* Each primary emitted gluon q_i in the initial-state branching processes of Fig. 1 undergoes a timelike coherent cascade according to the algorithm of Ref. [10]. The cascade is confined within an angular cone around the spatial direction of q_i of aperture θ_i , where $\xi_i = 1 - \cos \theta_i$ is obtained via Eq. (12) from the value of q'_{ti} generated in the initial-state branching. The cascade continues until the evolution variables of all final-state gluons fall below the cutoff scale Q_0 . This treatment takes soft gluon coherence in timelike parton evolution fully into account to leading order.

In general there will also be gluon radiation from the heavy quarks produced in the hard subprocess, which could be handled in a similar way to that from the primary emitted gluons [14]. This is expected to be important when the heavy quarks move relativistically, with large relative transverse momentum. However, the photon-gluon fusion cross section favours the kinematic region in which the heavy quarks are produced near threshold and have little phase space for gluon emission. We therefore neglect this component of the final state in the present treatment.

4 One-loop coherent branching

In the conventional treatment corresponding to the leading-order approximation, the following simplifications are made:

Phase space. For $x \rightarrow 0$ the phase space is given by q_t -ordering rather than angular ordering.

In terms of our variables, the ordering becomes

$$q'_{ti} > q'_{t,i-1} , \quad (22)$$

which preserves the angular ordering and coherence structure at large x . At low x this phase space is much smaller than that specified by Eq. (11).

Splitting function. The non-Sudakov form factor is neglected in this case, so that the splitting function is given by

$$P_g^i(z_i, q_{ti}^2, k_{ti}^2) = \frac{\bar{\alpha}_S(q_{ti}^2)}{1 - z_i} + \frac{\bar{\alpha}_S(k_{ti}^2)}{z_i} . \quad (23)$$

As a result of this and the phase space (22), the anomalous dimension γ_N is given by the one-loop expression and thus does not contain the Lipatov resummation of the leading contributions for N near 1.

Hard matrix element. In this approximation one neglects the dependence of the matrix element on the gluon virtuality and uses only the on-shell value. As explained before, in this case one does not include the resummation of the leading $N \rightarrow 1$ contributions to the coefficient function.

Stopping condition. The stopping condition for q'_{tn} in the leading-order QCD cascade is given by the hard subprocess scale μ defined in Eq. (7). As we saw in Fig. 2, this leads to an over-estimate of the cross section, especially at small Q^2 .

5 Results on heavy quark electroproduction

Figs. 3–13 show results from the new Monte Carlo program SMALLX which is based on the ideas outlined above. The results on the left in each Figure are for bottom quark production at HERA, while those on the right are for the hypothetical 1.6 TeV LHC machine. The input gluon structure function, at the starting scale of $\mu_0^2 = 1 \text{ GeV}^2$, is assumed to be

$$F(x, \mu_0^2) = 3(1 - x)^5 , \quad (24)$$

and at this scale a Gaussian distribution of k_t with width μ_0 is assumed.

It should be stressed that the behaviour of the gluon structure function below $x = 10^{-2}$ is completely unknown and therefore the input parametrization (24) is chosen only for its simplicity and for illustrative purposes. Very different forms, such as a $1/\sqrt{x}$ increase at small x , are possible. A comparison of results on b production at HERA for widely different input gluon distributions may be found in Ref. [15]. At present, our ignorance on this point is by far the greatest source of uncertainty about small x processes. Only after the gluon distribution has been determined at HERA will the dynamical details of QCD at small x , such as those considered here, become measurable.

The input structure function (24) is normalized to a gluon momentum fraction of 0.5. Since quarks are neglected in our treatment, the momentum fraction carried by gluons should remain fixed throughout the evolution of the structure function. In fact the approximate gluon splitting function (18) does not precisely satisfy the momentum sum rule, because of

the non-Sudakov form factor and the different arguments of α_S in the two terms. Therefore, as explained in Sect. 3, we renormalize the output structure function, and all the output distributions, by the ratio of the input and output gluon momentum fractions.

A leading-order QCD scale of $\Lambda = 200$ MeV and an infrared cutoff value of $Q_0 = 250$ MeV were used. In the all-loop evolution we find, for this choice of the cutoff, that in about 30% of the events one of the branches generates an exchanged gluon k_i which has a transverse momentum below Q_0 . In this case we reject the branching, while continuing the evolution in q'_t . Recall that there is no singularity for $k_{ti} \rightarrow 0$ at fixed α_S , but the requirement $k_{ti} > Q_0$ is necessary for running α_S . The average number of rejections per event is 0.28 at HERA and 0.38 at LEHC, out of about 9 and 13 initial-state branchings per event respectively, for our values of the cutoff Q_0 and starting scale μ_0 .

Simulations were performed using both the one-loop and all-loop approaches described above. The results are shown in Figs. 3–13 by the crosses and circles, respectively. As explained in Sect. 4, for the one-loop simulations, in addition to using the one-loop evolution algorithm, we replace the off-shell subprocess cross section by its on-shell value multiplied by a step-function.

Because of the complexity of the small- x branching process, the simulation is carried out by generating weighted events. Since the Sudakov form factor (20) involves only the part of the splitting function (18) that is singular at $z = 1$, there is a weighting factor of

$$\int_0^{1-Q_0/q'_t} \tilde{P}_g(z_i, q_{ti}^2, k_{ti}^2) \Big/ \int_0^{1-Q_0/q'_{ti}} \frac{dz}{1-z} \bar{\alpha}_S(q_{ti}^2) \quad (25)$$

for each initial-state branching. Next the heavy quark momenta are generated randomly within the phase space specified by the interacting gluon and photon momenta. Finally the event receives a further weighting factor equal to its hard subprocess cross section. The resulting weight distribution is rather broad and so the Monte Carlo efficiency of the program is low, as may be seen from the substantial fluctuations in some of the Figures, even though they are based on samples of 10^7 events.

We remind the reader also that the simulation neglects quarks and the non-singular part of the gluon splitting function in parton showers, does not generate any showers from the produced heavy quarks, and uses simplified kinematics. These simplifications are adequate for our primary purpose here, which is not to do detailed quantitative phenomenology, but rather to get an overall picture and to compare the results of the all-loop formulation and the one-loop approximation.

The curves at HERA energy in Figs. 3, 9, 12 and 13 show the predictions of Ellis and Kunszt [17], based on the on-shell Born diagram (γg) or on the Born contribution plus gluon bremsstrahlung ($\gamma g + \gamma gg$) as indicated. The predictions are not strictly comparable since we use the input gluon distribution (24) at $\mu_0^2 = 1$ GeV² and neglect quarks, whereas they used the structure functions of Eichten et al. (EHLQ set 1 [18]), which start at $\mu_0^2 = 5$ GeV² and take quarks into account in the (one-loop) evolution equations. However the EHLQ set 1 gluon structure function is similar to ours, if somewhat flatter at $\mu^2 = 5$ GeV², and quarks are not very important, so our one-loop results are roughly similar to those of Ellis and Kunszt where they can be compared.

We now describe our results in more detail in order of increasing exclusiveness, as follows.

1) *Luminosity*. This quantity, plotted in Fig. 3, depends only on the initial state branching, i.e. the structure function. As already discussed in Ref. [12], at this highly inclusive level the all-loop and the conventional one-loop branching give similar results, in spite of the large differences between the two approaches. This is partially due to the fact that in the expansion (1) of the anomalous dimension, there are no corrections to the one-loop expression until the order α_S^4 . As a result the steeper behaviour of the structure function is seen only for very low x . Moreover it has been pointed [16] that the steeper behaviour for α_S running is even more asymptotic than for fixed α_S . This is due to the presence of the cutoff in the exchanged transverse momenta $k_{ti} > Q_0$. Although this condition is asymptotically negligible, it has some effect in reducing the evolution of the branching in the first steps. As a result the distribution at small x is somewhat reduced (see [16] for a detailed discussion).

2) *Cross sections*. In the phase space region for $b\bar{b}$ production, $\sqrt{s_L} > 10$ GeV, the luminosities produced by one-loop and all-loop branching are similar. On the other hand, we saw in Fig. 2 that the one-loop approximation with the scale choice (7) tends to overestimate the subprocess cross section. Therefore the overall cross section predictions are lower in the all-loop case, even at LHC energy, as summarized in Table 1. The reduction of the cross section becomes less at higher energies, owing to the enhanced luminosity at smaller x and the tail of the subprocess cross section at high k_t .

$\sqrt{s}$ (GeV)	1-loop	All-loop
314	5.6	3.3
1600	41	31

Table 1: Cross sections (nb) for $b\bar{b}$ electroproduction.

Owing to the dynamical simplifications we have made, as discussed above, these cross sections should only be taken as an indication of the probable trend of the all-loop versus one-loop results. The one-loop values are however broadly consistent with those obtained from more complete one-loop calculations [17].

3) *Interacting gluon distributions*. In Figs. 4 and 5 we report the x - and k_t -distributions of the last spacelike gluon generated by the branching, corresponding to gluon k_n in Fig. 1.

For the x distribution the two branching algorithms give similar results, apart from the difference in normalization discussed above. There is the expected shift to slightly lower values of x in the all-loop case, as a consequence of the fact that the structure function is slightly steeper at small x than in the one-loop approximation.

The k_t -distributions in Fig. 5 show clearly several of the main results of our investigation. The predictions of the two algorithms start at similar values at small k_t , then the all-loop value falls below the one-loop approximation, until the latter is cut off rather sharply above 10 GeV, corresponding to the limit (7) on the virtuality. By the time this cutoff is reached, the cross section has fallen by more than one order of magnitude, and so the larger value of

the on-shell cross section at smaller k_t is more important than the high- k_t tail of the off-shell cross section at these energies.

4) *Final-state gluon distributions.* Some significant differences also occur in the associated multiplicity of produced gluons, Figs. 6 and 7. These differences arise from the additional phase space available for primary gluon emission in the all-loop evolution: the region of disordered transverse momenta is forbidden in one-loop evolution, while in the new treatment it is allowed, although suppressed at very small momentum fractions by the non-Sudakov form factor. Therefore the number of emitted gluons is enhanced, especially at small x and large angles, i.e. in the low-rapidity region (Fig. 7).

Studies of jet fragmentation in e^+e^- annihilation suggest that, for the value of the infrared cutoff Q_0 and the QCD scale we use, approximately one charged particle is produced for each gluon in the partonic final state. Therefore, in the spirit of local parton-hadron duality [19, 20], we may roughly interpret the distributions of produced gluons shown in Figs. 6 and 7 as being those of charged hadrons.

One quantity that is not sensitive to the cutoff Q_0 is the transverse energy carried off by the emitted gluons, shown in Fig. 8. We see that this is also enhanced at smaller rapidities, for the same reasons as the gluon multiplicity.

5) *Heavy quark distributions.* Apart from the normalization differences given in Table 1, the distributions of the produced b and $\bar{b}$ quarks, Figs. 9–13, do not show striking differences between the two algorithms, even at LHC energy. The two heavy quarks are usually produced at low relative momentum and therefore tend to be strongly correlated in rapidity (Fig. 10). The preferred azimuthal configuration remains back-to-back, although there is a significant probability of finding both quark and antiquark on the same side, when the fusing photon and gluon have significant transverse momentum. The enhanced all-loop cross section at high k_t is reflected in the resultant transverse momentum distribution of the heavy quark pair (Fig. 13).

The rather narrow rapidity and transverse momentum distributions of the heavy quarks, compared with the predictions of Ellis and Kunszt [17], may arise from our steeper input gluon x distribution, as discussed above.

6 Comments and conclusions

The similarity of the inclusive distributions generated by one-loop and all-loop evolution is mostly due to a cancellation of leading higher-order corrections in inclusive observables, reflected in the absence of leading singularities of the anomalous dimension (1) in second and third order. In less inclusive quantities such as the associated multiplicity and distributions of emitted gluons (Figs. 6 and 7), there are differences between the results of the two evolution algorithms, which are explicable in terms of the two new dynamical features of the all-loop algorithm, namely the enlarged phase space due to angular ordering and the screening of the $1/z$ singularity in the gluon splitting function by the non-Sudakov form factor. At present, these differences are small compared with uncertainties due to our lack of knowledge of the input gluon distribution. This underlines the importance of determining the gluon structure

function experimentally down to the lowest possible values of x .

The effects of the leading singularities of the coefficient function, represented by the off-shell photon-gluon subprocess cross section (3), are visible in the transverse momentum distribution of the interacting gluon (Fig. 5). However, the asymptotically dominant effect of the high- k_t tail is masked, even for bottom production at LHC energy, by the lower value of the off-shell cross section at $k_t \lesssim 2M$. For lower quark masses the masking is smaller and an enhancement of the charm cross section is indicated at LHC energy. Although the dynamical simplifications prevent firm predictions of the resulting cross sections, these results are confirmed qualitatively by recent analytical studies of sub-asymptotic effects [21].

The program SMALLX used to generate the Monte Carlo results is available from the authors in a version that outputs parton-level event data in the standard LEP format (the HEPEVT common block). However, it should be emphasised again that the program is not an event generator: it produces weighted events only and the weight distribution is broad. It also makes numerous kinematical and dynamical simplifications, as detailed above. Therefore it is not suitable for detailed quantitative phenomenology, but is intended rather as a theoretical tool to permit a first look at exclusive small- x phenomena.

Acknowledgements

We have benefited greatly from discussions with S. Catani, M. Ciafaloni, F. Fiorani, E.M. Levin, A.H. Mueller and M.G. Ryskin.

References

- [1] L.N. Lipatov, *Yad. Fiz.* 23 (1976) 642 [*Sov. J. Nucl. Phys.* 23 (1976) 338]; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Zh. Eksp. Teor. Fiz.* 72 (1977) 373 [*Sov. Phys. JETP* 45 (1977) 199]; Y.Y. Balitskii and L.N. Lipatov, *Yad. Fiz.* 28 (1978) 1597 [*Sov. J. Nucl. Phys.* 28 (1978) 822]; J. Bartels, *Nucl. Phys.* B151 (1979) 293; T. Jaroszewicz, *Acta Phys. Pol.* B11 (1980) 965; *Phys. Lett.* 116B (1982) 291.
- [2] L.V. Gribov, E.M. Levin and M.G. Ryskin, *Phys. Rep.* 100 (1983) 1; E.M. Levin and M.G. Ryskin, in *Proc. DESY Topical Meeting on the Small- x Behaviour of Deep Inelastic Structure Functions in QCD*, ed. A. Ali and J. Bartels [*Nucl. Phys. B (Proc. Suppl.)* 18C (1990) 92].
- [3] M. Ciafaloni, *Nucl. Phys.* B296 (1987) 249.
- [4] S. Catani, F. Fiorani and G. Marchesini, *Phys. Lett.* 234B (1990) 339; *Nucl. Phys.* B336 (1990) 18; S. Catani, F. Fiorani, G. Marchesini and G. Oriani, *Nucl. Phys.* B361 (1991) 645.
- [5] G. Curci, W. Furmanski and R. Petronzio, *Nucl. Phys.* B175 (1980) 27; W. Furmanski and R. Petronzio, *Zeit. Phys.* C11 (1982) 293; J. Kalinowski, K. Konishi, P.N. Scharbach and T.R. Taylor, *Nucl. Phys.* B181 (1981) 253; E.G. Floratos, C. Kounnas and R. Lacaze, *Phys. Lett.* 98B (1981) 89; I. Antoniadis and E.G. Floratos, *Nucl. Phys.* B191 (1981) 217.

- [6] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. 242B (1990) 97; in *Proc. DESY Topical Meeting on the Small- x Behaviour of Deep Inelastic Structure Functions in QCD*, ed. A. Ali and J. Bartels [Nucl. Phys. B (Proc. Suppl.) 18C (1990) 220]; Nucl. Phys. B366 (1991) 135.
- [7] J.C. Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3.
- [8] E.M. Levin, M.G. Ryskin, Yu.M. Shabelski and A.G. Shuraev, DESY preprints 91-054, 91-065.
- [9] A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rep. 100 (1983) 201; Yu.L. Dokshitzer, V.A. Khoze, S.I. Troyan and A.H. Mueller, Rev. Mod. Phys. 60 (1988) 373.
- [10] G. Marchesini and B.R. Webber, Nucl. Phys. B238 (1984) 1; B.R. Webber, Nucl. Phys. B238 (1984) 492.
- [11] G. Marchesini, Parma preprint to be published in *QCD at 200 TeV*, Proc. 17th INFN Eloisatron Project Workshop, Erice, Italy, June 1991.
- [12] G. Marchesini and B.R. Webber, Nucl. Phys. B349 (1991) 617.
- [13] D. Amati, A. Bassetto, M. Ciafaloni and G. Veneziano, Nucl. Phys. B173 (1980) 429.
- [14] G. Marchesini and B.R. Webber, Nucl. Phys. B310 (1988) 461.
- [15] B.R. Webber, Cambridge preprint Cavendish-HEP-91/9, to be published in *QCD at 200 TeV*, Proc. 17th INFN Eloisatron Project Workshop, Erice, Italy, June 1991.
- [16] E.M. Levin, G. Marchesini, M.G. Ryskin and B.R. Webber, Nucl. Phys. B357 (1991) 167.
- [17] R.K. Ellis and Z. Kunszt, Nucl. Phys. B303 (1988) 653.
- [18] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. 56 (1984) 579; *ibid.* 58 (1986) 1065.
- [19] D. Amati and G. Veneziano, Phys. Lett. 83B (1979) 87.
- [20] Yu.L. Dokshitzer and S.I. Troyan, Leningrad Nuclear Physics Institute preprint N922 (1984); Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, Phys. Lett. 165B (1985) 147; Zeit. Phys. C27 (1985) 65.
- [21] S. Catani, M. Ciafaloni and F. Hautmann, CERN preprint TH.6398 (1992), to be published in *Proc. Workshop on Physics at HERA*, Hamburg, October 1991.

Figure 4: Momentum fraction distribution of the interacting gluon.

Figure 5: Transverse momentum distribution of the interacting gluon.

Figure 6: Multiplicity distribution of gluons from initial-state radiation.

Figure 7: Rapidity distribution of gluons from initial-state radiation.

Figure 8: Rapidity distribution of transverse energy from initial-state radiation.

Figure 9: Rapidity distribution of produced b and $\bar{b}$ quarks.

Figure 10: Rapidity difference between produced b and $\bar{b}$ quarks.

Figure 11: Difference in azimuth between produced b and $\bar{b}$ quarks.

Figure 12: Transverse momentum distribution of produced b and $\bar{b}$ quarks.

Figure 13: Transverse momentum distribution of produced $b\bar{b}$ pairs.