Implications of LEP on Laboratory Searches for Dark Matter Neutralinos

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Abstract

We discuss how the parameter space of the Minimal Supersymmetric extension of the Standard Model could be probed by future searches for neutralinos with laboratory Dark Matter detectors of a given level of sensitivity. We take into account all the constraints already imposed on model parameters by negative searches for supersymmetry at LEPI, restricting our attention to neutralinos lighter than the W -boson. We find that at a level of sensitivity of $0.1 \text{ (kg}\cdot\text{day)}^{-1}$ targets of heavy nuclei would probe much more of the remaining parameter space than would targets of light nuclei. However, at a level of sensitivity of $0.01 \text{ (kg}\cdot\text{day)}^{-1}$, a target of heavy nuclei combined with a target of light nuclei such as ^{19}F would allow exploration of more parameter space than either one alone. We then consider the impact that a negative search for supersymmetry at LEPII would have on these conclusions. We find that when all the constraints that such searches would imply are imposed on model parameters, at a level of sensitivity of $0.1 \text{ (kg}\cdot\text{day)}^{-1}$ even targets of heavy nuclei would probe only a small fraction of the allowed parameter space, and at a level of sensitivity of $0.01 \text{ (kg}\cdot\text{day)}^{-1}$ a target of heavy nuclei would allow exploration of much more parameter space than a target of light nuclei. Our conclusions are generic to the extent that the large variety of nuclei that we consider cover the range of possibilities.

Experiments at LEP have provided some of the most restrictive constraints thus far on possible new physics beyond the Standard Model. In particular, they have significantly constrained the parameter space of the Minimal Supersymmetric extension of the Standard Model (MSSM), thus restricting the parameter space where the spin- $\frac{1}{2}$ lightest supersymmetric particle (LSP) can be a solution to the Dark Matter problem. Weakly-interacting Dark Matter particles such as this LSP can in principle be detected via their elastic scattering off massive nuclei [1], and with many groups now planning searches for Cold Dark Matter particles based on a variety of target nuclei (such as ^{73}Ge + ^{76}Ge , GaAs, NaI, CaF_2 , C_6F_6 , Xe, ^{29}Si , Nb, and Sapphire), it is of interest to analyze the implications of such restrictions for the rate of elastic scattering to be expected in these kinds of detectors. In this paper we evaluate the potential of such searches to detect or exclude this LSP as the main component of the halo of our galaxy, finding the parameter space allowed by LEP constraints where the scattering rate of the LSP in a large variety of nuclei could exceed a given level of sensitivity.

Recent analyses of astronomical data on the distribution and motion of galaxies on very large scales [2] indicate that the universe must contain a very large amount of Dark Matter, $\Omega \equiv \rho/\rho_{crit} = O(1)$, and are incompatible with a flat universe of large cosmological constant. In the absence of a cosmological constant, precisely a critical density $\Omega = 1$ is expected on the basis of naturalness arguments and specifically predicted by inflationary models of cosmology, and it is entirely consistent with the recent COBE observations of structure in the microwave background radiation [3]. Since the amount of visible matter in the universe is small, and primordial nucleosynthesis constraints [4] restrict the amount of

baryonic matter in the universe to no more than 10% of the critical density, we shall assume here a critical density of Dark Matter consisting of relic LSP's. Moreover, although it is compatible with these constraints that the Dark Matter in galactic halos be unseen baryonic matter, in the absence of theoretical motivation or observational indication that this could be the case, it is natural to hypothesize that this Dark Matter is also non-baryonic, and hence in our study that it also consist of relic LSP's.

The pure Cold Dark Matter model we assume here, while consistent with the COBE data by themselves, is not consistent with observations on the structure of the Universe on other scales. The combined data are consistent, however, for example with a Universe containing mainly Cold Dark Matter and an admixture of Hot Dark Matter, with $\Omega_{cold} \approx 0.7$ and $\Omega_{hot} \approx 0.3$ [5]. Although very tentative, this can be taken as an indication of the accuracy to which the pure Cold Dark Matter model we assume here might be reliable. Thus, we also discuss below how the elastic scattering rates we use would change in such mixed Dark Matter models.

We use the full composition of the LSP in the MSSM, expected to be some combination of electromagnetically-neutral and non-strongly-interacting gauginos and higgsinos, commonly called a neutralino. We choose the masses of the supersymmetric partners of the quarks and leptons, the sfermions, in such a way that the relic density of the LSP is critical, $\Omega_{LSP} = 1$ [6]. We restrict our attention to neutralinos lighter than the W-boson, since interaction rates are largest for smaller neutralino mass, and the many new annihilation channels that open up above the W-mass complicate the relic density calculation that fixes the sfermion masses. We incorporate the supersymmetric Grand Unified Theory (GUT) relations between the

sfermion and gaugino masses, and work with the full one-loop radiatively-corrected MSSM Higgs mass matrix and coupling mixing angle. To estimate the nucleonic matrix elements of the effective four-fermion LSP-quark interaction operator, we use the latest EMC estimates for the spin-dependent parts, and the standard sigma-term estimates for the spin-independent parts. To estimate the nuclear matrix elements, we use the odd group model, which has been shown in the past to agree well with more detailed shell model calculations wherever they are available. Finally, we model the mass and spin form factors similarly to the way we have discussed in ref. [7].

We find that when we incorporate all constraints from LEP experiments, a search with sensitivity down to $0.1 \text{ (kg-day)}^{-1}$ would probe much more of the remaining parameter space if the detector target were made of heavy nuclei. However, with sensitivity down to $0.01 \text{ (kg-day)}^{-1}$, a target of heavy nuclei combined with a light nucleus of large spin-dependent cross section, such as ^{19}F , would allow exploration of more parameter space than either one alone. We then consider the possibility that future searches for supersymmetry at the next phase of the LEP collider, LEP-II, could further restrict the parameter space of the MSSM. We find that when all the constraints that such searches would imply are imposed on the model, at a level of sensitivity of $0.1 \text{ (kg-day)}^{-1}$ even detectors with targets of heavy nuclei would probe only a small fraction of the allowed parameter space. At a level of sensitivity of $0.01 \text{ (kg-day)}^{-1}$, a target of heavy nuclei would allow exploration of much more parameter space than would a target of light nuclei. We have attempted to cover the range of possibilities by considering a large variety of

target nuclei, and choose a few nuclei to illustrate the main conclusions of our search.

There are four neutralinos in the MSSM that are mixtures of the standard gauginos ($\tilde{W}_3, \tilde{B}$) and Higgsinos ($\tilde{H}_1, \tilde{H}_2$). We have recently discussed the computation of elastic scattering rates of the LSP neutralino in this model [8], and here we just briefly summarize the main ingredients for notational purposes. The neutralino mixing matrix is characterized by the ratio of the hypercharge +1 to the hypercharge -1 Higgs vacuum expectation values $v_1/v_2 \equiv \tan\beta$, a superpotential $H_1 H_2$ mixing coefficient μ , and SU(2) and U(1) gaugino masses M_2 and M_1 respectively. We assume grand unification relations for the gaugino masses, $M_1 = \frac{5}{3}\tan^2\theta_W M_2$, and for the squark and slepton masses assuming a common sfermion mass m_ϕ at the unification scale. We include sfermion mixing for the stop and sbottom, characterized by μ and soft-supersymmetry-breaking parameters A_t and A_b respectively. We do not include sfermion mixing for the partners of the lightest quarks, which might be thought potentially important because it can lead to spin-independent interactions that are present even if the LSP is a pure gaugino [9], but we find it to have a negligible effect on the elastic scattering rates in the parameter space we survey [8]. An extra parameter is required to specify the Higgs spectrum and mixing angle α_H at the one-loop level, in addition to the top quark mass m_t ; the mass of the pseudoscalar Higgs boson, m_A . The one-loop Higgs mass matrix [10] is then specified by $\tan\beta, m_A, \mu, A_t, A_b$, and the top quark mass m_t . Our results are not very sensitive to the choice of parameters A_t and A_b . Current constraints [11] place the value of m_t in the range $90\text{--}200 \text{ GeV}$, and we choose $m_t = 140 \text{ GeV}$ as a representative value.

The principal experimental constraints on the parameters μ , M_2 and $\tan\beta$ entering the neutralino mass matrix are: (1) $M_{\text{chargino}} > 45 \text{ GeV}$ (LEP) [12]; (2) an upper limit on non-standard contributions to the Z decay width [13], which we implement assuming that the processes $Z \rightarrow \chi\chi'$ and $Z \rightarrow \chi'\chi'$ contribute to the hadronic decay width, while only $Z \rightarrow \chi\chi$ contributes to the invisible width (here χ' is a generic heavier neutralino, χ the LSP neutralino, and we note that the resulting bounds are independent of any assumption on the chargino and neutralino decay modes); (3) an upper limit on the branching ratio for $Z \rightarrow \chi\chi'$ and $Z \rightarrow \chi'\chi'$ decays [14] which, for simplicity, we implement assuming that $\chi' \rightarrow \chi\gamma$ predominantly since the results are not sensitive to this choice; (4) a lower bound $M_{\text{gluino}} > 70 \text{ GeV}$ (UA2 and CDF) [15]. In Figure 2 we show the envelope of these constraints in the (μ, M_2) plane for two representative choices $\tan\beta = 2$ and 8. There are in addition constraints on the parameters $\tan\beta$ and m_A that enter the one-loop scalar Higgs mass matrix set by the unsuccessful searches at LEP for the pseudoscalar (A) and lightest scalar (h) Higgs bosons in the decays $Z \rightarrow Z^* h$ and $Z \rightarrow hA$ [16]. We implement these constraints, which depend on the sfermion masses and μ via α_t , by finding the sfermion mass scale m_0 that gives $\Omega_{LSP} = 1$, and then finding whether α_t is allowed by the LEP constraints.

The rate of neutralino-nucleus elastic scattering, per unit mass of target material and for 100% isotopic abundance, is given by:

$$R = (R_{S.D.} + R_{S.I.}) \left(\frac{4 M_\chi M_N}{(M_\chi + M_N)^2} \right) \left(\frac{\rho}{.3 \text{ GeV} \cdot \text{cm}^{-3}} \right) \left(\frac{\langle \vec{v}_E \rangle}{320 \text{ km} \cdot \text{s}^{-1}} \right) \frac{\text{events}}{\text{kg} \cdot \text{day}} \quad (1)$$

where ρ is the density of dark matter in the solar neighborhood and $\langle \vec{v}_E \rangle$ is the average velocity with which the particles hit the detector. Here

$$R_{S.D.} = 580 \lambda^2 J(J+1) \zeta(r_{\text{spin}}) \left(10^4 \text{ GeV}^2 \sum_{q \in p(n)} (B - A) \Delta q \right)^2; \quad (1a)$$

$$R_{S.I.} = 210 M_N^2 M_Z^4 \zeta(r_{\text{charge}}) \left(\hat{f} \frac{m_u C_u + m_d C_d}{m_u + m_d} + f C_s + \frac{2}{27} (1 - f - \hat{f})(C_c + C_b + C_t) \right)^2. \quad (1b)$$

Explicit expressions for the various quantities appearing in (1), as well as references to other relevant work, are given in ref. [8]. In the Table we list a variety of nuclei of interest in Dark Matter searches and give squared expectation values of the nucleons' total spin $\langle N | \sum_{p \in n \in N} \vec{S}_{p(n)/N} = \lambda \langle N | \vec{J}/N \rangle$ in the nucleus $|N\rangle$, where $\vec{J}$ is the total nuclear angular momentum. We use the odd-group model [17] to estimate $\lambda^2 J(J+1)$ since it gives values generally in good agreement with more reliable estimates [17,18] available for some of the nuclei considered here, except for ^7Li , ^9Be and ^{127}I for which we list numerical estimates taken from [18]. The factor ζ corrects for the finite size of the nuclear mass [19] and spin [7,20] distributions, and depends on r_{mass} (r_{spin}), the rms mass (spin) radius. We approximate $r_{\text{mass}} \approx r_{\text{charge}}$, and take the rms charge radius from a fit to the data [21]: $r_{\text{charge}} = (0.30 + 0.89 A^{1/3}) \text{ fm}$. The spin-dependent cross section is corrected by $\zeta(r_{\text{spin}}) = \zeta(r_{\text{charge}}) \chi(r_{\text{spin}}/r_{\text{charge}})$, and we calculate the ratio $r_{\text{spin}}/r_{\text{charge}}$ using a harmonic well potential [8]. In the Table we list values of $r_{\text{spin}}/r_{\text{charge}}$.

We can now evaluate the elastic scattering rates for various nuclei and determine the regions of parameter space that a detector could probe at a given level of sensitivity. In order to evaluate the rate (1) we choose a scalar mass parameter m_ϕ such that the relic density of neutralinos is critical for a present-day Hubble expansion rate of $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We calculate the relic density for a general neutralino composition and include all channels kinematically allowed for $M_\chi \leq M_W$. We have recently studied in [8] the case of an experiment now being prepared that will use two equal-mass targets of pure ^{73}Ge ($J = \frac{9}{2}$) and ^{76}Ge ($J = 0$) [22], and found that such a detector could probe large regions of the (μ, M_2) parameter space where $\Omega_{\text{LSP}} = 1$ is possible for low values of m_A . Here we shall study detectors of other target nuclei, and in particular whether they could probe larger or different regions of this parameter space. We then discuss the chances of detecting Dark Matter neutralinos with such detectors should future searches for supersymmetry at the LEP-II collider further restrict the parameter space of the MSSM model.

The choice of target materials for cryogenic detectors is largely determined by practical and technological considerations [1]. However, in order to assess the chances of detecting Dark Matter neutralinos one may simply look at the nuclei that can maximize the rate in equation (1). The spin-independent contribution (1b) is proportional to the square of the nuclear mass M_N , and is maximized by choosing a target nucleus as heavy as possible. The spin-dependent contribution (1a) is proportional to the squared spin matrix element, and is not a simple function of M_N . In Figure 1 we show a spin figure of merit for the nuclei in the Table, $\frac{4}{3} \chi^2 J(J+1) \zeta(r_{\text{spin}}) \frac{M_Y M_N}{(M_X + M_N)^2}$. Note that we have chosen the figure of merit not to

include an isotopic abundance factor, as it may be technologically feasible to enrich any desirable isotope. As can be seen in Figure 1, the spin-dependent contribution (1a) is maximized by the light nucleus ^{19}F .

We compare three possible target nuclei in Figure 2, where we present our results for the elastic scattering rate in the (μ, M_2) plane for two representative values of $\tan\beta$ and m_A . We have chosen ^{19}F because of its large spin figure of merit, $^{203}\text{Tl}+^{205}\text{Tl}$ because of its large mass and relatively large spin figure of merit, and in order to compare with a realistic detector we have also presented in Figure 2 the case of a target composed of $^{23}\text{Na}+^{127}\text{I}$ [23]. We have recently [8] calculated rates for the $^{73}\text{Ge}+^{76}\text{Ge}$ detector [22], and we find that the rates in the NaI and Ge detectors would differ by less than 5% either way in regions of the (μ, M_2) parameter space where the elastic scattering rate exceeds $0.01 \text{ (kg-day)}^{-1}$. As can be seen in Figure 2, at a level of sensitivity of $0.1 \text{ (kg-day)}^{-1}$ the ^{19}F target would probe only small regions of the (μ, M_2) parameter space, and these are regions that could be probed with the $^{23}\text{Na}+^{127}\text{I}$ detector operating at the same level of sensitivity. A $^{203}\text{Tl}+^{205}\text{Tl}$ target, on the other hand, would only marginally extend the regions of parameter space that could be probed at this level of sensitivity.

The situation would be quite different, however, if one could build detectors operating at a higher level of sensitivity. It can be seen in Figure 2 that at a level of sensitivity of $0.01 \text{ (kg-day)}^{-1}$ a ^{19}F target would probe regions of the parameter space that could not be probed by the $^{23}\text{Na}+^{127}\text{I}$ detector operating at the same level of sensitivity. This complementarity significantly extends the parameter space that could be probed only for $m_A \gtrsim 100 \text{ GeV}$ and $\tan\beta \approx 1 - 3$. A target of

$^{203}\text{Tl}+^{205}\text{Tl}$, on the other hand, only marginally expands the parameter space that could be probed.

The situation could also be altered should future searches for supersymmetry at the next phase of the LEP collider further restrict the allowed parameter space of the MSSM. In Figure 2 we show with a dashed line the contour where the lightest chargino mass equals the Z-boson mass; regions inside this contour could be excluded since the collider is expected to operate at cms energies of 180–200 GeV. Also, we find here that the searches for Higgs bosons discussed above could exclude values of $\tan\beta \lesssim 3$ and $m_A \lesssim 90$ GeV, and further constrain the values of m_0 and, hence, the (μ, M_2) regions where $\Omega_{\text{LSP}} = 1$ is possible. Note that if values of $\tan\beta \lesssim 3$ were excluded the complementarity of light and heavy nuclei discussed above will not be relevant. We also find that detectors operating at a level of sensitivity of $0.1 \text{ (kg}\cdot\text{day)}^{-1}$ could not probe much of the cosmologically interesting parameter space, as shown in Figure 3. There we plot as a function of $\tan\beta$ and m_A the fraction of the $\Omega_{\text{LSP}} = 1$ parameter space where the elastic scattering rate would exceed $0.1 \text{ (kg}\cdot\text{day)}^{-1}$ and $0.01 \text{ (kg}\cdot\text{day)}^{-1}$ in the $^{23}\text{Na}+^{127}\text{I}$ detector. In the dark region this fraction is 100%, making possible the detection or exclusion of the $M_{\tilde{\chi}} \leq M_W$ LSP as the main constituent of the galactic halo. Thus, sensitivities down to a level of $0.01 \text{ (kg}\cdot\text{day)}^{-1}$ would be required to probe the Dark Matter neutralino parameter space.

We have assumed throughout this work a critical density of neutralinos. As mentioned in the introduction, such a Cold Dark Matter model is consistent with the COBE data alone, but it is disfavored by their combination with observations on the structure of the Universe on smaller scales. A better fit to the combined data, for

example, is a Universe containing mainly Cold Dark Matter and an admixture of Hot Dark Matter, with $\Omega_{\text{cold}} \approx 0.7$ and $\Omega_{\text{hot}} \approx 0.3$ [5]. There are two possible effects on the scattering rates for LSP's in such a "cocktail" model: one is that the local galactic halo density might be shared between the Cold and Hot particles, and the other is that the assumed properties of other particles must be changed in order to reduce the calculated cosmological relic LSP density, which will in general also change the elastic LSP–nucleus scattering cross-section.

Since Hot Dark Matter particles always have velocities at least as large as those of Cold Dark Matter particles, they cannot have a lumpier distribution than the LSP's, and will tend to smooth out at least on small scales due to free streaming. There are therefore two extreme possible scenarios for the galactic halo: one is that the halo formed relatively late and the Hot Dark Matter particles have not been able to accrete, and the other is that it formed relatively early and all Hot Dark Matter particles have accreted. In the former case, the local galactic halo density of LSP's would be $0.3 \text{ GeV}\cdot\text{cm}^{-3}$ as usual, and in the latter case it would be $0.3\Omega_{\text{cold}} \text{ GeV}\cdot\text{cm}^{-3} \approx 0.2 \text{ GeV}\cdot\text{cm}^{-3}$.

Reducing the overall cosmological relic LSP density requires increasing their annihilation cross-section, which in turn entails reducing the masses of the sfermions, for fixed values of the other model parameters. In domains of parameter space where sfermion exchange dominates the elastic LSP–nucleus scattering cross-section, as in the large μ/M_2 limit, this will increase the scattering rate by the inverse of Ω_{cold} . However, if Higgs boson exchange dominates, as for heavy spinless nuclei, there would be no net effect on the elastic scattering rate. There are also intermediate regions of M_2/μ where interference between sfermion and Z

exchanges is important and where reducing the sfermion masses can even reduce the elastic scattering rate. Searching parameter space, we have found that for $\Omega_{cold} = 0.7$ the increase in the scattering cross-section is at most 30%, whilst the scattering cross-section can be reduced by at most 10%.

We conclude on the basis of the previous two paragraphs that if $\Omega_{cold} \approx 0.7$ the elastic scattering rates calculated above could be altered in either direction by up to 30 – 40 % and, hence, would not alter our conclusions about the levels of sensitivity required to search for Dark Matter neutralinos.

References

- [1] See e.g. P. F. Smith and J. D. Lewin, *Phys. Rep.* **187** (1990) 204.
- [2] See A. Dekel, E. Bertchinger, A. Yahil, M. Strauss and M. Davis, *Ap. J.*, in press; and references therein.
- [3] G. F. Smoot *et al.* (1992), to be submitted to *Ap. J.*
- [4] See e.g. K. A. Olive, D. N. Schramm, G. Steigman and T. P. Walker, *Phys. Lett.* **B236** (1990) 454.
- [5] M. Davis, F. J. Summers and D. Schlegel, UC Berkeley preprint CIPA–TH–92–016 (1992).
- [6] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, *Nucl. Phys.* **B238** (1984) 453;
M. Srednicki, R. Watkins and K. A. Olive, *Nucl. Phys.* **B310** (1988) 693;
K. A. Olive and M. Srednicki, *Phys. Lett.* **B230** (1989) 78;
K. Griest, M. Kamionkowski, and M. S. Turner, *Phys. Rev.* **D41** (1990) 3565;
- K. Griest and D. Seckel, *Phys. Rev.* **D43** (1991) 3191;
- J. Lopez, D. V. Nanopoulos and K. Yuan, *Phys. Lett.* **B267** (1992) 219;
Nucl. Phys. **B370** (1992) 445;
- J. McDonald, K. A. Olive and M. Srednicki, *Phys. Lett.* **B283** (1992) 80.
- [7] J. Ellis and R. Flores, *Phys. Lett.* **B263** (1991) 259.
- [8] J. Ellis and R. Flores, CERN preprint CERN–TH–6483/92 (1992).
- [9] M. Srednicki and R. Watkins, *Phys. Lett.* **B225** (1989) 140.
- [10] Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys. Lett.* **85** (1991)1;

Table Caption

Shell Model predictions for $\lambda^2 J(J+1)$ and r_{spin}/r_{charge} for nuclei of interest in dark matter searches. Tables (a) and (b) contain the nuclei which are described by one unpaired proton or neutron respectively. Column 1 identifies the nucleus by chemical symbol and atomic number $A = Z + N$. Column 2 gives the nuclear spin J . Column 3 gives the shell model description of the nucleus. Column 4 contains the measured value of the magnetic dipole moment. Column 5 gives odd-group model predictions for $\lambda^2 J(J+1)$, except for those marked with an asterisk for which numerical estimates taken from [18] are listed. Finally, column 6 gives the value of the ratio of spin and charge rms radii in the harmonic well shell model.

Figure Captions

Figure 1: Spin Figure of Merit, $\frac{4}{3} \lambda^2 J(J+1) \zeta(r_{spin}) \frac{M_Z M_N}{(M_Z + M_N)^2}$, of the nuclei in the

Table for a neutralino mass in the middle of the range of masses discussed here, $M_Z = 20 - 80 \text{ GeV}$. A nucleus with low (high) atomic number moves roughly parallel to the upper (lower) line if M_Z changes.

Figure 2: Elastic scattering rate, eq. (7), in the (μ, M_Z) plane for three different targets: ^{19}F (a,b), $^{23}\text{Na} + ^{127}\text{I}$ (c,d) and $^{203}\text{Tl} + ^{205}\text{Tl}$ (e,f). Here $m_A = 70 \text{ GeV}$ (a,c,e) and 200 GeV (b,d,f), $m_{top} = 140 \text{ GeV}$, and $A_i = A_b = 100 \text{ GeV}$. For the left (right) panels $\mu < 0$ ($\mu > 0$) and for the upper (lower) panels $\tan\beta = 2$ (8). The elastic scattering rate $R(\chi N \rightarrow \chi N)(\text{kg-day}) > 1$ (0.1, 0.01) in the dark (cross-hatched, hatched) regions. The light-shaded region is excluded by experiments at LEP

- J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett. B* **257** (1991) 83;
H. E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.
[11] See e.g. P. Langacker, University of Pennsylvania preprint UPR-0492T (1992).
[12] Particle Data Group, *Phys. Lett. B* **239** (1990) p. IX.5.
[13] ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett. B* **236** (1990) 511;
J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett. B* **237** (1990) 423.
[14] ALEPH Collaboration, CERN preprint CERN-PPE/91-149 (1991).
[15] Particle Data Group, ref. [11], p. IX.8.
[16] ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett. B* **265** (1991) 475.
[17] J. Engel and P. Vogel, *Phys. Rev. D* **40** (1989) 3132.
[18] A. F. Pacheco and D. Strotman, *Phys. Rev. D* **40** (1989) 2131;
F. Iachello, L. Krauss and G. Maino, *Phys. Lett. B* **254** (1991) 220.
[19] A. Gould, *Ap. J.* **321** (1987) 571.
[20] J. Engel, *Phys. Lett. B* **264** (1991) 114.
[21] G. Eder, "Nuclear Forces" (MIT Press, Cambridge, 1968) p. 56.
[22] See e.g. B. Cabrera, talk presented at the IFT Conference on Dark Matter, Gainesville, FL (1992); to be published in the Proceedings.
[23] See e.g. P. F. Smith, review talk presented at the Cosener's House meeting on "Implications of the COBE Data", June 20-21 (1992).

Figure 3: Fraction of the (μ, M_2) parameter space that could be probed with the $^{23}\text{Na}+^{127}\text{I}$ target, as a function of $\tan\beta$ and m_A . A level of sensitivity of $0.1 (0.01 \text{ } \chi^2\text{-day})^{-1}$ and $\mu < 0$ ($\mu > 0$) is assumed in the upper (lower) panel of 3(a) (3(b)). For $M_\chi \leq M_W$, in the dark region the fraction is 100% of the $\Omega_{\text{LSP}} = 1$ parameter space that would remain allowed by negative chargino and Higgs boson searches at LEP II, and in the progressively lighter-shaded regions the fraction is at least 50%, 25% and 12.5%.

Table (a)

Isotope	J	Shell Model	μ_{ex}	$\lambda^2 J(J+1)$	$\frac{r_{\text{spin}}}{r_{\text{charge}}}$
Element		nL_J		odd group	harmonic potential
^1H	1/2	1S _{1/2}	2.793	0.750	1
^7Li	3/2	1P _{3/2}	3.256	0.411*	1.17
^{11}B	3/2	1P _{3/2}	2.689	0.112	1.09
^{15}N	1/2	1P _{1/2}	-0.283	0.087	1.06
^{19}F	1/2	2S _{1/2}	2.629	0.647	1.21
^{23}Na	3/2	1D _{5/2}	2.218	0.041	1.16
^{27}Al	5/2	1D _{5/2}	3.642	0.087	1.13
^{31}P	1/2	2S _{1/2}	1.132	0.057	1.11
^{35}Cl	3/2	1D _{3/2}	0.822	0.036	1.10
^{51}V	7/2	1F _{7/2}	5.151	0.167	1.19
^{55}Mn	5/2	1F _{7/2}	3.450	0.060	1.17
^{59}Co	7/2	1F _{7/2}	4.620	0.077	1.15
^{69}Ga	3/2	2P _{3/2}	2.017	0.021	1.13
^{71}Ga	3/2	2P _{3/2}	2.562	0.089	1.13
^{75}As	3/2	2P _{3/2}	1.439	0.000	1.12
^{79}Br	3/2	2P _{3/2}	2.106	0.029	1.11
^{81}Br	3/2	2P _{3/2}	2.271	0.047	1.11
^{93}Nb	9/2	1G _{9/2}	6.171	0.162	1.20
^{107}Ag	1/2	2P _{1/2}	-0.114	0.054	1.06
^{109}Ag	1/2	2P _{1/2}	-0.131	0.057	1.06
^{121}Sb	5/2	2D _{5/2}	3.363	0.050	1.15
^{123}Sb	7/2	1G _{7/2}	2.550	0.055	1.15
^{127}I	5/2	2D _{5/2}	2.813	0.023*	1.15
^{133}Cs	7/2	1G _{7/2}	2.582	0.052	1.14
^{139}La	7/2	1G _{7/2}	2.783	0.031	1.13
^{191}Ir	3/2	2D _{3/2}	0.146	0.145	1.08
^{193}Ir	3/2	2D _{3/2}	0.160	0.142	1.08
^{203}Tl	1/2	3S _{1/2}	1.622	0.180	1.07
^{205}Tl	1/2	3S _{1/2}	1.638	0.185	1.07
^{209}Bi	9/2	1H _{9/2}	4.200	0.005	1.16

Table (b)

Isotope	J	Shell Model	μ_{ex}	$\lambda^2 J(J+1)$	$\frac{r_{spin}}{r_{charge}}$
Element		nL_J		odd group	harmonic potential
³ He	1/2	1S _{1/2}	-2.128	0.928	1.00
⁹ Be	3/2	1P _{3/2}	-1.178	0.288*	1.12
¹⁷ O	5/2	1D _{5/2}	-1.890	0.342	1.25
²⁹ Si	1/2	2S _{1/2}	-0.555	0.063	1.12
⁴⁷ Ti	5/2	1F _{7/2}	-0.788	0.059	1.20
⁴⁹ Ti	7/2	1F _{7/2}	-1.104	0.107	1.20
⁶⁷ Zn	5/2	1F _{5/2}	0.875	0.073	1.13
⁷³ Ge	9/2	1G _{9/2}	-0.879	0.065	1.25
⁹¹ Zr	5/2	2D _{5/2}	-1.304	0.163	1.21
⁹⁹ Ru	5/2	2D _{5/2}	-0.640	0.039	1.19
¹⁰¹ Ru	5/2	2D _{3/2}	-0.719	0.049	1.19
¹¹¹ Cd	1/2	3S _{1/2}	-0.594	0.072	1.17
¹¹³ Cd	1/2	3S _{1/2}	-0.622	0.079	1.17
¹¹⁵ Sn	1/2	3S _{1/2}	-0.919	0.173	1.16
¹¹⁷ Sn	1/2	3S _{1/2}	-1.001	0.205	1.16
¹²⁹ Xe	1/2	3S _{1/2}	-0.778	0.124	1.14
¹³¹ Xe	3/2	2D _{3/2}	0.692	0.055	1.14
¹⁵⁵ Gd	3/2	3P _{3/2}	-0.260	0.008	1.22
¹⁵⁷ Gd	3/2	3P _{3/2}	-0.340	0.013	1.22
¹⁸³ W	1/2	3P _{1/2}	0.118	0.003	1.19
¹⁹⁹ Hg	1/2	3P _{1/2}	0.506	0.052	1.17
²⁰¹ Hg	3/2	3P _{3/2}	0.560	0.036	1.17
²⁰⁷ Pb	1/2	3P _{1/2}	0.593	0.072	1.17

Figure 1

Figure 2(a)

Figure 2(b)

Figure 2(c)

Figure 2(d)

Figure 2(e)

Figure 2(f)

Figure 3(b)

Figure 3(a)