Three Jet plus Photon Events as a Reference Process to Study the Gluon Self Interaction

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Abstract

By making use of the similarity of photon and gluon emission, a method is proposed to isolate the gluon self coupling contribution, which is rather independent of model assumptions. From a comparison of $e^+e^-$ annihilations into four jets and 3 jets and a photon, both the strength and the structure of the triple gluon vertex can be measured. Several systematic uncertainties hampering the corresponding analysis of four jet production alone are reduced.

1 Introduction

Experimental tests of the Standard Model have found convincing agreement with its predictions for both the electroweak and the strong sector [1]. However, they probe almost exclusively the fermion-boson coupling. Another essential ingredient of the Standard Model are the three gauge boson interactions where only little experimental data is available.

The study of the electroweak $ZZ$ and $\gamma\gamma$ couplings is in an early stage. A significant test will only be possible at LEP2 or even a next generation linear colliders [2]. Eventually the existence, strength and structure of the electroweak gauge boson interaction will be measured to very high precision [3]. This situation can be contrasted with the three gluon interactions of the strong sector: by comparing the QCD predictions with models having an Abelian strong interaction, plenty of evidence has been collected to support its existence [4, 5, 6]. On the other hand, testing its structure with an accuracy that is only vaguely similar to the one expected for the electroweak counterpart is difficult with both theoretical and experimental uncertainties limiting the potential precision. However, instead of lacking the gluon self coupling completely, as assumed in Abelian models, strong interactions may be only slightly different from standard model expectations. E.g. in analogy to models for the electroweak three boson coupling modifications have been suggested for strong interactions [7]. To be sensitive to such changes a precise measurement is needed. Given the complications of such an analysis it is important to study the gluon self coupling with several methods. In this paper we suggest a potential new experimental approach which allows one to disentangle the three gluon couplings with little dependence on models.

2 The Triple Gluon Vertex in $e^+e^-$ Collisions

One of the most clear experimental tests of the triple gluon vertex (TGV) has been performed in $e^+e^-$ interactions on the $Z^0$ resonance [5, 6] where the angular structure of the gluon self interaction can be inferred from the production of four jets.

Four jets in $e^+e^-$ interactions are due to the processes depicted in fig. 1. The four quark final state (1c) contributes only $5\%-6\%$ of the four jet rate [8], the double bremsstrahlung and TGV contributions interfere. Within QCD the latter dominates due to the larger colour charge of the gluon. Whereas the properties of the four quark final state are quite distinct, the double bremsstrahlung contribution
and the one from the triple gluon vertex are similar. This makes it difficult to disentangle the two dominant contributions.

The precision of these studies is also limited by the absence of exact calculations of higher than $\alpha_s^2$ contributions. To estimate those one usually resorts to QCD shower models which provide an approximation of higher order corrections. These models are not unambiguous and they predict properties of the four jet topology that differ from each other. Given these complications it would be desirable to compare the experimental four jet distributions to data which have the four jet structure without the triple gluon vertex. Such data are (almost) provided by events with three jets and a photon. Those can only be produced by the diagram 1d which is, apart from the coupling strength, identical to 1b with just one gluon replaced by a photon.

Given a variable $x$ to express the structure of the four jet and three jet plus photon topology, like the Bengtsson-Zerwas angle (see below) the respective cross sections are given for a certain maximum scaled parton mass $y_c = M_j^2/E^2_{cm}$ in $O(\alpha_s^2)$ and $O(\alpha_s\alpha)$ respectively

\begin{equation}
\sigma_{4jet}(y_c, x) = A(\frac{\alpha_s}{2\pi})^2 \left[ C_F \frac{N_f}{2} H_{q\bar{q}q\bar{q}}(y_c, x) + C_F N_c H_{TGV}(y_c, x) + C_F^2 H_{DB}(y_c, x) \right] \tag{1}
\end{equation}

\begin{equation}
\sigma_{3jet+\gamma}(y_c, x) = A \cdot A' \left( \frac{\alpha_s}{2\pi} \right)^2 C_F^2 H_{DB}(y_c, x) \tag{2}
\end{equation}

where $A, A'$ are given by the electroweak processes, $\alpha_s$ and $\alpha$ are the strong and electroweak coupling strengths, $C_F$ the QCD colour factor ($C_F = 4/3$), $N_f$ the number of flavours and $N_c$ the number of colours. The functions $H_{q\bar{q}q\bar{q}}, H_{TGV}, H_{DB}$, which are calculated, denote the contributions from the four quark final state, the triple gluon vertex and the double bremsstrahlung diagrams. $H_{TGV}$ also includes the interference of the latter two. The two cross sections can be combined, e.g. by calculating the ratio

\begin{equation}
\frac{\sigma_{4jet} - \frac{1}{A} \sigma_{3jet+\gamma}}{\sigma_{4jet}} = \frac{C_F \frac{N_f}{2} H_{q\bar{q}q\bar{q}}(y_c, x) + C_F N_c H_{TGV}(y_c, x)}{\sigma_{4jet}} \sim \frac{\sigma_{TGV}}{\sigma_{4jet}} \tag{3}
\end{equation}

The latter approximation uses the fact that the four quark contribution $H_{q\bar{q}q\bar{q}}$ is small and is perhaps even measurable [9]. To simplify the expression we in addition assumed the $\alpha_s$ values in equations (1)
Table 1: Expected numbers of three jet plus photon events as a function of $y_c$. The numbers are extrapolations from OPAL data and include experimental cuts.

<table>
<thead>
<tr>
<th>$y_c$</th>
<th>LEP PhaseI $5$ Mio $Z^0$'s</th>
<th>High Lumi LEP $100$ Mio $Z^0$'s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>4000</td>
<td>80,000</td>
</tr>
<tr>
<td>0.02</td>
<td>2200</td>
<td>44,000</td>
</tr>
<tr>
<td>0.04</td>
<td>500</td>
<td>10,000</td>
</tr>
</tbody>
</table>

and (2) to be the same. This assumption can be easily relaxed. Thus, by comparing the two processes the triple gluon contribution can be isolated.

However, there is no such thing as a free lunch: in addition to the missing four quark contribution that we will assume to be known, the colour flow, and thus the hadronisation correction is different in the two processes and the rate of three jets plus photon events is low. These two latter problems will be addressed now.

3 A Model of the Measurement

Events with final state photons are not very abundant. The yield is further reduced by realistic selection criteria to obtain a good signal to background ratio. Current experiments impose severe cuts on the photon energy and the isolation with respect to other particles (see A.Firestone in these proceedings). An extrapolation from the measurement of OPAL [10] based on about 150,000 hadronic $Z^0$ decays leads to the expected number of events listed in table 1. With a better understanding of the photon identification and the background rejection it may be possible to increase the number of events with final state photons by a factor of two.

To become less sensitive to higher order corrections in $\alpha_s$, e.g. due to events with more jets, a large $y_c$ is preferable. However, this has to be balanced against the required statistics suggesting a $y_c=0.02$ for the first phase of LEP. This will be the sample used in the following discussion. Obviously, if LEP runs in the high luminosity option [11], the analysis can be performed at higher $y_c$.

Given these numbers we can estimate the accuracy with which the gluon self coupling contribution to the cross section can be measured. To do this we use equation (3) modified to allow for different coupling strengths of the various contributions. Such a measurement is a test of the gauge structure of strong interactions which implies the coupling strengths at the $qgq$ and $ggg$ vertices to be identical (e.g. [12]). It is analogous to the determination of $q\mu W$ in the electroweak sector. Assuming 2,000 three jet plus photon events, the statistical and systematic uncertainty will be about 3%. The value of $\alpha_s$ for this comparison is fixed by the ratio of three jet over two jet plus photon events which has a statistical error of 2%. A slight complication arises from the fact that the $\alpha_s$ as determined from this ratio includes corrections of $O(\alpha_s^2)$ which themsevelf depend on the gluon self coupling. Since these are unambiguously predicted within QCD they can be taken into account and thus a comparison of four jet and three jet plus photon production allows therefore a direct test of the equality of the strengths of the $ggg$ and $qgq$ vertices.

For more detailed tests of the three gluon vertex the angular structure of the four jets can be scrutinised. Several variables have been proposed which disentangle mainly the four quark final state and the triple gluon vertex/double bremsstrahlung contributions [13], the angle between the gluon jets [6] has been found to be particularly sensitive to differences in the contributions between gluon self interaction and double bremsstrahlung.

Since it is experimentally difficult to identify the gluon jets, the four jets are ordered in energy $E_1 > E_2 > E_3 > E_4$ and the two lowest energy jets (3,4) are treated as gluon jets. Similarly we
will treat the photon as a jet and apply the same prescription. This procedure reduces somewhat the discrimination power of these variables but allows one to exploit the full statistics. This ordering can in principle be relaxed for the three jet plus photon case since the photon 'replaces' a gluon jet. For a comparison to the four jet case this is only meaningful if a similar constraint can be introduced. As discussed in [8] this can be achieved at the expense of losing 98% of the events by identifying a quark jet via a heavy quark decay. We will not discuss this option.

In the following we will consider the Bengtsson-Zerwas angle $\chi_{BZ}$, the angle between the planes given by the two highest energy jets (1,2) and the two lowest energy jets (3,4), and the angle $\alpha_{34}$ between jets 3 and 4.

$$\chi_{BZ} = \cos \left[ \frac{\vec{p}_1 \times \vec{p}_2 \cdot (\vec{p}_3 \times \vec{p}_4)}{|\vec{p}_1 \times \vec{p}_2| \cdot |\vec{p}_3 \times \vec{p}_4|} \right]$$

$$\cos \alpha_{34} = \frac{\vec{p}_3 \cdot \vec{p}_4}{|\vec{p}_3| \cdot |\vec{p}_4|}$$

Since for the measurements only photons above a minimum energy are selected, we require each jet to have $x = 2E/E_{cm} > 0.15$. The distributions of four jet events and three jet plus photon events in these variables will now be discussed. We start with the parton level using an $\alpha_s^2$ [14] and $\alpha_s$ matrix calculation. We will then compare these distributions with the predictions of the QCD models JETSET [15] and ARIADNE [16] which provide an estimate of higher order and hadronisation corrections. Finally we will discuss the effects of cuts on the photon isolation.

The expectations for the (normalised) distributions in the angle $\chi_{BZ}$ and $\alpha_{34}$ are shown in figs. 2a,b. Whereas the distributions of the four jets and the 3-jets plus photon events are almost identical in $\chi_{BZ}$, they differ in $\cos \alpha_{34}$: the 'gluons' from the three gluon vertex are closer together than those from double bremsstrahlung.

These ideal distributions are distorted by effects of higher order in QCD and hadronisation. To estimate their importance, QCD shower models are employed. With respect to the comparison of three jet plus photon and four jet events the interesting question is, whether or not the corrections
for the two processes are similar and thus cancel out. In fig. 3a-d we display the ratio between the ARIADNE and JETSET expectations and the matrix element calculation in $\chi^{R2}$ and $\alpha_{34}$ for the four jet production and the three jet plus photon events. This gives an estimate of the distortions due to higher order corrections. In both cases the QCD shower models are significantly different from the matrix element calculation. However, according to both models, these higher order contributions affect the four jet and three jet plus photon distributions in a similar way. This suggests that in comparing these processes significant parts of these corrections cancel out.

Finally we address the biases introduced by the experimental requirements. They are hardly a problem for the four jet case but since the photon is required to be isolated, they may distort the three jet plus photon events. Similar to the analysis of [10] we required no particle to be within a cone of half opening angle of 15 degrees around the photon candidate. We point out that such a requirement may be relaxed in the future. As shown in fig.4 these experimental corrections are rather uniform and thus do not introduce a major problem.

4 Conclusion

Three jet plus photon events are a generic process to study the strength and the structure of a double bremsstrahlung process. They are thus a reference process that can be compared to the four jet events of QCD allowing one to isolate the contributions due to the gluon self interaction both in absolute yield and in shape. The attractive feature of this method is its relative freedom from any model calculations. This is supported by the studies of higher order and hadronisation effects using the QCD shower models JETSET and ARIADNE. As a result, a measurement like that indicated by the error bars in figure 2 may either provide a confirmation of the gauge structure with increased precision, or may point to deviations as evidence for new physics.

5 Acknowledgement

Useful comments from N.Watson and W.Zeuner are gratefully acknowledged.

References

Figure 3: Ratio of the predictions of JETSET and ARIADNE over the matrix element expectation or the Bengtsson - Zerwas angle $\chi_{BZ}$ (a,c) and $\cos(\alpha_{34})$ (b,d).

The ratios are shown for four jet events (a,b) and three jet plus photon events (c,d).
Figure 4: Experimental efficiencies for the process $e^+e^- \rightarrow 3$ jets plus photons after applying cuts on the polar angle ($|\cos\theta| < 0.72$), on the photon and jet energies ($x > 0.15$) and on the isolation (15 degree cone). (a) the Bengtsson - Zerwas angle $\chi_{BZ}$, (b) $\cos(\alpha_{34})$.


[10] OPAL collaboration, P. Acton et al., CERN-PPE/91-189


