The Neutrino Electromagnetic Moments and Charge
Radius Confront Kamiokande II and Homestake
Experimental Results

Ana M. Mourão

CFN-Av. Prof. Gama Pinto, 2, P-1699 Lisboa
Dept. Fisica - IST, Av. Rovisco Pais, P-1000 Lisboa
PORTUGAL

João Pulido

Theory Division, CERN - Geneva
and
IFM-Av. Prof. Gama Pinto, 2, P-1699 Lisboa
PORTUGAL

John P. Ralston

Theory Division, CERN - Geneva
and
Dept. of Physics and Astronomy, University of Kansas
Lawrence, Kansas 66045-2151 USA

ABSTRACT

An analysis is done comparing the main results from the Kamiokande II and Chlorine solar neutrino experiments to test the possible existence of a neutrino electric or magnetic moment and charge radius. We find an upper bound on the charge radius of $5.9 \times 10^{-11} \text{ MeV}^{-2}$ at 90% C.L., an improvement with respect to the previous experimental value by a factor greater than 2. We show that the maximum permissible range for the proper combination of electromagnetic moments has a mild dependence on the theoretical solar model and is reached at the experimental lower bound of the charge radius. Within experimental errors, the apparent discrepancy observed between the data of the two experiments is consistent both with a vanishing magnetic moment and charge radius, and with a Dirac magnetic moment as large as $5.4 \pm 1.4 \times 10^{-10} \mu_B$ for zero charge radius.

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Introduction

A recent revival of interest in the long-standing solar neutrino problem, the fact that the too few solar neutrinos are observed^{1),2)} as compared to the theoretical prediction,³⁾ has been motivated by a close examination of the data over recent years from two experiments: the Homestake mine experiment of Davis¹⁾ and Kamiokande II.²⁾ A third experiment, SAGE,⁴⁾ started collecting data in early 1990 but so far has only brought an apparent confirmation of the solar neutrino deficit.

Attempted explanations for this deficit based on the possible existence of a neutrino magnetic moment were put forward by Cisneros in 1971⁵⁾ and again in a different form by Voloshin, Vysotsky and Okun in 1986.⁶⁾ In fact, if neutrinos have a magnetic moment, the neutrinos produced in the core of the Sun may flip their helicity due to interaction with the solar magnetic field. They then become sterile for ordinary weak interactions. The authors of ref. 6) were mainly motivated by the anti-correlation with sun-spot activity that the Davis data seem to suggest. An increase in sunspot activity is directly related to an increase in the solar magnetic field in the Sun's outer layers. If the observed neutrino flux decreases during the periods of more intense sunspot activity and increases at times of less activity, the neutrino magnetic moment would no doubt provide an interesting explanation for the whole puzzle. While the anti-correlation effect of the Davis data is far from clear, there are no indications so far that such a time variation is observed by Kamiokande II.

Recent theoretical efforts in solar neutrino physics have been mainly concentrated on model building⁷⁾ and parameter range investigations,^{8),9)} in an attempt to reconcile and/or explain the possible time dependence observed by Davis with the Kamiokande II data. It is currently believed⁷⁾⁻⁹⁾ that the Kamiokande II experiment sees a nearly constant neutrino flux whereas Davis observes the above mentioned anti-correlation. If one takes, however, a close look at the KII data,¹⁰⁾ one realizes that their neutrino signal may not be constant in time, so that averaging over data points may conceal important short term information over a relatively extended observation period. A comparison between the data from the two experiments during the period January 1987, through April 1990 is shown in table I. Each point corresponds to the same data taking time interval in the two experiments. It is seen that the neutrino flux measurements in both cases vary with time and the Davis readings always lie below the KII ones.

The Davis experiment is looking at a purely weak process, namely

$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^- \quad (1),$$

whereas Kamiokande is based on the elastic scattering

$$\nu_e + e^- \rightarrow \nu_e + e^- \quad (2)$$

whose amplitude may contain an additional electromagnetic contribution.

In this paper we endow the neutrino with electric and magnetic dipole moments and a charge radius. We derive the range of electromagnetic form factors compatible with the results available from the two experiments. Unlike the authors of ref. 9), who averaged over data points, we compare the KII and Davis results point by point. Thus the time dependence of the KII data is not concealed and is confronted directly with the Davis results. As will be seen, the difference between the two sets of data can be explained either on the basis of the neutrino dipole moments, the charge radius, or both.

2. Neutrino Scattering and Parameter Ranges

The elastic scattering (2) with the inclusion of an electromagnetic interaction via neutrino form factors has been studied by Kerimov *et al.*¹¹⁾ For Dirac electron neutrinos one has the following differential cross section with respect to the recoil electron energy T :

$$\frac{d\sigma}{dT} = \frac{1 + \xi\xi'}{2} \left(\frac{d\sigma_W}{dT} + \frac{d\sigma_{+EM}}{dT} + \frac{d\sigma_{-EM}}{dT} \right) + \frac{1 - \xi\xi'}{2} \frac{d\sigma_{-EM}}{dT} . \quad (3)$$

We list the weak cross section,

$$\frac{d\sigma_W}{dT} = (1 - \xi) \frac{G_F^2 m_e}{4\pi} \left[(g_V + g_A)^2 + (g_V - g_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - (g_V^2 - g_A^2) \frac{m_e T}{E_\nu^2} \right] , \quad (4a)$$

the electromagnetic cross sections,

$$\frac{d\sigma_{+EM}}{dT} = \frac{\pi\alpha^2}{m_e} \left(\frac{f_{1\nu}}{T} \right)^2 \left[1 + \left(1 - \frac{T}{E_\nu}\right)^2 - \frac{m_e T}{E_\nu^2} \right] , \quad (4b)$$

$$\frac{d\sigma_{-EM}}{dT} = \frac{\pi\alpha^2 (f_{2\nu}^2 + g_{2\nu}^2)}{m_e^2} \left(\frac{1}{T} - \frac{1}{E_\nu} \right) , \quad (4c)$$

and the interference term

$$\frac{d\sigma_{int}}{dT} = \frac{1 - \xi}{\sqrt{2}} \alpha G_F \frac{f_{1\nu}}{T} \left[g_V \frac{m_e T}{E_\nu^2} - (g_V + g_A) - (g_V - g_A) \left(1 - \frac{T}{E_\nu}\right)^2 \right] . \quad (4d)$$

Here $\xi(\xi')$ is the initial (final) neutrino helicity ($\xi, \xi' = -1$ for left handed and $\xi, \xi' = +1$ for right handed neutrinos). The quantity $f_{1\nu}(q^2)$ is the Dirac form factor multiplying γ^μ in

the electromagnetic current, so that (4b) is the non-helicity flip part of the electromagnetic cross section.^{F1}

The neutrino charge radius $\langle r^2 \rangle$ is related to $f_{1\nu}(q^2)$ by¹²⁾¹³⁾

$$f_{1\nu}(q^2) = 6 \langle r^2 \rangle q^2 \quad (5)$$

In this paper we will consider $\langle r^2 \rangle$ as the parameter. The magnetic and electric dipole form factors $f_{2\nu}(q^2), g_{2\nu}(q^2)$ multiply $\sigma^{\mu\nu}$ in the current so they represent the helicity flip part of the cross section (4c). We use their values at zero momentum transfer, giving the magnetic and electric dipole moments of the neutrino in units of the Bohr magneton μ_B : $\mu_\nu = f_{2\nu}(0)\mu_B, d_\nu = g_{2\nu}(0)\mu_B$. Since the cross section (4c) depends on the combination $f_{2\nu}^2 + g_{2\nu}^2$, the electric and magnetic moments cannot be distinguished. We will call $\sqrt{f_{2\nu}^2 + g_{2\nu}^2}$ the dipole moment. The predicted event rate for Kamiokande (N_{KII}) under the assumption that the neutrino possesses electromagnetic properties, is then

$$N_{KII} = \int_{T_m}^{T_M} e(T) dT \int_{E_{\nu m}}^{E_{\nu M}} f(E_\nu) \left[R_D \frac{d\sigma_W}{dT} + \frac{d\sigma_{+EM}}{dT} + \frac{d\sigma_{-EM}}{dT} + R_D \frac{d\sigma_{int}}{dT} \right] dE_\nu \quad (6).$$

Here the quantity R_D is the ratio between the number of left handed neutrinos present in the solar neutrino flux and their number as predicted by the solar standard model. We take this to be given by the Davis result as in Table I. It is seen from eq. (4d) that left handed neutrinos are also the only ones entering the interference part of the cross section. The functions $f(E_\nu)$ and $e(T)$ denote respectively the energy dependent neutrino flux³⁾ (in neutrinos $\cdot$ (area)⁻¹ $\cdot$ (time)⁻¹ $\cdot$ (energy⁻¹)) and the detector efficiency.¹⁰⁾ Both left and right handed neutrinos equally feel the electromagnetic interaction, so the factor multiplying the electromagnetic cross section is unity. The recoil electron kinetic energy T measured in the experiment ($T = E_e - m_e$) is related to the neutrino energy E_ν by:

$$T \leq \frac{2E_\nu^2}{2E_\nu + m_e} \quad (7).$$

Solving for E_ν one obtains

$$E_\nu \geq \frac{T + \sqrt{T^2 + 2m_e T}}{2} \quad (7a).$$

The right hand side of this inequality is the minimum energy for a given neutrino to be capable of producing a detected electron with kinetic energy T . It is therefore the lower

^{F1} Theoretical predictions for $f_{2\nu}(q^2)$ can be found in refs. (13,14).

integration limit for E_ν in (6), which in the range of parameters considered is well approximated by

$$E_{\nu m} \simeq T + \frac{m_e}{2} \quad (8).$$

For the upper limits in (6), we take³⁾

$$E_{\nu M} = 14 \text{ MeV}, \quad T_M = E_{\nu M} - \frac{m_e}{2} \quad (9).$$

The lower limit for $T(= T_m)$ is determined from the experiment's threshold energy:^{2),10)} $E_{eth} = 9.3 \text{ MeV}$ (data points I, II, III(a), IV(a) and V(a) and $E_{eth} = 7.5 \text{ MeV}$ (data points III, IV, V).

The standard solar model (SSM) prediction for the KII event rate (N_{SSM}) assumes all neutrinos to be left handed and to possess no electromagnetic properties so

$$N_{SSM} = \int e(T) dT \int f(E_\nu) \frac{d\sigma_W}{dT} dE_\nu \quad (10)$$

On the other hand the measured rate is the product of this quantity by the ratio R_{KII} (table I). Then if we consider that the discrepancy between the SSM prediction and the observed rate is due to neutrino electromagnetic properties, equating N_{KII} (6) to the measured rate,

$$N_{KII} = R_{KII} \int e(T) dT \int f(E_\nu) \frac{d\sigma_W}{dT} dE_\nu \quad (10a),$$

we obtain a relationship between the possible neutrino dipole moments and the charge radius. All integration limits are as in (6), (8) and (9). Introducing the notation

$$A_W = \int e(T) dT \int \frac{d\sigma_W}{dT} f(E_\nu) dE_\nu; \quad (f_{2\nu}^2 + g_{2\nu}^2) B_{1EM} = \int e(T) dT \int f(E_\nu) \frac{d\sigma_{-EM}}{dT} dE_\nu; \\ < r^2 > A_{int} = \int e(T) dT \int \frac{d\sigma_{int}}{dT} f(E_\nu) dE_\nu; \quad < r^2 >^2 B_{2EM} = \int e(T) dT \int f(E_\nu) \frac{d\sigma_{+EM}}{dT} dE_\nu, \quad (11)$$

using (5) and the kinematical relation $q^2 = -2m_e T$, we get

$$f_{2\nu}^2 + g_{2\nu}^2 = \left[(R_{KII} - R_D) - R_D < r^2 > \frac{A_{int}}{A_W} \right] \frac{A_W}{B_{1EM}} - < r^2 >^2 \frac{B_{2EM}}{B_{1EM}} \quad (12)$$

This is the basic equation for our analysis on the charge radius and dipole moment parameter space. The current experimental limits on these quantities are, for the charge radius (90% C.L.)

$$-7.13 \times 10^{-11} \text{ MeV}^{-2} \leq < r^2 > \leq 1.27 \times 10^{-10} \text{ MeV}^{-2} \quad (13)$$

from a recent neutrino electron scattering experiment,¹⁵⁾ and

$$\mu_\nu \leq 4 \times 10^{-10} \mu_B \quad (13a)$$

for the magnetic dipole moment.¹⁶⁾ (This appears to actually be a limit on $\sqrt{\mu_\nu^2 + d_\nu^2}$, but it also follows that it is a limit on μ_ν). In the following we first calculate (12) with a vanishing charge radius and next treat $\langle r^2 \rangle$ as a free parameter. Requiring (12) to be positive, upper and lower bounds will be derived for $\langle r^2 \rangle$ and confronted with the experimental ones.

For the numerical estimates we consider separately $E_{eth} = 9.3 MeV$ (data points (I, II)) and $E_{eth} = 7.5 MeV$ (III, IV, V).

(i) $E_{eth} = 9.3 MeV$. This corresponds to the initial period of 450 days from January 1987 to May 1988. The electron threshold kinetic energy is $T_{th} = 9.3 MeV - m_e = 8.79 MeV$. The results obtained for the integrals (11) are

$$\begin{aligned} A_W &= 2.8 \times 10^{-23} MeV^{-1} & B_{2EM} &= 4.0 \times 10^{-4} MeV^{-1} \\ A_{int} &= 2.1 \times 10^{-13} MeV^{-1} & B_{1EM} &= 1.0 \times 10^{-5} MeV^{-1} \end{aligned} \quad (14)$$

which, after using (12) and table I, give

$$\begin{aligned} \left(f_{2\nu}^2 + g_{2\nu}^2 \right)_I^{1/2} &= (4.4 \pm 4.8) \times 10^{-10} \\ \left(f_{2\nu}^2 + g_{2\nu}^2 \right)_{II}^{1/2} &= (2.0 \pm 12) \times 10^{-10} \end{aligned} \quad (15)$$

in units of μ_B and up to 1σ deviation.

(ii) $E_{eth} = 7.5 MeV$. This corresponds to the 590-day period from June 1988 to April 1990, after an improvement in the detector performance.^{2),10)} The integrals (11) are slightly larger, as one is considering a larger energy span, ($T_{th} = 7.5 MeV - m_e = 6.99 MeV$);

$$\begin{aligned} A_W &= 7.1 \times 10^{-23} MeV^{-1} & B_{2EM} &= 1.0 \times 10^{-3} MeV^{-1} \\ A_{int} &= 5.4 \times 10^{-13} MeV^{-1} & B_{1EM} &= 4.2 \times 10^{-5} MeV^{-1} \end{aligned} \quad (16)$$

Similarly one obtains

$$\begin{aligned} \left(f_{2\nu}^2 + g_{2\nu}^2 \right)_{IV}^{1/2} &= (3.6 \pm 3.6) \times 10^{-10} \\ \left(f_{2\nu}^2 + g_{2\nu}^2 \right)_V^{1/2} &= (5.7 \pm 2.1) \times 10^{-10} \end{aligned} \quad (17)$$

in units of μ_B . Regarding point III, where the Chlorine result is rather low, there is obviously a large discrepancy between the two experiments. Such a discrepancy is not seen for any other data point. It coincides with the fact that in the corresponding runs of the Chlorine experiment the characteristic half-life of the excited Argon decaying into the measured electron is not observed, nor is the characteristic $2.8 keV$ peak in the electron energy.¹⁷⁾ Such

a puzzling feature casts doubts on the reliability of this data point and in general we will discard it. For reference we note that the corresponding dipole moment calculated from (12) (central point) would be:

$$(f_{2\nu}^2 + g_{2\nu}^2)_{III}^{1/2} = 8.4 \times 10^{-10}$$

at $\langle r^2 \rangle = 0$. This is far larger than the experimental upper limit on the neutrino magnetic dipole moment.¹⁶⁾

It is more instructive to examine the last two data points with recoil electron threshold energy $E_{eth} = 9.3 \text{ MeV}$, also reported by Kamiokande II (table I). The numerical values of the integrals are the same as in (14) and we obtain up to 1σ deviation for points IVa, Va

$$\begin{aligned} \left(f_{2\nu}^2 + g_{2\nu}^2\right)_{IVa}^{1/2} &= (1.2 \pm 1.3) \times 10^{-10} \\ \left(f_{2\nu}^2 + g_{2\nu}^2\right)_{Va}^{1/2} &= (6.1 \pm 2.3) \times 10^{-10} \end{aligned} \quad (18)$$

in units of μ_B . All values (15), (17), (18) correspond to $\langle r^2 \rangle = 0$. Taking their weighted average and 1σ deviation, then

$$\sqrt{\mu_\nu^2 + d_\nu^2} = (5.4 \pm 1.4) \times 10^{-10} \mu_B \quad (19)$$

for zero neutrino charge radius. This is to be directly confronted with the upper bound for μ_ν (13a). The limits are compatible, but just barely. Cosmological limits¹⁸ on d_ν do not resolve the ambiguity between measuring μ_ν and d_ν , so we have chosen to report the observable combination $\sqrt{\mu_\nu^2 + d_\nu^2}$ in (18).

An alternative way to obtain an estimation of μ_ν at $\langle r^2 \rangle = 0$ is to use the mean values of R_{KII}, R_D in eq. (12). Averaging over all points of table I with the exception, as before, of point III in the Davis data we get

$$R_{KII} = 0.390 \pm 0.006, \quad R_D = 0.341 \pm 0.051 \quad (20)$$

and, with 1σ deviation,

$$\sqrt{\mu_\nu^2 + d_\nu^2} = (3.7 \pm 1.9) \times 10^{-10} \mu_B \quad (21).$$

If one takes instead only the KII data points corresponding to the 9.3 MeV energy threshold (that is I, II, IIIa, IVa, Va) the values (20), (21) remain unchanged up to three decimal places. This is because the common points IIIa, IVa, Va, bearing the smallest error bars, have the largest weights. We note that the 1σ intervals (19) and (21) have a large intersection range.

This is consistent with the results from ref. 9. Probably less reliable is the result one gets by considering instead points I, II, III, IV, V for Kamiokande as these have the largest error bars: $R_{KII} = 0.447$ and $\sqrt{\mu_\nu^2 + d_\nu^2} = (5.4 \pm 1.8) \times 10^{-10} \mu_B$.

As mentioned above we next let $\langle r^2 \rangle$ be a free parameter and investigate the parameter space $(\langle r^2 \rangle, \sqrt{\mu_\nu^2 + d_\nu^2})$ on the basis of eq. (12). We perform the analysis for the 9.3 MeV energy threshold, the case for which more data points are available, and also the more accurate ones are included. The values of the integrals are given in (14) and with R_{KII}, R_D given by the central values in (20) one obtains the dashed curve shown in Fig. 1. Letting in eq. (12) R_{KII}, R_D cover their whole parameter ranges (20) leads to the shaded area represented in Fig. 1 in the plane $(\langle r^2 \rangle, \sqrt{\mu_\nu^2 + d_\nu^2})$. It is seen that there are upper and lower limits for the charged radius corresponding to a vanishing dipole moment. In fact for $\sqrt{\mu_\nu^2 + d_\nu^2} = 0$, either

$$-2.8 \times 10^{-12} \text{MeV}^{-2} \leq \langle r^2 \rangle \leq 3.8 \times 10^{-11} \text{MeV}^{-2} \quad (22a)$$

or

$$-2.1 \times 10^{-10} \text{MeV}^{-2} \leq \langle r^2 \rangle \leq -1.9 \times 10^{-10} \text{MeV}^{-2} \quad (22b)$$

Range (22a) contains the point where both parameters vanish. The upper limit in (22a) is also an upper bound on the charge radius. Up to 2σ deviations it becomes $5.9 \times 10^{-11} \text{MeV}^{-2}$, an improvement with respect to the experimental one by a factor 2–2.5 (see eq. (13)). Range (22b) is clearly excluded by the experimental constraint (13). The central value in (22) is at

$$\langle r^2 \rangle = 1.7 \times 10^{-11} \text{MeV}^{-2} \quad (23)$$

The Kamiokande and Davis data are therefore compatible within the error bars with a vanishing neutrino charge radius and dipole moment. On the other hand, even the coincidence of the two sets of data ($R_{KII} = R_D$) would not exclude $\langle r^2 \rangle, \sqrt{\mu_\nu^2 + d_\nu^2} \neq 0$ as can be seen from eq. (12).

We remark that these results are based on integrating over $d\sigma/dT$, which gives the best limits due to the limited statistics. In principle the energy distribution itself also contains useful information. In particular, the electromagnetic contribution (4c) has a $1/T$ dependence which would cause the energy distribution to stand out significantly at small enough T . We studied this to see if it could be seen in the KII data. Unfortunately the region of T to produce a sizable effect ($T \lesssim \text{few MeV}$) is not yet available from experiments, making a “smoking-gun” signal of the magnetic moment impractical at present.

To conclude this section, we examine the sensitivity of these results with respect to the theoretical solar model. It is well known³⁾ that the neutrino energy flux distribution $f(E_\nu)$ is practically the same in any solar model, up to an overall normal normalization. That is, it is the total number of neutrinos that is model dependent. So choosing another solar model amounts to changing R_{KII} and R_D in eq. (12). Considering for instance a 60% change in both, one can re-do the analysis with

$$R'_{KII} = 0.62 \quad , \quad R'_D = 0.55 \quad . \quad (24)$$

(For instance in the model of Turck-Chièze the KII reading is about 0.7).

In Table II we show the result $(\langle r^2 \rangle, \sqrt{\mu_\nu^2 + d_\nu^2})$ without error bars. The main difference with respect to the prediction obtained from Bahcall's model is that the vertex of the curve raises by 50% and is slightly shifted to the left of the plane. The sharpest conclusion from the analysis, namely the upper bound (22a) on $\langle r^2 \rangle$, is practically unchanged. As a general conclusion, one may say that the results of the present analysis are hardly sensitive to the solar model normalization.

3. Outlook

We have derived the constraints between the neutrino magnetic moment and its charge radius following from a comparison of Kamiokande II and Davis data on solar neutrinos. The possible range of the two parameters is represented by the shaded area of Fig. 1. For a vanishing dipole moment the charge radius is maximized. The 1σ interval for the charge radius at vanishing dipole moment is $(1.7 \pm 2.1) \times 10^{-10} MeV^{-2}$, thus corresponding to an upper bound of $3.8 \times 10^{-11} MeV^{-2}$ for $\langle r^2 \rangle$ (68% C.L.). Hence the discrepancy between Kamiokande and Davis data is compatible with a simultaneously vanishing charge radius and dipole moment. After all this discrepancy is within a 1σ error only (see eq. (20)). Of course, in such a case the neutrino deficit would remain unexplained and one must resort, for instance, to neutrino oscillations.²⁰⁾ On the other hand, using the lower experimental bound on the charge radius, $\langle r^2 \rangle \geq -7.13 \times 10^{-10} MeV^{-2}$, one can maximize $\sqrt{\mu_\nu^2 + d_\nu^2}$, obtaining $\sqrt{\mu_\nu^2 + d_\nu^2} = (6.7 \pm 0.4) \times 10^{-10} \mu_B$. We emphasize that this interval for $\sqrt{\mu_\nu^2 + d_\nu^2}$ corresponds to a finite charge radius close to its lower experimental bound and should not be directly compared to (13a). The results presented were derived using Bahcall's theoretical solar model.³⁾ Their model dependence is quite mild and does not affect the range close to $\sqrt{\mu_\nu^2 + d_\nu^2} = 0$ as can be seen from comparing table II with Fig. 1.

So far the Kamiokande II measurements have always been larger than the Davis ones and if this relative order of magnitude changes in future data, the neutrino charge radius is, from eq. (12), implied to be negative. It was shown in this paper that a comparison of the two experiments leads solely to a constraint between $\langle r^2 \rangle$ and $\sqrt{\mu_\nu^2 + d_\nu^2}$. Further information can be obtained from calculations of the conversion probability in the Sun²¹⁾ and comparing this to the experimental prediction. However, the main difficulties here are related to the uncertainties in the solar magnetic field and the existence or non-existence of oscillations. Laboratory searches for evidence of neutrino electromagnetic interactions are therefore in themselves sufficiently important to deserve effort from experimenters. Over most of the interesting range discussed here, such interactions would indicate new physics to be investigated beyond the minimal standard model.

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Data/SSM	Kamiokande II		Davis
	$E_e \geq 9.3 MeV$	$E_e \geq 7.5 MeV$	
I	0.405 ± 0.13	—	0.335 ± 0.085
II	0.53 ± 0.13	—	0.515 ± 0.115
III	0.46 ± 0.01 (a)	0.505 ± 0.105	0.085 ± 0.085
IV	0.33 ± 0.01 (a)	0.40 ± 0.10	0.325 ± 0.115
V	0.365 ± 0.015 (a)	0.42 ± 0.10	0.23 ± 0.10

Table I – The ratio of data/SSM prediction for the Kamiokande II (R_{KII}) and Davis (R_D) experiments during the period January 1987 to April 1990 referred to the same time intervals (ref. 10). Kamiokande II data points III, IV and V contain two readings: III (a), IV (a) and V (a) refer to the previous recoil electron threshold energy of 9.3 MeV used in the first three points, while III, IV and V refer to the improved threshold of 7.5 MeV.

$\langle r^2 \rangle (MeV^{-2})$	$\sqrt{\mu_\nu^2 + d_\nu^2}(\mu_B)$
1.784×10^{-11}	7.20×10^{-12}
0	4.60×10^{-10}
-1.40×10^{10}	1.01×10^{-9}
-3.038×10^{-10}	3.03×10^{-11}

Table II – Recalculated values of the neutrino charge radius $\langle r^2 \rangle$ in units of MeV^{-2} and the observable combination of electric (d_ν) and magnetic (μ_ν) dipole moments in units of μ_B . Values have been calculated using the ratios R'_{KII} and R'_D as in eq. (24) which correspond to a 60% increase relative to R_{KII} and R_D in Bahcall's model. The values in this table may be compared to the dashed curve of Fig. 1.

Figure Caption

Fig. 1 – The shaded area represents the parameter range $(\langle r^2 \rangle, \sqrt{\mu_\nu^2 + d_\nu^2})$ compatible with the data from Kamiokande II and Homestake experiments including the error bars. The dashed curve corresponds to the central values. Units are in $10^{-10} \text{ MeV}^{-2}$ for $\langle r^2 \rangle$ and $10^{-10} \mu_B$ for $\sqrt{\mu_\nu^2 + d_\nu^2}$.

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$$1 - \frac{1}{2} \mu^2$$