On the Handedness in Polarized $\tau \rightarrow a_1 \nu \rightarrow 3\pi \nu$ DecayA.V. Efremov[†], L. Mankiewicz[‡] and N.A. Törnqvist[§]

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Abstract

We discuss the polarization asymmetries in the reactions $e^+e^- \rightarrow \tau^+\tau^-$ and $\tau \rightarrow a_1 \nu_\tau \rightarrow 3\pi \nu_\tau$ at the Z^0 . The $a_1 \rightarrow 3\pi$ decay is particularly interesting since it is the lightest hadronic decay in which the helicity of the initial state can be determined from the distribution of the final particle momenta. Thus it is the first example of a helicity self-analysing strong decay, as $\Lambda \rightarrow N\pi$ is for weak decays. The relevant observable is the "handedness" asymmetry introduced earlier. It can be used together with two other asymmetries to determine both polarization of τ and left-handedness of ν_τ , and to check the self-consistency of the analysis.

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1. Introduction

Recently we introduced the concept of handedness [1] in jet fragmentation or in strong particle decay, as a measure of the polarization of the initial parton or hadron. For strong interactions, parity invariance requires that there are at least three particles (spinless or spin-averaged) in the final state in order to have a correlation in the decay distribution with the initial helicity. Namely, from three particle momenta one can construct a pseudovector $n_\mu = \epsilon_{\mu\nu\rho\sigma} k_1^\nu k_2^\rho k_3^\sigma$ which gives, when contracted with the initial polarization pseudovector, a scalar component in the strong process. Thus the average of the vector n_μ can give information on the initial polarization, provided the correlation (or analysing power α) is large enough to be measurable. We called this quantity the handedness vector ($\vec{H} = \frac{3}{2}\alpha\langle\vec{n}\rangle$).

In fact, an idea very similar to handedness was first proposed by Nachtmann [2], and was also mentioned in an early work by one of us [3]. Its application to certain heavy-quark decays was studied in Ref. [4]. Similar techniques are also discussed in a recent work of Collins et al. [5]. Most of these papers deal with the asymmetries, which can be identified as longitudinal handedness $H_{||}$.

Because of the complicated process of jet fragmentation much, of the correlation (i.e. the analysing power $\bar{\alpha}_W$) between polarization and handedness is expected to be washed away when averaging over phase space and/or summing over different choices of the final three momenta k_i . Therefore we find it useful and instructive to look at the simpler situation of particle decay, where one can even calculate the expected handedness and analysing power α .

Looking at the lightest hadronic decays the process $a_1 \rightarrow 3\pi$ is unique in the sense that it is the lightest strong decay where one can measure a handedness, which can then reveal the helicity of the initial state. Thus this strong decay is similar to the well-known weak decay $\Lambda \rightarrow \pi N$ in the sense that it is "self-analysing" as to polarization.

One might first think that the 3π decay of a vector meson (ω , ϕ , J/ψ) could also be self-analysing, since we do have three final-state particles. However, these decays do not fulfil a second requirement: that we must have at least two amplitudes which depend differently on the initial polarization, since it is the interference between the two amplitudes which gives the helicity-dependent term. For $\omega \rightarrow 3\pi$ we have only one amplitude $\propto \epsilon_{\mu\nu\rho\sigma} \epsilon_\omega^\mu k_1^\nu k_2^\rho k_3^\sigma$, and therefore we can only measure the symmetric (tensor) part of the spin-density matrix ($\rho_{ij} \propto \text{Re}\epsilon_i\epsilon_j^*$), where ϵ_i is the polarization vector of the helicity i state. Note that it is the antisymmetric part of $\rho_{ij} \propto \text{Im}\epsilon_i\epsilon_j^*$ which is sensitive to the helicity or circular polarization vector $\mathcal{P}_\mu = i\epsilon_{\mu\nu\rho\sigma} p_\nu \epsilon_\rho \epsilon_\sigma^*/m$. The latter is the true Pauli-Lubanski pseudovector for angular momentum of spin 1.

We shall consider a_1 produced in e^+e^- annihilations at Z^0 decay to $\tau^+\tau^-$ and $\tau \rightarrow a_1\nu_\tau$. Since the τ is polarized (on the average $\approx 14\%$) at the Z^0 , owing to the weak interaction, this polarization is measurable through the one it induces on the a_1 . Therefore a detailed study of the polarization asymmetries in the reaction $\tau \rightarrow a_1\nu_\tau \rightarrow 3\pi\nu_\tau$ is of experimental interest. However, the a_1 is polarized also by the parity violation in the decay $\tau \rightarrow a_1\nu_\tau$, even if the τ is unpolarized. This was

studied by Kühn and Wagner [6] and by Feindt [7], and was used by the ARGUS group [8] to measure the helicity of the ν_τ to be left-handed, as expected in the standard model for a sequential third-generation tau neutrino.

As we shall show, the unique property of the $\tau \rightarrow 3\pi\nu_\tau$ decay is the possibility to separate measurements of the effects due to the τ polarization, i.e. the effects of parity violation in the neutral current, from the effects of left-handedness of the ν_τ , i.e. the parity violation in the charged current. The latter have to be assumed in studies of τ polarization with other decay channels [9]–[12].

2. General framework

Let us consider the matrix element for τ^- production in e^+e^- collisions at Z^0 energy, followed by its decay into ν and $\pi^+\pi^-\pi^-$ through the a_1 intermediate state. In the approximation, when the width of the τ can be neglected as compared to energy resolution, the matrix-element squared can be conveniently decomposed as

$$|M|^2 = \frac{1}{2} \left(\bar{W}_P W_D^{\mu\nu} + W_P^\sigma W_D^{\sigma,\mu\nu} \right) H_{\mu\nu} . \quad (1)$$

In this formula $\bar{W}_P$ is the standard matrix-element squared for the production of unpolarized τ^- in the $e^+e^- \rightarrow \tau^+\tau^-$ reaction. The function W_P^σ corresponds to the production of τ in a state characterized by a spin vector S (the choice of S most convenient for our purposes will be specified below); $W_D^{\mu\nu}$ and $W_D^{\sigma,\mu\nu}$ are leptonic tensors for the decay of unpolarized and polarized τ into ν and a_1 respectively (note that because of parity violation in this decay both symmetric and antisymmetric parts of $W_D^{\mu\nu}$ and $W_D^{\sigma,\mu\nu}$ are different from zero); $H_{\mu\nu}$ is the hadronic tensor describing the transition $a_1 \rightarrow 3\pi$. If we introduce the average polarization of τ^- along S^μ , denoted by Λ

$$\Lambda = \frac{W_P^\sigma}{\bar{W}_P} , \quad (2)$$

then $|M|^2$ assumes the intuitive form

$$|M|^2 = \frac{1}{2} \bar{W}_P (W_D^{\mu\nu} + \Lambda W_D^{\sigma,\mu\nu}) H_{\mu\nu} . \quad (3)$$

We discuss below each of the above terms in more detail.

3. Reaction $a_1^- \rightarrow \pi^-\pi^-\pi^+$

We introduce the following notation. Let k_1 , k_2 , and k_+ denote momenta of the two π^- and π^+ , respectively, and let $k = k_1 + k_2 + k_+$ be the momentum of a_1 . The squared invariant masses of the two possible $\pi^+\pi^-$ systems are denoted as $s_1 = (k_1 + k_+)^2$ and $s_2 = (k_2 + k_+)^2$. The decay of a_1 into $\pi^+\pi_1^-\pi_2^-$ can proceed through two paths: $a_1 \rightarrow \pi_1^-\rho \rightarrow \pi_1^-\pi^+\pi_2^-$ or $a_1 \rightarrow \pi_2^-\rho \rightarrow \pi_2^-\pi^+\pi_1^-$. The matrix element J^μ can be written as the sum of two amplitudes:

$$J^\mu = \rho_a^{-1} (K_{(1)}^\mu \rho_1^{-1} + K_{(2)}^\mu \rho_2^{-1}) , \quad (4)$$

where ρ_i is given by the Breit-Wigner formula $\rho_i = s_i - m_\rho^2 - im_\rho\Gamma_\rho$ and $\rho_a = k^2 - m_a^2 - im_a\Gamma_a$. Here Γ_ρ , m_ρ , and Γ_a , m_a denote widths and masses of ρ and a_1 , respectively. Assuming PCAC, the most general expression for the current K_i^μ is given by

$$K_i^\mu = \left(g^{\mu\alpha} - \frac{k^\mu k^\alpha}{k^2} \right) T_{\alpha\beta}^i (k_+ - k_i)^\beta F_{a_1}(k^2). \quad (5)$$

The $W \rightarrow a_1$ transition form factor F_{a_1} is not relevant for our discussion and will be omitted in the following. The tensor $T_{\alpha\beta}^i$ contains two possible forms of $a_1 \rightarrow \rho\pi$ coupling [7]

$$T_{\alpha\beta}^i = h g_{\alpha\beta} + g(k_+ + k_i)_\alpha k_\beta. \quad (6)$$

From previous experiments, it is known [13] that the S-wave Born term proportional to $g^{\alpha\beta}$ dominates; therefore, the D-wave contribution will be neglected in the following.

The resulting hadronic tensor $H^{\mu\nu} = J^\mu J^{\nu*}$ can be decomposed into symmetric and antisymmetric parts $\bar{H}^{\mu\nu}$ and $\hat{H}^{\mu\nu}$:

$$\bar{H}^{\mu\nu} = (BW) \left(K_{(1)}^\mu K_{(1)}^\nu |\rho_2|^2 + K_{(2)}^\mu K_{(2)}^\nu |\rho_1|^2 + (K_{(1)}^\mu K_{(2)}^\nu + K_{(1)}^\nu K_{(2)}^\mu) \text{Re} \rho_1^* \rho_2 \right) \quad (7)$$

and

$$\hat{H}^{\mu\nu} = (BW) \left(K_{(2)}^\mu K_{(1)}^\nu - K_{(2)}^\nu K_{(1)}^\mu \right) \text{Im} \rho_1^* \rho_2 \quad (8)$$

where (BW) denotes a string of Breit-Wigner factors

$$(BW) = (|\rho_a| |\rho_1| |\rho_2|)^{-2}. \quad (9)$$

The non-zero antisymmetric part reflects the sensitivity of the decay cross section to the circular polarization of the initial a_1 . Note that it requires both an interference between $K_{(1)}^\mu$ and $K_{(2)}^\mu$ and the presence of the imaginary part in the propagator of the ρ meson to produce a non-vanishing $\hat{H}^{\mu\nu}$. In this case a_1 decays in a self-analysing way which, as we discuss below, allows us to determine its polarization from the momenta distribution of the decay products.

4. Reaction $\tau \rightarrow a_1 \nu_\tau$

We denote the τ momentum by p and the neutrino momentum by q , $p = q + k$. Since the spin of a_1 is 1 the hadronic tensor $H^{\mu\nu}$ obeys $k_\mu H^{\mu\nu} = H^{\mu\nu} k_\nu = 0$; in the following we shall therefore omit terms proportional to k^μ or k^ν in the leptonic tensors $W_D^{\mu\nu}$ and $W_D^{\sigma,\mu\nu}$. As in the case of $H^{\mu\nu}$, we decompose both tensors into their symmetric and antisymmetric parts,

$$W_D^{\mu\nu} = \bar{W}_D^{\mu\nu} + \hat{W}_D^{\mu\nu}$$

and

$$W_D^{\sigma,\mu\nu} = \bar{W}_D^{\sigma,\mu\nu} + \hat{W}_D^{\sigma,\mu\nu},$$

where

$$\begin{aligned} \bar{W}_D^{\mu\nu} &= (G_A^2 + G_V^2)(2p^\mu p^\nu - g^{\mu\nu} q \cdot p) \\ \hat{W}_D^{\mu\nu} &= 2i G_A G_V \epsilon^{\mu\nu\alpha\beta} p_\alpha k_\beta, \end{aligned} \quad (10)$$

and for the τ -spin-dependent part $W_D^{\sigma,\mu\nu}$ we find

$$\begin{aligned}\bar{W}_D^{\sigma,\mu\nu} &= 2G_A G_V m_\tau (S^\mu p^\nu + S^\nu p^\mu + g^{\mu\nu} S \cdot k) \\ \hat{W}_D^{\sigma,\mu\nu} &= i(G_A^2 + G_V^2) m_\tau \epsilon^{\mu\nu\alpha\beta} q_\alpha S_\beta ,\end{aligned}\tag{11}$$

where G_A and G_V denote vector and axial-vector coupling constants of τ to the charged weak current. Since G_A is different from zero, parity is violated in the $\tau \rightarrow a_1 \nu$ decay and a_1 is created polarized, even when the parent τ is in an unpolarized state. This is evident from the presence of the antisymmetric part $\hat{W}_D^{\mu\nu}$ of the leptonic tensor. By the same reason, the τ -polarization vector S^μ contributes to the symmetric part $\bar{W}_D^{\sigma,\mu\nu}$ as well.

5. Polarization of τ in $e^+e^- \rightarrow \tau^+\tau^-$

Until now we have not given the definition of the pseudovector S^μ , but it is understood that it has to obey usual constraints $S^2 = -1$ and $S \cdot p = 0$. Now, we choose S^μ explicitly in the form

$$S^\mu = \left(\frac{\vec{p} \cdot \vec{e}_a}{m_T}, \vec{e}_a \frac{p^0}{m_T} \right),$$

where $\vec{e}_a$ is the direction of the three momentum of a_1 in the e^+e^- CM frame, and $m_T = \sqrt{m_\tau^2 + \vec{p}^2 - (\vec{p} \cdot \vec{e}_a)^2}$. This choice of S^μ is advantageous because, contrary to the τ momentum p , the a_1 momentum k can be easily reconstructed from the momenta of the three measured pions. In the following we will find it easier to write down our expressions in the CM frame of a_1 with the z -axis parallel to $\vec{e}_a$. Note that m_T is invariant under the boost from the electron CM frame to this frame. The average polarization Λ in Eqs. (1) and (3) can now be easily calculated for a fixed polar angle of a_1 using standard techniques:

$$\Lambda \approx -\lambda_e \frac{m_\tau}{m_\perp} \frac{(1 + \cos^2 \Theta_a)(\lambda_\tau/\lambda_e) + 2 \cos \Theta_a}{1 + \cos^2 \Theta_a + 2 \cos \Theta_a \lambda_e \lambda_\tau},\tag{12}$$

where

$$\lambda_i = \frac{2g_a^i g_v^i}{g_a^{i2} + g_v^{i2}}$$

is given in terms of the lepton i , i.e. electron or τ axial and vector couplings to the weak neutral current. Lepton universality would imply $\lambda_\tau = \lambda_e$. To derive Eq. (12) we use the relation between the polar angles Θ_τ and Θ_a of τ and a_1 , respectively, with respect to the e^+e^- axis (see Fig. 1).

$$\cos \Theta_\tau = \cos \Theta_a \cos \Theta_{\tau a} + \sin \Theta_a \sin \Theta_{\tau a} \cos \phi ,\tag{13}$$

where $\Theta_{\tau a}$ is the angle between τ and a_1 in the e^+e^- CM frame. Because $\Theta_{\tau a} = \sqrt{(m_\tau^2 + m_a^2)/E E_a}$ is small (E and E_a denote the electron and a_1 energies in the e^+e^- CM frame), it is justified to replace $\cos \Theta_\tau$ by $\cos \Theta_a$ in (13). The correction, linear in $\Theta_{\tau a}$, drops out after averaging over the azimuthal angle ϕ because it is

proportional to $\cos \phi$. As a result the error is of the order of the $\Theta_{\tau a}^2$, and therefore completely negligible at LEP energies considered here.

6. τ^- decay into $\nu_\tau \pi^+ \pi^- \pi^-$

In this section we shall work in the CM frame of a_1 . In this frame currents $K_{(i)}^\mu$ have only space-like components

$$K_{(i)}^\mu = (0, \vec{K}_{(i)}) = (0, \vec{k}_+ - \vec{k}_i) .$$

We also keep in mind that owing to the unknown azimuthal directions of τ and neutrino, one should average the matrix-element squared over the azimuthal angle ϕ . In the following, we first present the full covariant expressions and then the result obtained after the averaging in the a_1 CM frame.

The total matrix element $|M|^2$, see Eq. (1), can be written as

$$|M|^2 = \frac{1}{2}(G_A^2 + G_V^2)\bar{W}_P(\bar{M}_1 + \Pi\Lambda\bar{M}_2 + \Lambda\hat{M}_1 - \Pi\hat{M}_2) , \quad (14)$$

where $\Pi = 2G_A G_V / (G_A^2 + G_V^2)$. ‘Bar’ or ‘hat’ over M_i points to its origin from the contraction of symmetric or antisymmetric parts of the leptonic and hadronic tensors, as described above in Sections 3 and 4. Note that the common factor of (BW) , see Eq. (9), has been omitted for simplicity, but must be added in front of each expression. In the covariant notation we obtain

$$\begin{aligned} \bar{M}_1 &= \left(2(p \cdot K_{(1)})^2 - K_{(1)}^2 p \cdot q\right) |\rho_2|^2 \\ &+ \left(2(p \cdot K_{(2)})^2 - K_{(2)}^2 p \cdot q\right) |\rho_1|^2 \\ &+ 2\left(2p \cdot K_{(1)} p \cdot K_{(2)} - K_1 \cdot K_2 p \cdot q\right) \text{Re}\rho_1^* \rho_2 , \\ \bar{M}_2 &= m_\tau \left[\left(2S \cdot K_{(1)} K_{(1)} \cdot q + K_{(1)}^2 S \cdot k\right) |\rho_2|^2 \right. \\ &+ \left(2S \cdot K_{(2)} K_{(2)} \cdot q + K_{(2)}^2 S \cdot k\right) |\rho_1|^2 \\ &\left. + \left(S \cdot K_{(1)} K_{(2)} \cdot q + S \cdot K_{(2)} K_{(1)} \cdot q + K_{(1)} \cdot K_{(2)} S \cdot k\right) \text{Re}\rho_1^* \rho_2 \right] , \end{aligned} \quad (15)$$

$$\hat{M}_1 = 2m_\tau \epsilon_{\mu\nu\alpha\beta} K_{(2)}^\mu K_{(1)}^\nu S^\alpha q^\beta \text{Im}\rho_1^* \rho_2 ,$$

$$\hat{M}_2 = 2\epsilon_{\mu\nu\alpha\beta} K_{(2)}^\mu K_{(1)}^\nu p^\alpha k^\beta \text{Im}\rho_1^* \rho_2 .$$

The same expressions have the following form after averaging over the azimuthal angle of the τ momentum in the CM system of a_1 :

$$\begin{aligned} \bar{M}_1 &\Rightarrow m_a |\vec{p}| \left\{ \left[\left(1 + \frac{2|\vec{p}|}{m_a} \cos^2 \Psi\right) (K_{(1)}^z)^2 + \left(1 + \frac{|\vec{p}|}{m_a} \sin^2 \Psi\right) (K_{(1)}^\perp)^2 \right] |\rho_2|^2 \right. \\ &+ \left[\left(1 + \frac{2|\vec{p}|}{m_a} \cos^2 \Psi\right) (K_{(2)}^z)^2 + \left(1 + \frac{|\vec{p}|}{m_a} \sin^2 \Psi\right) (K_{(2)}^\perp)^2 \right] |\rho_1|^2 \\ &\left. + 2 \left[\left(1 + \frac{2|\vec{p}|}{m_a} \cos^2 \Psi\right) K_{(1)}^z K_{(2)}^z + \left(1 + \frac{|\vec{p}|}{m_a} \sin^2 \Psi\right) K_{(1)}^\perp \cdot K_{(2)}^\perp \right] \text{Re}\rho_1^* \rho_2 \right\} , \end{aligned}$$

$$\begin{aligned}
\bar{M}_2 &\Rightarrow \frac{m_a m_\tau}{m_T} |\vec{p}| \cos \Psi \left\{ \left[2 \left(K_{(1)}^z \right)^2 \frac{p^0}{m_a} - \vec{K}_{(1)}^2 \right] |\rho_2|^2 \right. \\
&\quad + \left[2 \left(K_{(2)}^z \right)^2 \frac{p^0}{m_a} - \vec{K}_{(2)}^2 \right] |\rho_1|^2 \\
&\quad \left. + 2 \left(2 K_{(1)}^z K_{(2)}^z \frac{p^0}{m_a} - \vec{K}_{(1)} \cdot \vec{K}_{(2)} \right) \text{Re} \rho_1^* \rho_2 \right\} , \\
\hat{M}_1 &\Rightarrow 6n^z |\vec{p}| \frac{m_\tau}{m_T} (p^0 - |\vec{p}| \cos^2 \Psi) \text{Im} \rho_1^* \rho_2 , \\
\hat{M}_2 &\Rightarrow 6n^z |\vec{p}| m_a \cos \Psi \text{Im} \rho_1^* \rho_2 ,
\end{aligned} \tag{16}$$

where Ψ is the τ polar angle in the a_1 CM frame, see Fig. 1. In the convention of this paper the orientation of this frame is chosen such that the direction of the z -axis is defined by $\vec{e}_a$. In this case $n^z = \vec{e}_a \cdot (\vec{k}_1 \times \vec{k}_2)$ denotes the third component of the normal to the decay plane. We recall also that in this frame $p^0 = (m^2 + m_a^2)/2m_a$, $|\vec{p}| = (m^2 - m_a^2)/2m_a$ and that $m_a = \sqrt{k^2}$ everywhere except for ρ_a . Finally, $\text{Im} \rho_1^* \rho_2 = m_\rho \Gamma_\rho (s_1 - s_2)$ and $\text{Re} \rho_1^* \rho_2 = (s_1 - m_\rho^2)(s_2 - m_\rho^2) + m_\rho^2 \Gamma_\rho^2$, and the angle Ψ can be expressed through the laboratory energy of the 3π system:

$$\cos \Psi = \frac{2m_a^2 E_a - (m_\tau^2 + m_a^2) E_a}{(m_\tau^2 - m_a^2) \sqrt{E_a^2 - m_a^2}} .$$

The contribution to (16) that is proportional to n^z has an interesting physical interpretation. It arises because the decay $a_1 \rightarrow 3\pi$ is the (rare) example of a self-analysing strong decay. Note that both terms $\hat{M}_1$ and $\hat{M}_2$ are due to the antisymmetric part $\hat{H}^{\mu\nu}$ of the hadronic tensor. Let us contract $\hat{H}^{\mu\nu}$ with $E_{\mu\nu} = \epsilon_\mu \epsilon_\nu^*$, the polarization tensor of a_1 . The antisymmetric part of $E_{\mu\nu}$ equals $\hat{E}_{\mu\nu} = 2i \text{Im} \epsilon_\mu \epsilon_\nu^* = i \epsilon_{\mu\nu\alpha\beta} \mathcal{P}_a^\alpha k^\beta / m_a$, where $\mathcal{P}_a^\alpha$ is the Pauli-Lubanski vector of circularly polarised a_1 . In the a_1 CM frame, one obtains $\hat{H}^{\mu\nu} \hat{E}_{\mu\nu} \propto 6 \vec{\mathcal{P}}_a \cdot \vec{n} \text{Im} \rho_1^* \rho_2$, to compare with Eq.(16). Dividing by the symmetric part $\bar{H}^{\mu\nu} \bar{E}_{\mu\nu}$ we arrive at the asymmetry parameter which is antisymmetric with respect to the components of the vector $\vec{n}$, and proportional to the polarization vector $\vec{\mathcal{P}}_a$

$$H^i = \frac{N(n^i > 0) - N(n^i < 0)}{N(n^i > 0) + N(n^i < 0)} = \alpha(s_1, s_2) \mathcal{P}_a^i , \tag{17}$$

where N is the number of events observed, and H^i is just the handedness vector introduced in [1, 2, 4].

In conclusion of this section, note that the matrix-element squared (14) which determines the angular distribution of decay products is linear in $n^z = \cos \Theta$, where Θ is the polar angle of the normal $\vec{n}$, see Fig. 1. This seems to differ from the analysis presented in Ref. [14], which gives an expression quadratic in $\cos \Theta$. However, it was obtained by integration over the whole Dalitz plot and therefore the interference

terms $\hat{M}_1$ and $\hat{M}_2$ which are linear in n^z are averaged to zero [15]. In addition, one can check that the contributions to $\bar{M}_1$ and $\bar{M}_2$ quadratic in $K_{(i)}^z$ appear proportional to $\sin^2 \Theta$ after averaging over the azimuthal rotation around the normal $\vec{n}$.

7. Asymmetries

The cross section (14)–(16) depends on the weak charged and neutral currents couplings to τ . We note that each of the four terms in (14) exhibits different symmetries with respect to p^z (or $\cos \Psi$) and n^z (or $\cos \Theta$); $\bar{M}_1$ is even in both p^z and n^z , $\bar{M}_2$ is even in n^z but odd in p^z , $\hat{M}_1$ is even in p^z but odd in n^z , while $\hat{M}_2$ is odd in both. Hence, in the ideal case it should be possible to construct three different asymmetries for simultaneous determination of both Π and Λ . It is non-trivial that the same set of data can be used. The third independent asymmetry allows one to check the consistency of the analysis.

Let us consider now the three possible asymmetries. The first one can be interpreted as the component of the handedness vector of Eq. (17) in the direction of $\vec{e}_a$, averaged over $\cos \Psi$ in a symmetric way:

$$\mathcal{H} = \frac{N(n^z(s_1 - s_2) > 0) - N(n^z(s_1 - s_2) < 0)}{N(n^z(s_1 - s_2) > 0) + N(n^z(s_1 - s_2) < 0)}. \quad (18)$$

The second one is the n^z, p^z correlation function:

$$\mathcal{C} = \frac{N(n^z p^z(s_1 - s_2) > 0) - N(n^z p^z(s_1 - s_2) < 0)}{N(n^z p^z(s_1 - s_2) > 0) + N(n^z p^z(s_1 - s_2) < 0)}, \quad (19)$$

while the third one can be chosen as the p^z asymmetry:

$$\mathcal{A} = \frac{N(\cos \Psi > 0) - N(\cos \Psi < 0)}{N(\cos \Psi > 0) + N(\cos \Psi < 0)}. \quad (20)$$

The handedness $\mathcal{H}$ determines the parameter Λ , which equals the τ polarization, and can also be shown to be proportional to the a_1 polarization averaged over $\cos \Psi$:

$$\mathcal{H} = \Lambda(\Theta_a) \frac{\int d\Omega(BW) |\hat{M}_1|}{\int d\Omega(BW) \bar{M}_1} = \alpha \Lambda(\Theta_a). \quad (21)$$

Had we not performed the average over $\cos \Psi$, the handedness would have been proportional just to the polarization of a_1 ; this, however, depends on both Λ and Π .

The correlation $\mathcal{C}$ determines the parameter Π

$$\mathcal{C} = \Pi \frac{\int d\Omega(BW) |\hat{M}_2|}{\int d\Omega(BW) \bar{M}_1}. \quad (22)$$

The two above asymmetries arise because of the antisymmetric part of the hadronic tensor. The third asymmetry probes the symmetric part of $H^{\mu\nu}$:

$$\mathcal{A} = \Pi \Lambda \frac{\int d\Omega(BW) |\bar{M}_2|}{\int d\Omega(BW) \bar{M}_1}. \quad (23)$$

It can be used to check the consistency of the determination of Π and Λ from (21) and (22). In the above expressions, $d\Omega$ is the Lorentz-invariant phase space for three final-state pions. Note also that we have restored explicitly the Breit-Wigner factors of Eq. (9).

8. Conclusions

The main purpose of this paper was to demonstrate that the concept of handedness indeed allows the polarization of an undetectable particle to be determined, and that the analysing power need not be small. It is easy to see that the analysing power α determined by Eq. (21) can be of the order of unity when the relative phase between two amplitudes in Eq. (4) is close to 90° , i.e. when $s_1 = m_\rho^2 + m_\rho \Gamma_\rho$ and $s_2 = m_\rho^2 - m_\rho \Gamma_\rho$. In this sense $a_1 \rightarrow 3\pi$ decay is the unique example of self-analysing strong decay, and handedness of the decay products is proportional to the polarization of a_1 . We discussed how this channel can be used to determine weak currents couplings to τ . One can form three independent asymmetries, an overcomplete set of equations for two unknown parameters, and therefore self-consistency of the analysis can be investigated as well.

The corrections due to the D-wave Born-term contribution to the decay $a_1 \rightarrow \rho\pi$ and due to decay via the $\epsilon\pi$ channel are known to be small, according to the discussion of Ref. [7].

We point out that there is no reason to assume that a situation similar to that discussed above cannot occur in the case of a polarized quark fragmenting into a jet of light hadrons. In particular, the interference between amplitudes of subsequent emission of, e.g. ρ and π , from the quark leg can produce final state with non-zero handedness. We note that similar considerations for the case of fragmentation of heavy quarks were presented in Ref. [4].

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Figure caption

Fig. 1 Rapidity space view of the kinematics of the process $e^+e^- \rightarrow \tau^+\tau^-$ followed by the decay chain $\tau^- \rightarrow a_1\nu_\tau \rightarrow 3\pi\nu_\tau$.

Fig. 1