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**THE SIMULTANEOUS EVOLUTION OF MASSES AND
COUPLINGS: CONSEQUENCES ON SUPERSYMMETRY
SPECTRA AND THRESHOLDS**

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Abstract

We use the Renormalization Group Equations to work out, at the one-loop level, the simultaneous evolution of all masses and couplings and show explicitly the self-consistency of the whole scheme. A thorough examination is performed of the light Supersymmetry threshold in the Minimal Supersymmetric extension of the Standard Model (MSSM). All fundamental quantities α_{GUT} , M_{GUT} , $\alpha_3(m_Z)$, $\alpha_2(m_Z)$, $\alpha_1(m_Z)$ [consequently $\sin^2\theta(m_Z)$] are given in terms of the detailed spectrum of all particle and sparticle thresholds. Examples of Supersymmetry spectra are given as function of $\alpha_3(m_Z)$ and of the other essential parameters. The results of this study, where the evolution of masses is extended to all possible masses, confirm our previous conclusions on the EGM effect for the Supersymmetry threshold lower bound. Examples of the predictive power of our method are given.

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1. Introduction

This work is in the line of our previous study [1] on the light threshold for Supersymmetry breaking [2], where a range for the primordial parameter $m_{1/2}$ was derived by comparing the experimental value $\sin^2\theta(m_Z)_{\text{exp}}$ with $\sin^2\theta(m_Z)_{\text{th}}$, i.e. the two-loop-corrected minimal SUSY-SU(5) theoretical prediction, without threshold effects. The difference between these two quantities was then accounted for by the light threshold contribution, thus establishing, by means of the light spectrum, the range of energy where $m_{1/2}$ should lie.

In our previous work [1] we introduced the EGM effect, i.e. the evolution of the gaugino masses, in the light threshold study for SUSY breaking. Here we extend the light threshold effects to all unification parameters like M_{GUT} and α_{GUT} and obtain entirely analytic solutions for the one-loop evolution equations of the gauge couplings. The two-loop analytical formulation of threshold effects is being completed with a single discontinuity in the two-loop β -function coefficients b_{ij} at $m_{1/2}$ [3].

The structure of this paper is as follows. In section 2 we give the analytic formulae for M_{GUT} and α_{GUT} with the detailed contribution of the light threshold. Then the evolution equations for $\alpha_1, \alpha_2, \alpha_3$ down to the Z-mass value (m_Z) are given, starting from M_{GUT} , with due account of the complete light SUSY spectrum. Consistency checks of the whole formalism are given.

In section 3 we indicate the level of approximation used when dealing with the evolution of the various parameters. We follow the proposal of Ellis et al. [2], for the mass degeneracy of all squarks except stops: $\tilde{t}_R, \tilde{t}_L$. The main points where we depart from [2] are: i) to account for the evolutions of the gaugino masses, as already done in [1], and to extend these evolutions to all masses; ii) to relax the fixed input values for both the higgsino and the Higgs masses, $m_{\tilde{H}}$ and m_H , respectively; these are let to evolve according to known equations [4]; furthermore we transfer the inputs for $m_{\tilde{H}}$ and m_H into more general primordial parameter ratios.

In section 4 we summarize the results obtained by our group about the SUSY threshold problem and discuss the trends which emerge from our studies. In section 5 we present the conclusions.

2. Formulae for α_{GUT} , M_{GUT} and the simultaneous evolutions for couplings and masses

We consider a spectrum where all Standard Model particles are included with the addition of a second Higgs doublet (H) and the Supersymmetric partners of the Standard particles. This spectrum, in spite of its complexity, is somewhat simplified as we treat all squarks in the same way, with an exception for the left stop and the right stop. The sleptons, left and right, are obviously included. The simplification in the squark and slepton spectra follows Ellis et al. [2], who introduced this treatment in computing the threshold contribution to $\sin^2\theta(m_Z)$. The estimate of this contribution by Ellis et al. [2] is confirmed in our work for the whole range of evolution of the squark and slepton masses.

Since the squarks and sleptons contribute to $\sin^2\theta(m_Z)$ by less than 5×10^{-5} , to neglect them in our previous study [1] when computing the EGM effect on the SUSY lower bound was obviously justified. Moreover this study [1], based on a single discontinuity threshold, is confirmed by our present exhaustive iterative method involving all evolutions of couplings and masses. Our previous approximation is therefore numerically confirmed.

Let us remind the reader that the key point of our work is to derive a range for the primordial SUSY parameters by comparing [2] the experimental value $\sin^2\theta(m_Z)_{\text{exp}}$ with the two-loop-theoretical calculation based on the minimal SUSY-SU(5), without threshold effects:

$$\sin^2\theta(m_Z)_{\text{th}} = 0.2 + \frac{7\alpha_{\text{em}}^{\text{exp}}(m_Z)}{15 \alpha_3^{\text{exp}}(m_Z)} + 0.0029.$$

This is the starting point for all predictions on where the SUSY spectrum could be. These predictions, started by Ellis et al. [2], were improved by the introduction of the EGM effect [1], and are now worked out through a numerical method which automatically adjusts the whole light SUSY spectrum in such a way that the light threshold contributions, ΔT_L , to $\sin^2\theta(m_Z)$ make the theoretical prediction to coincide exactly with the experimental value. Notice that the present method would be straightforwardly modified if a light sparticle were found below the Z mass.

The solutions of the evolution equations for α_{GUT} and M_{GUT} , modified by the threshold effects, together with all couplings and masses

involved, have been worked out at the one-loop level with the appropriate spectrum of thresholds.

The detailed formulae grouped into "couplings" and "masses" are as follows:

COUPLINGS

$$\begin{aligned}
\frac{1}{\alpha_{\text{GUT}}} = & \frac{3}{20} \left(\frac{1}{\alpha_{\text{em}}^{\text{exp}(m_Z)}} + \frac{4}{\alpha_3^{\text{exp}(m_Z)}} \right) + \\
& + \frac{1}{2\pi} \left[\frac{6}{5} \log\left(\frac{m_{\tilde{g}}}{m_Z}\right) + \frac{65}{48} \log\left(\frac{m_{\tilde{q}}}{m_Z}\right) + \frac{43}{240} \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + \right. \\
& + \frac{1}{6} \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) + \frac{3}{20} \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + \frac{3}{20} \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) + \\
& + \frac{1}{5} \log\left(\frac{m_{\tilde{H}}}{m_Z}\right) + \frac{1}{20} \log\left(\frac{m_H}{m_Z}\right) + \frac{1}{5} \log\left(\frac{m_{\tilde{W}}}{m_Z}\right) + \\
& \left. + \frac{83}{120} \log\left(\frac{m_t}{m_Z}\right) \right] \quad (1)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\alpha_1(\mu)} = & \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[\frac{33}{5} \log\left(\frac{M_{\text{GUT}}}{\mu}\right)^* - \frac{49}{60} \log\left(\frac{m_{\tilde{q}}}{\mu}\right)^* - \right. \\
& - \frac{17}{30} \log\left(\frac{m_t}{\mu}\right)^* - \frac{1}{60} \log\left(\frac{m_{\tilde{t}_L}}{\mu}\right)^* - \\
& - \frac{4}{15} \log\left(\frac{m_{\tilde{t}_R}}{\mu}\right)^* - \frac{3}{10} \log\left(\frac{m_{\tilde{t}_L}}{\mu}\right)^* - \\
& - \frac{3}{5} \log\left(\frac{m_{\tilde{t}_R}}{\mu}\right)^* - \frac{2}{5} \log\left(\frac{m_{\tilde{H}}}{\mu}\right)^* - \\
& \left. - \frac{1}{10} \log\left(\frac{m_H}{\mu}\right)^* \right] \quad (2)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\alpha_2(\mu)} = & \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[\log\left(\frac{M_{\text{GUT}}}{\mu}\right)^* - \log\left(\frac{m_{\tilde{q}}}{\mu}\right)^* - \right. \\
& - \log\left(\frac{m_t}{\mu}\right)^* - \frac{1}{2} \log\left(\frac{m_{\tilde{t}_L}}{\mu}\right)^* - \\
& - \frac{1}{2} \log\left(\frac{m_{\tilde{t}_L}}{\mu}\right)^* - \frac{2}{3} \log\left(\frac{m_{\tilde{H}}}{\mu}\right)^* - \\
& \left. - \frac{1}{6} \log\left(\frac{m_H}{\mu}\right)^* - \frac{4}{3} \log\left(\frac{m_{\tilde{W}}}{\mu}\right)^* \right] \quad (3)
\end{aligned}$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[-3 \log\left(\frac{M_{\text{GUT}}}{\mu}\right)^* - 2 \log\left(\frac{m_{\tilde{g}}}{\mu}\right)^* - \right. \\ \left. - \frac{2}{3} \log\left(\frac{m_{\tilde{q}}}{\mu}\right)^* - \frac{5}{3} \log\left(\frac{m_{\tilde{u}_L}}{\mu}\right)^* - \right. \\ \left. - \frac{1}{6} \log\left(\frac{m_{\tilde{t}_L}}{\mu}\right)^* - \frac{1}{6} \log\left(\frac{m_{\tilde{t}_R}}{\mu}\right)^* \right] \quad (4)$$

where: $\log(x)^* \equiv \Theta(x-1) \cdot \log(x)$

$$\Theta(y) \equiv \begin{cases} 0 & \text{if } y < 0 \\ 1 & \text{if } y > 0 \end{cases}$$

$$\mu = \sqrt{q^2}.$$

MASSES

$$\frac{M_{\text{GUT}}}{(M_{\text{GUT}})_0} = \left(\frac{m_{\tilde{w}}}{m_Z}\right)^{1/15} \cdot \left(\frac{m_{\tilde{g}}}{m_Z}\right)^{-4/15} \cdot \left(\frac{m_{\tilde{h}}}{m_Z}\right)^{1/15} \cdot \left(\frac{m_H}{m_Z}\right)^{1/60} \cdot \\ \cdot \left(\frac{m_{\tilde{q}}}{m_Z}\right)^{-5/48} \cdot \left(\frac{m_{\tilde{t}_L}}{m_Z}\right)^{1/240} \cdot \left(\frac{m_{\tilde{t}_L}}{m_Z}\right)^{1/20} \cdot \\ \cdot \left(\frac{m_{\tilde{t}_R}}{m_Z}\right)^{1/20} \cdot \left(\frac{m_t}{m_Z}\right)^{1/120} \quad (5)$$

$$\log\left(\frac{m_{1/2}}{m_Z}\right) = F(X) + \frac{28}{19} \log\left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{\text{GUT}}}\right) - \frac{32}{19} \log\left(\frac{\alpha_2(m_{\tilde{w}})}{\alpha_{\text{GUT}}}\right) + \\ + \frac{3}{19} \left\{ \frac{5}{2} \log\left(\frac{m_{\tilde{q}}}{m_Z}\right) - \frac{19}{6} \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + \frac{5}{3} \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) - \right. \\ \left. - 3 \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + 2 \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) - 3 \log\left(\frac{m_t}{m_Z}\right) + \right. \\ \left. + 6 \log\left(\frac{\alpha_2(m_{\tilde{h}})}{\alpha_{\text{GUT}}}\right) + \frac{2}{11} \log\left(\frac{\alpha_1(m_{\tilde{h}})}{\alpha_{\text{GUT}}}\right) - 4 \log\left(\frac{m_4}{m_{1/2}}\right) + \right. \\ \left. + 6r^2 - \frac{1}{2} \log\left[\frac{3}{2} \left[1 - \left(\frac{\alpha_2(m_H)}{\alpha_{\text{GUT}}}\right)^2\right] + \frac{1}{22} \left[1 - \left(\frac{\alpha_1(m_H)}{\alpha_{\text{GUT}}}\right)^2\right] + \right. \right. \\ \left. \left. + \left(\frac{m_0}{m_{1/2}}\right)^2 + \left(\frac{\alpha_{\text{GUT}}}{\alpha_2(m_H)}\right)^3 \cdot \left(\frac{\alpha_{\text{GUT}}}{\alpha_1(m_H)}\right)^{1/11} \cdot \left(\frac{m_4}{m_{1/2}}\right)^2 \right] \right\} \quad (6)$$

where: $F(X) = \frac{3}{19} \left\{ \frac{20\pi}{\alpha_{\text{em}}^{\text{exp}}(m_Z)} (0.2029 + \right. \\ \left. + \frac{7\alpha_{\text{em}}^{\text{exp}}(m_Z)}{15 \alpha_3^{\text{exp}}(m_Z)} - \sin^2\theta(m_Z)_{\text{exp}}) \right\} \quad (7)$

$$\log \left(\frac{(M_{\text{GUT}})_0}{m_Z} \right) = \frac{\pi}{10} \left(\frac{1}{\alpha_{\text{em}}^{\text{exp}}(m_Z)} - \frac{8}{3\alpha_3^{\text{exp}}(m_Z)} \right) \quad (8)$$

$$m_{\tilde{g}} = \frac{\alpha_3(m_{\tilde{g}})}{\alpha_{\text{GUT}}} m_{1/2} \quad (9)$$

$$m_{\tilde{W}} = \frac{\alpha_2(m_{\tilde{W}})}{\alpha_{\text{GUT}}} m_{1/2} \quad (10)$$

$$\left(\frac{m_{\tilde{q}}}{m_{1/2}} \right)^2 = -\frac{8}{9} \left[1 - \left(\frac{\alpha_3(m_{\tilde{q}})}{\alpha_{\text{GUT}}} \right)^2 \right] + \left(\frac{m_0}{m_{1/2}} \right)^2 \quad (11)$$

$$\begin{aligned} \left(\frac{m_{\tilde{t}_L}}{m_{1/2}} \right)^2 = & -\frac{8}{9} \left[1 - \left(\frac{\alpha_3(m_{\tilde{q}})}{\alpha_{\text{GUT}}} \right)^2 \right] + \\ & + \frac{w_1}{6} \left[1 - \left(\frac{\alpha_3(m_{\tilde{q}})}{\alpha_{\text{GUT}}} \right)^2 \right] + \left(\frac{m_0}{m_{1/2}} \right)^2 \end{aligned} \quad (12)$$

where: w_1 = fine structure constant of squarks

$$\begin{aligned} \left(\frac{m_{\tilde{t}_R}}{m_{1/2}} \right)^2 = & -\frac{8}{9} \left[1 - \left(\frac{\alpha_3(m_{\tilde{q}})}{\alpha_{\text{GUT}}} \right)^2 \right] + \\ & + \frac{w_1}{3} \left[1 - \left(\frac{\alpha_3(m_{\tilde{q}})}{\alpha_{\text{GUT}}} \right)^2 \right] + \left(\frac{m_0}{m_{1/2}} \right)^2 \end{aligned} \quad (13)$$

$$\begin{aligned} \left(\frac{m_{\tilde{\tau}_L}}{m_{1/2}} \right)^2 = & \frac{3}{2} \left[1 - \left(\frac{\alpha_2(m_{\tilde{\tau}_L})}{\alpha_{\text{GUT}}} \right)^2 \right] + \\ & + \frac{1}{22} \left[1 - \left(\frac{\alpha_1(m_{\tilde{\tau}_L})}{\alpha_{\text{GUT}}} \right)^2 \right] + \left(\frac{m_0}{m_{1/2}} \right)^2 \end{aligned} \quad (14)$$

$$\left(\frac{m_{\tilde{\tau}_R}}{m_{1/2}} \right)^2 = \frac{2}{11} \left[1 - \left(\frac{\alpha_1(m_{\tilde{\tau}_R})}{\alpha_{\text{GUT}}} \right)^2 \right] + \left(\frac{m_0}{m_{1/2}} \right)^2 \quad (15)$$

$$m_{\tilde{H}} = m_4 \left[\frac{\alpha_{\text{GUT}}}{\alpha_2(m_{\tilde{H}})} \right]^{3/2} \cdot \left[\frac{\alpha_{\text{GUT}}}{\alpha_1(m_{\tilde{H}})} \right]^{1/22} \cdot \exp\left(-\frac{3r^2}{2}\right) \quad (16)$$

$$m_H = \sqrt{m_{\tilde{H}}^2 + m_{\tilde{\tau}_L}^2} \quad (\text{at scale } m_H) \quad (17)$$

where:
$$r^2 = \frac{1}{4\pi} \int_0^T \alpha_t(\mu') d\log\left(\frac{\mu'^2}{\mu^2}\right) \quad (18)$$

$$\alpha_t = \frac{h_t^2}{4\pi} = \text{top-Yukawa coupling}$$

$$T = \log\left(\frac{M_{\text{GUT}}^2}{\mu^2}\right) . \quad (19)$$

Three are the primordial masses, $m_{1/2}$, m_0 and m_4 , whose role, at M_{GUT} , is as follows:

$m_{1/2}$ parametrizes the gauginos;
 m_0 parametrizes squarks and sleptons;
 m_4 parametrizes the Higgsino;
 m_0 and m_4 parametrize the Higgs.

In the course of the evolution, squarks, sleptons, Higgs and Higgsino get additional contributions parametrized also by $m_{1/2}$.

Notice that, in the degenerate case when all $\tilde{m}_i = m_{1/2}$, we find our previous fomulae for M_{GUT} and α_{GUT} [1]:

$$M_{\text{GUT}} = (M_{\text{GUT}})_0 \cdot \left(\frac{m_{1/2}}{m_Z}\right)^{-7/60} \quad (20)$$

$$\frac{1}{\alpha_{\text{GUT}}} = \frac{1}{(\alpha_{\text{GUT}})_0} + \frac{73}{40\pi} \log\left(\frac{m_{1/2}}{m_Z}\right) . \quad (21)$$

A self-consistency check reads as follows: inserting for instance (1) and (5) into (3) for $\mu = m_Z$, we get:

$$\frac{1}{\alpha_2(m_Z)} = \frac{1}{\alpha_{\text{em}}^{\text{exp}}(m_Z)} \left(0.2 + \frac{7\alpha_{\text{em}}^{\text{exp}}(m_Z)}{15 \alpha_3^{\text{exp}}(m_Z)} + \Delta T_L\right) \quad (22)$$

or

$$\frac{\alpha_{\text{em}}^{\text{exp}}(m_Z)}{\alpha_2(m_Z)} = (\sin^2\theta(m_Z))_0 + \Delta T_L \quad (23)$$

with

$$(\sin^2\theta(m_Z))_0 = 0.2 + \frac{7\alpha_{\text{em}}^{\text{exp}}(m_Z)}{15 \alpha_3^{\text{exp}}(m_Z)} \quad (24)$$

and

$$\begin{aligned} \Delta T_L \equiv & \frac{\alpha_{em}^{exp}(m_Z)}{20\pi} \cdot \\ & \cdot \left[\frac{5}{2} \log\left(\frac{m_{\tilde{q}}}{m_Z}\right) - \frac{19}{6} \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + \frac{5}{3} \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) - \right. \\ & - 3 \log\left(\frac{m_{\tilde{t}_L}}{m_Z}\right) + 2 \log\left(\frac{m_{\tilde{t}_R}}{m_Z}\right) - \log\left(\frac{m_H}{m_Z}\right) - \\ & - 4 \log\left(\frac{m_{\tilde{h}}}{m_Z}\right) - 3 \log\left(\frac{m_t}{m_Z}\right) + \frac{28}{3} \log\left(\frac{m_{\tilde{g}}}{m_Z}\right) - \\ & \left. - \frac{32}{3} \log\left(\frac{m_{\tilde{w}}}{m_Z}\right) \right] \end{aligned} \quad (25)$$

which is our formula (10) in [1].

Another complementary test can be obtained inserting (1) and (5) into (2) for $\mu = m_Z$. One gets, with obvious notations:

$$\frac{5}{3} \frac{\alpha_{em}^{exp}(m_Z)}{\alpha_1(m_Z)} = \left(\cos^2 \theta(m_Z) \right)_0 - \Delta T_L. \quad (26)$$

Formulae (2)-(4) and (8)-(26) (or their equivalent) can be found in the literature (in [4] and refs. therein). The others are our responsibility.

3. Primordial ratios and mass spectra

To study the full evolution of all Supersymmetric masses is the main task of this paper. As we have already shown [1], the evolution of the gaugino masses (EGM) has important effects. For example it lowers by more than two orders of magnitude the value of the primordial mass, $m_{1/2}$, predicted without the EGM effect [2]. In contrast with the case of the gauginos, the evolution of squark and slepton families has practically no effect. This was already implicit in [2] and the reason for it was discussed in [1]. These features are by now well known and need not be further discussed. The only relevant members of the squarks to be kept in the evolution equations are the left and right stops: $\tilde{t}_L, \tilde{t}_R$. These squark states have their evolution partly dictated by the top-Yukawa coupling, which is expected to be potentially large. This coupling is the one which drives negative the squared mass of the Higgs, thus producing the electroweak symmetry breaking. If this scenario has to be considered seriously, it may happen that for large primordial m_0 and/or

m_4 , which determine the Higgs mass at M_{GUT} , one needs a large top-Yukawa coupling.

It becomes therefore important to check that, in the case of the stop-squarks, the Yukawa-driven part will not win over the gauge contribution to their evolution. If this were the case, colour breaking would occur because the Yukawa part would make negative the mass squared of the squarks. This aspect of our study to predict the $m_{1/2}$ mass must always remain under control.

Having accounted for the evolutions of gaugino masses [1], the other delicate point is to relax the fixed input values for $m_{\tilde{g}}$ and m_H , and to use new inputs, namely the primordial mass ratios:

$$\frac{m_0}{m_{1/2}} \quad (27)$$

and

$$\frac{m_4}{m_{1/2}} \quad (28)$$

This does not introduce any new additional input, as we know, at any energy, the connection between $m_{\tilde{g}}$ and m_4 , and also between m_H and m_0 , $m_{1/2}$, m_4 . In fact the evolutions of $m_{\tilde{g}}$ and m_H are given in terms of the primordial mass ratios (27) and (28). The range where $m_{1/2}$ varies is very sensitive to m_4 . If the value of $m_{\tilde{g}}$ is taken fixed at 100 GeV, this corresponds to a value of m_4 of the same order of magnitude. It seems to us not very consistent – as some authors do – to keep m_4 fixed at low values even if, as a result of the analysis, $m_{1/2}$ turns out to be very large. However there are no compelling arguments to prevent $m_{\tilde{g}}$ to be small. Often in the literature high values for $m_{\tilde{g}}$ and m_H are limited by arguments based on “naturalness” and “fine-tuning”. We stress that we simply ignore these criteria and let the upper limits open, keeping under control the stop evolutions. Indeed we think that if “naturalness” and/or limits on “fine-tuning” are advocated, one is in the position to postulate the result wanted: namely the scale of $m_{1/2}$ in the low energy range, on the basis of the adopted value for $\alpha_3(m_Z)$. This is equivalent to say that $m_{1/2}$ is wanted to be between 0.1 and 1 TeV, thus any further calculation to predict the range of $m_{1/2}$ becomes unnecessary.

The first seventeen formulae reported in section 2 represent the system of coupled non-linear equations for couplings and masses which

describe in great detail the evolution of particles and sparticles interacting via $SU(3) \times SU(2) \times U(1)$ gauge forces.

In the system of coupled equations (1)-(17) we have neglected the heavy threshold, ΔT_H , i.e. the spectrum associated with the Supergroup breaking into SUSY [$SU(3) \times SU(2) \times U(1)$] at very high masses. This threshold can be described in terms of three basic masses [5] (M_V , M_H , M_Σ). Our method*) allows to treat in detail ΔT_H and its influence on the SUSY spectrum [6].

To the system of coupled equations (1)-(17) we apply our iterative procedure [1] in order to work out the quantitative results wanted. The first of these results is that the EGM effect on the SUSY threshold lower bound [1] is confirmed, i.e. the lower bound on $m_{1/2}$ is in the Z-mass range.

The next result refers to the spectrum of SUSY particles. For example let us focus our attention on the value of the SUSY threshold for a typical sparticle, such as the W-ino. In the following we ignore particle mixing. The first remark is that the W-ino mass is insensitive to the ratio $(m_0/m_{1/2})^2$ as shown in Fig. 1. Over four orders of magnitude for this ratio, from 10^{-2} to 10^2 , the W-ino mass slightly decreases by no more than 20%. On the contrary – as shown in Fig. 2 – the W-ino mass is very sensitive to $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and to the ratio $(m_4/m_{1/2})^2$. The range of variation for the measured quantities $\alpha_3(m_Z)$ and $\sin^2\theta(m_Z)$ has been taken in the $\pm 2\sigma$ interval around their world average (WA) values, while for $(m_4/m_{1/2})^2$ we have computed the W-ino mass letting this ratio vary by four orders of magnitude: from 10^{-2} to 10^2 . In Fig. 2, each curve is computed keeping constant the other variables. The fixed values are indicated by the arrows in Fig. 2. For example the W-ino mass versus $\alpha_3(m_Z)$ is computed keeping $\sin^2\theta(m_Z)$ at its world average value (0.2334) and the ratios $(m_0/m_{1/2})^2 = (m_4/m_{1/2})^2 = 1$. It is interesting to observe that – as shown in Fig. 2 – the W-ino mass decreases with increasing values of $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$. All these results are obtained with $m_t = 125$ GeV. We have also studied how the W-ino mass varies with m_t in the range ($90 \leq$

*) Using a simple treatment of the heavy threshold (only one heavy mass) we have studied the α_1 , α_2 , α_3 evolutions [7] in order to have a smooth convergence towards M_{GUT} and a unified evolution of the couplings above M_{GUT} . In this evolution [7] no light threshold effects were included.

$m_t \leq 180$) GeV, once all other parameters are kept fixed as indicated in Fig. 2a.

We have then computed the detailed behaviour of the gluino, squark and slepton masses.

For the gluino mass we find again that it is insensitive to the ratio $(m_0/m_{1/2})^2$ (range studied: from 10^{-2} to 10^2) while it decreases (as for the W-ino case) with increasing values of $\alpha_3(M_Z)$, $\sin^2\theta(M_Z)$ and $(m_4/m_{1/2})^2$. The results are reported in Fig. 3.

The squark mass is found to be insensitive to the ratio $(m_0/m_{1/2})^2$ in the range from 10^{-2} to 10^0 . From 10^0 to 10^2 it increases by less than an order of magnitude. The squark mass is very sensitive to $\alpha_3(M_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$: it decreases with increasing values of $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$. The results are shown in Fig. 4.

The slepton (left and right) masses are more sensitive than the squark one to the ratio $(m_0/m_{1/2})^2$ for the obvious reason that they carry no colour. The slepton masses increase when the ratio $(m_0/m_{1/2})^2$ increases: in the range from 10^{-2} to 10^2 , by more than two orders of magnitude, as shown in Fig. 5. On the other hand the trend versus $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$ is like the W-ino, gluino and squark cases: the slepton (left and right) masses decrease with increasing values of $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$, as shown in Fig. 6. This is the effect of the coupled evolutions.

These detailed studies are the basis for our overall analysis of the SUSY-mass spectra shown in Fig. 7, where for the primordial masses the value $(m_0/m_{1/2})^2 = (m_4/m_{1/2})^2 = 1$ has been chosen. The spectra have been worked out, taking for the experimentally measured quantities, $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$, three values, i.e. the world average and the corresponding $\pm 2\sigma$ values [8, 9, 10]:

$$\begin{aligned}\alpha_3(m_Z)^{WA} &= 0.118 \pm 0.008 \\ \sin^2\theta(m_Z)^{WA} &= 0.2334 \pm 0.0008 \\ 1/\alpha_{em}(m_Z)^{WA} &= 127.9 \pm 0.2\end{aligned}$$

It is interesting to remark that the results of these detailed studies confirm our first conclusion [8] – obtained using only a single M_{SUSY} – namely that within the minimum confidence level of our knowledge, i.e. $\pm 2\sigma$, the SUSY particle masses can be in the very wide range which goes from m_Z to PeV [8]. A more detailed study of the SUSY spectrum

shows that the mass sequence between all members ($\tilde{W}$, $\tilde{g}$, $\tilde{q}$, $\tilde{t}$) changes when the ratios $(m_0/m_{1/2})^2$, $(m_4/m_{1/2})^2$ change. The W-ino is the lightest member of the spectrum for the example shown in Fig. 8. We have investigated the effect on the W-ino mass when the experimentally measured quantities are allowed to take values within the $\pm 2\sigma$ range. The most important quantity is $\alpha_3(M_Z)$. From $[\alpha_3(M_Z)^{WA} - 2\sigma]$ to $[\alpha_3(M_Z)^{WA} + 2\sigma]$ the W-ino mass changes by more than four orders of magnitude. For $\sin^2\theta(M_Z)$ the variation is less than two orders of magnitude. For $\alpha_{em}(M_Z)$ the corresponding effect is a variation of about 25 %. These results remain practically unchanged when we allow the ratios of the primordial masses, $(m_0/m_{1/2})^2$ and $(m_4/m_{1/2})^2$, to go from 10^{-2} to 10^2 (see Table 1). The quantitative details concerning the W-ino mass are as follows: if we allow all parameters to change within $\pm 2\sigma$ [for the experimentally measured quantities, $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$] and within 10^{-2} and 10^2 [for the ratios of the primordial masses, $(m_0/m_{1/2})^2$ and $(m_4/m_{1/2})^2$], the maximum value for the W-ino mass is 4.2 PeV and the minimum is below m_Z (although we should recall that our formulae are only valid above the Z^0 mass). The search for SUSY particles should therefore be encouraged where right now there are colliders at work, i.e. LEP-I and Tevatron. Moreover, the forthcoming HERA and LEP-II are to be considered excellent tools to search for SUSY, whose mass spectrum (see Fig. 7) scans nearly an order of magnitude, once the lightest member of the sparticles has been identified.

For the sake of completeness we show in Figs. 9a, b, c, d how the primordial parameters m_0 , $m_{1/2}$, m_4 are coupled together for different values of $\alpha_3(m_Z)$: 0.115, 0.118, 0.123 and 0.125. The value chosen for $\sin^2\theta(m_Z)$ is the world average and for the top mass $m_t = 160$ GeV. These Figures 9a, b, c, d synthesize the predictive power of our iterative method, in terms of primordial masses, while Fig. 8 gives an example of this power in terms of SUSY mass spectrum.

Notice that, in the example shown, the W-ino is the lightest particle. We emphasise the relevance of this spectrum: it has been computed using the best of our experimental and theoretical knowledge. It is as reliable – and in fact by far more reliable – than other claims towards higher SUSY mass values which could be reached only when the new generation of colliders (SSC or LHC) will be available. Let us repeat: the search for SUSY needs not to wait for the new generation of colliders. Those available now (LEP-I, Tevatron) and in the near future (LEP-II, HERA) are in the energy range “predicted” by the best of our present knowledge. In this connection it is interesting to discuss

the trend which emerges from our various approaches to this key problem of finding where the SUSY threshold could be.

4. Discussion on the trends

Let us start recalling a few definitions:

- ΔT_H : indicates that the heavy threshold, when SUSY $[SU(5)]$ is broken into SUSY $[SU(3) \times SU(2) \times U(1)]$, has been taken into account (using only one single heavy mass).
- ΔT_L : indicates that we have taken a detailed SUSY spectrum to account for the threshold effects when SUSY $[SU(3) \times SU(2) \times U(1)]$ is broken into $SU(3) \times U(1)_{em}$.
- EGM: indicates that the simultaneous evolution of the couplings and of the gaugino masses has been worked out.
- EGM*: indicates that the analysis has been done working out the simultaneous evolution of the couplings and of all the masses (gauginos, squarks, Higgsino, etc.)
- A: stands for accuracy. There are in fact cases where we have worked out identical quantities using one-loop and two-loop approximations. The purpose being to find out the trend when the theoretical accuracy is improved.

Here are the main results from our group in order to contribute understanding the SUSY problem:

I. In our first paper [8] we have introduced a numerical solution to the coupled differential equations (not to an approximate analytical solution of these coupled differential equations) for $\alpha_1, \alpha_2, \alpha_3$ evolutions. We have used only one step function for SUSY breaking and no ΔT_L , no ΔT_H , no EGM. We have compared one-loop and two-loop results. We find that, when A improves, the value of M_{GUT} diminishes. On the other hand, we also find that when the value of $\alpha_3(m_Z)$ increases, the value of M_{GUT} increases and the value of M_{SUSY} decreases.

II. In a second step [1] we have introduced an iterative method to take into account the evolution of the gaugino masses: the EGM effect. In the present work we extend the simultaneous evolutions of couplings

and masses to all other masses (EGM*); ΔT_L is in, not ΔT_H . The formulae are at the one-loop level. We find that increasing values of $\alpha_3(m_Z)$ produce higher values of M_{GUT} and lower values of M_{SUSY} .

III. Our third approach [7] is based on a straightforward comparison between one-loop and two-loop calculations, using for ΔT_H only one single heavy mass in the 10^{16} GeV range. The only input is $\alpha_3(m_Z)$. The purpose of this approach was to see what the theoretical accuracy prefers as values of $\alpha_3(m_Z)$ once a χ^2 is defined in terms of the measured quantities $\sin^2\theta(m_Z)$ and $\alpha_{em}(m_Z)$. The results are that increasing accuracy prefers $\alpha_3(m_Z)$ values on the high side (i.e. above the world average) and M_{GUT} values on the low side. On the other hand it is found that higher values of $\alpha_3(m_Z)$ increase M_{GUT} and decrease M_{SUSY} .

A synthesis of the trends is shown in Fig. 10. A remark is in order: the fact that the proton lifetime limit is high ($\tau_{p \rightarrow e\pi} > 5.5 \times 10^{32}$ years) indicates that M_{GUT} cannot be too low. But our study on the trends indicates that high values of M_{GUT} imply low values for M_{SUSY} . In fact a possible scenario for the SUSY light threshold has been shown in Fig. 8.

5. Conclusions

The exhaustive treatment of the light threshold confirms the results obtained with the EGM on the SUSY-threshold lower bound. Our iterative method has been shown to be very powerful in predicting SUSY spectra as a function of the experimentally measured quantities $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$ and of the ratios of the primordial parameters $m_{1/2}$, m_0 and m_4 .

From our approaches to the SUSY problem we conclude that the following trends appear:

- 1) When the theoretical accuracy (from one loop to two loops) improves, the values of $\alpha_3(m_Z)$ are preferred to be on the high side (i.e. above the world average), and the values of M_{GUT} on the low side (20% decrease).
- 2) Higher values of $\alpha_3(m_Z)$ are correlated with higher values of M_{GUT} (formula (8)).
- 3) Higher values of M_{GUT} are correlated with lower values of M_{SUSY} (formulae (5) and (20)).

4) Higher values of $\alpha_3(m_Z)$ are correlated with lower values of M_{SUSY} (formulae (6) and (7)).

At present, increasing theoretical accuracy favours $\alpha_3(m_Z)$ to be on the high side. Consequently M_{SUSY} is lowered. The experimental searches for SUSY particles at LEP-I, at the Tevatron and in the forthcoming HERA and LEP-II colliders should be firmly encouraged.

$\frac{m_{\tilde{W}}(\alpha_3(m_Z)^{WA} - 2\sigma)}{m_{\tilde{W}}(\alpha_3(m_Z)^{WA} + 2\sigma)} \simeq 2 \times 10^4$ <p><i>[WA values for the experimental inputs: $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$]</i></p>
$\frac{m_{\tilde{W}}(\sin^2\theta(m_Z)^{WA} - 2\sigma)}{m_{\tilde{W}}(\sin^2\theta(m_Z)^{WA} + 2\sigma)} = 20 \div 50$ <p><i>[WA values for the experimental inputs: $\alpha_3(m_Z)$, $\alpha_{em}(m_Z)$]</i></p>
$\frac{m_{\tilde{W}}(1/\alpha_{em}(m_Z)^{WA} - 2\sigma)}{m_{\tilde{W}}(1/\alpha_{em}(m_Z)^{WA} + 2\sigma)} = 1.2 \div 1.3$ <p><i>[WA values for the experimental inputs: $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$]</i></p>
<p>Allowed range of variation for primordial SUSY breaking mass ratios:</p> $(m_0/m_{1/2})^2 = 10^{-2} \div 10^2$ $(m_4/m_{1/2})^2 = 10^{-2} \div 10^2$

TABLE 1

The variation of the W-ino mass corresponding to $\pm 2\sigma$ variation of the experimental inputs, $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$, with respect to their world average values, and to the variation of $(m_0/m_{1/2})^2$ and $(m_4/m_{1/2})^2$ in the indicated range. The data are derived from our system of seventeen coupled evolution equations (see section 2). The dominant effect is clearly due to $\alpha_3(m_Z)$. Please note that our equations are valid only above the Z^0 mass. Nevertheless the results in terms of $m_{\tilde{W}}$ ratios are given, for illustrative purposes, even when the $+2\sigma$ limit of the experimental inputs pushes $m_{\tilde{W}}$ below m_Z .

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Please note that for notations, formulae and other details not discussed in the present paper, we refer the reader to this work.
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Figure Captions

Figure 1: The value of the W-ino mass, $m_{\tilde{W}}$, versus the ratio of the primordial SUSY breaking parameters $(m_0/m_{1/2})^2$. The other parameters $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$ are fixed as indicated. Two extreme values, 10^{-2} and 10^2 , have been chosen for the latter.

Figure 2: The value of the W-ino mass, $m_{\tilde{W}}$, versus $\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$ and $(m_4/m_{1/2})^2$. Each curve is the result of our iterative system of seventeen evolution equations (see section 2). The curve showing the variation of $m_{\tilde{W}}$, versus $\alpha_3(m_Z)$, or $\sin^2\theta(m_Z)$, or $(m_4/m_{1/2})^2$, is computed keeping the other parameters fixed at the values indicated by the arrows. For the experimentally measured values, $\alpha_3(m_Z)$ and $\sin^2\theta(m_Z)$, the world averages have been chosen. For the ratio of primordial masses the value $(m_4/m_{1/2})^2 = 1$ has been taken. The top mass is kept at $m_t = 125$ GeV. Please note that our analysis is valid only above m_Z . The $m_{\tilde{W}}$ values shown below this limit are for illustrative purposes.

Figure 2a: The value of the W-ino mass versus m_t for given values of the other inputs, as indicated. This is the only case ($m_{\tilde{W}}$) where the dependence on m_t is shown. We do not show the results for $m_{\tilde{g}}$, $m_{\tilde{q}}$, $m_{\tilde{l}}$.

Figure 3: Same as Fig. 2 when the gluino mass, $m_{\tilde{g}}$, is computed.

Figure 4: Same as Fig. 2 when the squark mass, $m_{\tilde{q}}$, is computed.

Figure 5: Same as Fig. 1, when the slepton case is worked out. Note that the left (upper curve) and right (lower curve) slepton masses are very close.

Figure 6: Same as Fig. 2, when the slepton masses are computed. The left (upper curve) and right (lower curve) slepton masses almost coincide.

Figure 7: The predicted SUSY mass spectrum for three cases, when the measured quantities ($\alpha_3(m_Z)$, $\sin^2\theta(m_Z)$, $\alpha_{em}(m_Z)$) are taken at their world average values and at $\pm 2\sigma$. The ratios of the primordial parameters are kept equal to one. Note the large range where the SUSY spectra could be, on the basis of our best experimental and theoretical knowledge.

Figure 8: A detailed spectrum of SUSY particles showing that presently existing colliders (LEP-I and Tevatron) and forthcoming ones (HERA and LEP-II) have the same chance as future colliders (SSC and LHC) to discover SUSY.

Figure 9: Three-dimensional plots of the primordial parameters $m_{1/2}$, m_0 , m_A for different values of $\alpha_3(m_Z)$, i.e. 0.115 (a), 0.118 (b), 0.123 (c) and 0.125 (d), as indicated. For $\sin^2\theta(m_Z)$ and $\alpha_{em}(m_Z)$, the world average values are used. The top mass is kept at $m_t = 160$ GeV.

Figure 10: The trend emerging from our three approaches, as discussed in section 4. When the theoretical accuracy increases, A ($\uparrow$), high values of $\alpha_3(m_Z)$ are preferred: $\alpha_3(m_Z)$ ($\uparrow$). When $\alpha_3(m_Z)$ increases ($\uparrow$), M_{GUT} increases ($\uparrow$) and M_{SUSY} decreases ($\downarrow$). These trends have been studied allowing the measured quantities to go from the minimum (world average minus 2σ) to the maximum (world average plus 2σ) values.

$$\alpha_s(M_Z)=0.115$$

$$\sin^2\vartheta(M_Z)=0.2334$$

Figure 1

Figure 2

$$\alpha_s(M_Z) = 0.118 \text{ (W.A.)}$$

$$\sin^2\theta(M_Z) = 0.2334 \text{ (W.A.)}$$

$$m_0/m_{1/2} = 1$$

$$m_A/m_{1/2} = 1$$

Figure 2a

Figure 3

Figure 4

$$\alpha_s(M_Z)=0.115$$

$$\sin^2\theta(M_Z)=0.2334$$

Figure 5

Figure 6

Figure 7

SUSY PARTICLE MASS SPECTRUM

Figure 8

$$\alpha_s(M_Z)=0.115 \quad \sin^2\theta(M_Z)=0.2334 \quad m_t = 160 \text{ GeV}$$

Figure 9a

$$\alpha_s(M_Z)=0.118$$

$$\sin^2\theta(M_Z)=0.2334$$

$$m_t = 160 \text{ GeV}$$

Figure 9b

$$\alpha_s(M_Z)=0.123$$

$$\sin^2\theta(M_Z)=0.2334$$

$$m_t = 160 \text{ GeV}$$

Figure 9c

$$\alpha_s(M_Z)=0.125 \quad \sin^2\theta(M_Z)=0.2334 \quad m_t = 160 \text{ GeV}$$

Figure 9d

BASIC TRENDS

$$A \quad (\uparrow) \Rightarrow \alpha_3(M_Z) \quad (\uparrow)$$

$$\alpha_3(M_Z) \quad (\uparrow) \Rightarrow M_{GUT} \quad (\uparrow)$$

$$\alpha_3(M_Z) \quad (\uparrow) \Rightarrow M_{SUSY} \quad (\downarrow)$$

$$M_{GUT} \quad (\uparrow) \Rightarrow M_{SUSY} \quad (\downarrow)$$

Figure 10