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Experimental evidences for the box anomaly in η/η' decays and the electric charge of quarks

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Abstract : Motivated by nearly unanimous claim that the observed mass spectrum $\eta' \rightarrow \pi^+\pi^-\gamma$ is not that expected from pure ρ^0 dominance, we reanalyze all the published data on $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays. We find non-resonant contributions. In order to amount precisely the magnitude of these terms, we also refit the $e^+e^- \rightarrow \pi^+\pi^-$ data to get reliably the ρ^0 parameters and decay amplitude. We show that identifying the non-resonant contribution with the box anomaly predicted by QCD and chiral theories, one achieves a consistent picture. Several checks are performed which show the consistency of our fitted parameters. From four measurements, we determine f_1 , f_8 and θ_{PS} on purely experimental grounds, and we find them in the expected range. Presently available data allow for two solutions, one is consistent with QCD, the other favours integral charge quark models. We show that presently available data do not allow to choose among these two solutions. We discuss the interrelation of uncertainties in $e^+e^- \rightarrow \pi^+\pi^-$ physics with our two solutions.

Keywords : Decay, photon-photon interactions, rho-765 resonances, eta mesons, eta-958 resonances, quarks.

Free keywords : ρ parameters, ρ decay width, box anomaly, triangle anomaly, electric charge of quarks.

1 Introduction

Since the original formulation of the quark model and the introduction of colour as an internal degree of freedom, special emphasis has been put on the need to study carefully the η and η' decays into both $\gamma\gamma$ and $\pi^+\pi^-\gamma$ final states, mainly by Chanowitz[1–3]. With these four decays at hand we have in principle four equations to determine experimentally the pseudoscalar decay constants f_1 and f_8 , the singlet–octet mixing angle θ_{PS} and also a parameter named ξ which is equal to 1 if QCD holds. This parameter can be equal to 1 or 2 in standard realizations of integral charge quark models¹. Therefore finding $\xi = 1$ would provide only an educated guess that QCD is the right theory of strong interactions, while finding $\xi = 2$ would definitely discard it.

Another key issue is connected with the anomaly phenomenon in chiral field theories which is believed to explain the $\pi^0 \rightarrow \gamma\gamma$ decay. These anomalies are expected to exist in QCD, even if it seems difficult to estimate their magnitude from within QCD[3]. It is much easier to compute them using the Wess–Zumino lagrangian[4] which allows the recovery of the classical triangle anomaly[5] (see figure (1a)), but also predicts higher order anomalies, for instance the box anomaly (see figure (1b)). Using this lagrangian one can derive the predictions[6][7] $f_\pi/f_8 \simeq 0.80$ and $\theta_{PS} \simeq -19^\circ$, which account for SU(3) symmetry breaking. Notice also that the Gell Mann–Okubo mass formulae give $\theta_{PS} \simeq -10^\circ$ or $\theta_{PS} \simeq -20^\circ$ depending on whether one uses squared masses or simply masses. Thus, by mixing experimental results and theoretical guesses (the value of f_π/f_8 and $\xi = 1$), one is able to achieve a determination of the pseudoscalar nonet parameters. It is therefore interesting to replace these theoretical guesses by the experimental data on $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ and solve the four equations. We can expect in this way to find an experimental answer to the value of ξ , which is a key issue in hadronic physics.

Unfortunately, both decays of η and η' into $\pi^+\pi^-\gamma$ are dominated by ρ^0 production and decay, and the existence of the box anomaly beneath the ρ^0 is not easy to exhibit. To account reliably for this last contribution, one needs to know as precisely as possible what is the ρ^0 distribution and also how to use it.

In a recent report[8], it was argued that there is a large discrepancy between current algebra predictions for the decay $\eta \rightarrow \pi^+\pi^-\gamma$ and the experimental value ; therefore one concluded[8] that these decays could not be used reliably. In fact, this conclusion relies on using the prescription given by Chanowitz[3] to account for the dominance of the ρ^0 meson. To reach another conclusion, one needs to replace this prescription by another motivated procedure. Our ansatz consists in treating the resonance and the box anomaly contributions as independent and in adding the corresponding amplitudes coherently.

Firstly, a summary of what data is available is necessary. Concerning the decay $\eta \rightarrow \pi^+\pi^-\gamma$ the available data are scarce and rather old[9][10] but their statistics make them useful (7250 and 18150 events respectively) and *a priori* precise enough. No more recent data are quoted in the Review of Particle Properties[11]. These two experiments, which published the photon spectrum in the η decay also showed that a simple phase space distribution was unable to account for the general shape of the spectrum ; using some crude shape for the ρ^0 distribution, they also concluded that the ρ^0 dominates the observed photon spectrum.

¹We refer the interested reader to Chanowitz's quoted papers for a detailed analysis.

Data for $\eta' \rightarrow \pi^+\pi^-\gamma$ decay have been collected over a long period of time. Since 1970 it has been known[12–14] that the $\rho^0 \rightarrow \pi^+\pi^-$ decay dominates the η' ($\pi\pi$)–spectrum. This decay mode has been widely used to tag the η' meson in all experiments[15–22] aiming at an accurate measurement of the decay width $\eta' \rightarrow \gamma\gamma$ in photon–photon collision experiments. As a by-product of this measurement some collaborations[18–21] have also published a fit to the η' ($\pi\pi$)–spectrum assuming the presence of only the ρ^0 meson to account for it. Recently a collaboration using the Lepton F facility at Serpukhov[23] has produced the $\pi^+\pi^-$ spectrum observed in the process $\pi^-N \rightarrow \eta'N$ at $p_{lab} \simeq 32$ GeV. Other informations from PLUTO, MARK II, CELLO and ARGUS about the η' ($\pi\pi$)–spectrum have been given within PhD theses generally published only as preprints[24–27]. Data coming from the WA76 collaboration[28] are also now available.

From inspecting all these data, we find that only one[19] or maybe two[14] collaboration(s) find that the shape of the ρ^0 invariant mass distribution in η' decay is the expected one, while the rest claims[18][20–24][26][27] deviations from the expected distribution. The fact that the ρ distribution can differ from what is expected is surprising since the corresponding distribution is fully determined by very well known functions : the $\eta'\rho^0\gamma$ coupling which has been known for a long time[12][13][29]², the ρ^0 Breit–Wigner and its decay amplitude[30][31]. The effects reported in this case are always alike : the ρ^0 peak is found 20 to 30 MeV above the predicted value.

Of course, it is known that the broadness of the ρ^0 meson provides some uncertainties in the analytical shape of the functions describing its propagation and decay, however these effects have been found limited and generally curable[31]. Among the latest fits of the ρ^0 distributions where this meson is found to behave normally let us quote the invariant mass distribution in $\mu^+\mu^-$ [32][33] (which is sensitive to the ρ^0 propagator only), the $\pi^+\pi^-$ invariant mass distribution in J/ψ decays[34] or in muoproduction[35]. It is also known that the ρ^0 shape in photoproduction experiments appears to be skewed[36][37][38] ; this phenomenon is considered as being entirely due to a splitting of the incoming photon into a $\pi^+\pi^-$ pair (Drell amplitude). Such a phenomenon cannot be invoked in the case of η' decay.

Generally, the quoted disagreement is simply mentioned[18][20][24][26]. Only a very few attempts have been tried to understand it : reference [27] suggests a slight shift of the ρ^0 mass and width together with an interference between the ρ^0 and ω mesons ; reference [23] argues convincingly that this interference alone is unable to explain the phenomenon and suggests the existence of a non resonant term which can account for the observed ρ mass shift through its interference with the ρ part of the decay amplitude. To our knowledge, this is the first attempt to introduce (without naming it) the box anomaly[1–3].

The main aim of this paper is to study the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays using the existing experimental data. We plan to show that the box anomaly is present in the available data at the level expected from the two-photon decay widths of the η and η' mesons and from a small SU(3) symmetry breaking.

As a consequence, four main topics have to be addressed. We need :

²These last references quote also A. Rittenberg, UCRL-18863 (1969) which we have not found.

- i/ a good understanding of the ρ^0 physics,
- ii/ to address the physics of anomalies, especially the connection between quantities measured in $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays and the pseudoscalar nonet parameters,
- iii/ to consistently fit all available data on these reactions,
- iv/ to discuss the values found for ξ and the pseudoscalar nonet parameters.

This imposes the outline of the paper.

Because of the ρ^0 dominance in the mass spectra, we first discuss in section 2 rather thoroughly the status of ρ physics and data. We explain why it is better to define the ρ parametrization we shall use in e^+e^- data rather than in hadronic physics. This leads us to refit the corresponding cross section providing this way a new determination of the mass, width and decay amplitude of the ρ^0 meson. We show that there is an ambiguity connected with the possible existence on a non-VDM component to the pion electromagnetic form factor F_π . We thus perform the following analysis with two different parametrizations for the ρ resonance.

In section 3, we review briefly the physics of anomalies and mainly recall the equations of Chanowitz. The effects of the ρ^0 meson are discussed and our way to deal with it is presented. We also give information concerning the expected magnitude of the box anomaly constants for η and η' decays.

Section 4 is devoted to our dealing with the available experimental data. We also discuss here the difficulties encountered with some data sets.

In section 5 we perform our study of the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays. We show that using the two ρ^0 models defined in section 2, we are able to find the corresponding solutions for the box anomaly. Using Chanowitz equations and the experimental data on two-photon decays, we prove in section 6 that our results for the η and η' box anomalies are self consistent. This implies a reliable dealing with systematic errors.

Section 7 is devoted to a brief study of the geometrical properties of the anomaly problem.

In section 8, we compute the Chanowitz parameter ξ using our two ρ models. It comes out from here that one model is consistent with $\xi = 2$ in contradiction with QCD, while the other provides $\xi = 1$ and is therefore consistent with QCD predictions.

Since we are able to single out the ρ^0 contribution in the decay amplitudes $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$, we are also in a position to compare the ρ^0 component in this η decay with that deduced from experimental data on $\rho^0 \rightarrow \eta\gamma$. This comparison allows for another positive cross check of the numerical values coming from our fits. This is the matter of section 9.

In section 10, we finally use the box anomaly constants determined by fitting the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ spectra to achieve the first determination of the pseudoscalar nonet parameters f_1 , f_8 , θ_{PS} on purely experimental grounds.

In section 11, we show that the vector dominance hypothesis (VDM) allows for a new cross check of our fitted values. The test is positive for the model which supports QCD, while it fails by a factor of 2 for the model which contradicts QCD.

Section 12 is devoted to summary and conclusions. In appendix A and B, we display technical details concerning the models used to fit the $e^+e^- \rightarrow \pi^+\pi^-$ cross section and the decay amplitudes $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$. Appendix C is devoted to a brief review of some VDM results of relevance for our problem.

2 The $\rho^0 \rightarrow \pi^+\pi^-$ connection

In order to describe the η and η' decays a parametrization of the ρ^0 meson is required. The usual way to deal with the ρ^0 propagator and decay amplitude is to use theoretical formulae as the ones given by Jackson in ref.[30]. Another way is to use more phenomenologically motivated expressions like that fitted by Pisut and Roos[31] or Chabaud *et al*[39]. However, as we shall argue, the only way by which the ρ^0 meson propagator and decay amplitude can be reliably obtained is by fitting the $e^+e^- \rightarrow \pi^+\pi^-$ data in the range of dipion invariant mass m between $2m_\pi$ and $M_{\eta'}$. Contrary to what is usually believed, we shall show that this procedure is not free from ambiguities.

2.1 Left-hand singularities and ρ^0 decay amplitude

Because the ρ^0 decays nearly 100% to $\pi^+\pi^-$, it is legitimate to treat it as being purely elastic. Let us first consider the decay amplitude $\rho^0 \rightarrow \pi^+\pi^-$, and denote by m the (running) ρ^0 mass, p_1 and p_2 the pion 4-momenta. Let us also define $r = p_1 - p_2$; it can easily be shown that the most general amplitude to describe the decay $\rho^0 \rightarrow \pi^+\pi^-$ can be written :

$$T_0 = G_\rho \varepsilon_\mu^{j*} r^\mu , \quad (1)$$

where ε^j is the ρ^0 polarization vector associated with the helicity state $j = 0, \pm 1$, and G_ρ is an arbitrary function of m . $|G_\rho|^2$ is connected with the ρ^0 width $\Gamma(\rho^0 \rightarrow \pi\pi)$. There exists two ways to perform this connection practically.

First, using expression (1) we can compute the decay width using standard methods[40] and obtain :

$$|G_\rho|^2 = 6\pi \frac{m^2}{q_\pi^3} \Gamma(\rho^0 \rightarrow \pi\pi) , \quad (2)$$

where $q_\pi = \sqrt{m^2 - 4m_\pi^2}/2$ is the pion momentum in the ρ^0 rest frame.

A second way is to compute the amplitude $\pi\pi \rightarrow \rho \rightarrow \pi\pi$ using the coupling (1) and the usual ρ^0 inverse propagator[30] :

$$D(m) = (m_\rho^2 - m^2) - im_\rho \Gamma_\rho(m) , \quad (3)$$

where m_ρ is the (fixed) central mass of the ρ meson. The amplitude T_1 which describes this elastic scattering is :

$$T_1 = \frac{|G_\rho|^2}{D(m)} \sum_{i=1}^3 \varepsilon_\mu^i \varepsilon_\nu^{i*} r^\mu r^\nu = \frac{4|G_\rho|^2 q_\pi^2 \cos \theta}{D(m)} , \quad (4)$$

where θ is the pion decay angle in the ρ^0 rest frame with respect to the ρ^0 line of flight. The corresponding differential cross section is :

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2 m^2 |D(m)|^2} |G_\rho|^4 q_\pi^4 \cos^2 \theta \equiv |a_{l=1}|^2 \cos^2 \theta , \quad (5)$$

where $a_{l=1}$ denotes the P -wave amplitude. Integrating cross section (5), we can compare it with the usual formula[11][30] :

$$\sigma(m) = \frac{4\pi}{3} \frac{1}{(2\pi)^2} \frac{|G_\rho|^4 q_\pi^4}{m^2 |D(m)|^2} \equiv \frac{4\pi}{q_\pi^2} \frac{3m_\rho^2 \Gamma_\rho^2(m)}{|D(m)|^2} . \quad (6)$$

From these equations, we get :

$$|G_\rho|^2 = 6\pi \frac{m m_\rho}{q_\pi^3} \Gamma_\rho(m) . \quad (7)$$

Relations (2) and (7) follow from each other by simply redefining the width. However, because the last expression recalled in relation (6) is the most customarily used in particle physics, we shall prefer definition (7) to definition (2). With this choice, a comparison with refs.[30][31][39] will be more direct.

The partial P -wave amplitude as given in rel. (5) is connected with the partial wave S -matrix element $\exp[2i\delta_l]$ by :

$$a_l = \frac{\exp[2i\delta_l] - 1}{2iq_\pi} .$$

Using rels.(5) and (7), the partial P -wave amplitude of the $\pi^+\pi^-$ elastic process can be rewritten in a more suitable way :

$$a_{l=1} = \frac{1}{q_\pi} \frac{m_\rho \Gamma_\rho(m)}{D(m)} . \quad (8)$$

This expression can be identified with a N/D expression of the partial-wave amplitude[31] which has definite analyticity and unitarity properties³. The function D is nothing but the inverse propagator in expression (3) ; the function N :

$$N = \frac{1}{q_\pi} m_\rho \Gamma_\rho(m) \quad (9)$$

is an analytic function of m^2 in the complex m^2 plane cut from 0 (*i.e.* the crossed threshold) to $-\infty$, while the D function is analytic in the complex m^2 plane cut from $4m_\pi^2$ to $+\infty$ (*i.e.* it has only the unitarity cut). To fulfill this condition, it is clear from relation (9) that $\Gamma_\rho(m)$ must contain a factor carrying an odd positive power of q_π . This power must be 3 in order to keep track of spin conservation. All other dependences tend only to various approximations of the left-hand dynamical singularities of the partial wave amplitude. Therefore, using *a priori* expressions like :

$$\Gamma_\rho(m) = \Gamma_0 \left[\frac{q_\pi(m)}{q_\pi(m_\rho)} \right]^3 \frac{g(m)}{g(m_\rho)} , \quad (10)$$

³There exists a wide literature on this subject which was an important issue in particle physics in the years between 60 and 70 ; the interested reader can refer, for instance, to the book of Chew[41].

with some given $g(m)$ is not harmless⁴ ; it requires some arbitrary assumption about the structure of the left-hand singularities. In most experiments this does not actually matter very much, but in the present case it could well shadow the meaning of our results.

To conclude this section, we stress that the shape of the ρ decay amplitude is connected with the left-hand singularities of the $l = 1$ partial wave amplitude. It is therefore better to fit their influence in the physical region using experimental data, rather than relying on a partial theoretical knowledge of them.

2.2 The ρ^0 meson as measured in hadronic processes

There are very few attempts to perform the fit of the ρ^0 decay amplitude⁵ ; moreover the existing ones[31][39] have all been performed on $\pi N \rightarrow \rho^0 N$ data ; no attempt to perform this fit using e^+e^- data is currently known. We plan first to mention briefly, closely following Pisut and Roos[31], why the ρ^0 decay amplitude obtained in this way cannot be fully trusted and, second, what is the status of the ρ^0 mass and width.

It is also worth noting that all the commonly accepted ρ decay amplitudes and parameters are obtained in πN interactions ; in this case one has either to integrate over the momentum transfer t carried by the virtual pion or to extrapolate to 0 (instead of m_π^2). This kind of considerations are of course absent if the ρ meson is produced in the $e^+e^- \rightarrow \pi^+\pi^-$ process.

One of the main aims in ref.[31] was to exhibit the sharp dependence of the ρ^0 shape upon the assumed production cross section of that resonance. Subsequently, the resonance mass and width show also some dependence⁶. As argued by Pisut and Roos[31], since the production mechanism was (and is still) unknown to a large extent, it is not possible to believe that any among the 27 fits performed by them gives the correct ρ decay amplitude. By “correct decay amplitude”, we mean that it is free of influence from the production cross section.

Moreover, as can be seen from their figure (2), there exist two solutions for the ρ^- mass, one around 765 MeV, one around 778 MeV, which is summarized by their result :

$$m_\rho = 764.3^{+1.9}_{-1.8} \begin{smallmatrix} (19.2) \\ (3.3) \end{smallmatrix} \text{ MeV}$$

and :

$$\Gamma_\rho = 147.3^{+4.0}_{-3.9} \begin{smallmatrix} (8.2) \\ (7.8) \end{smallmatrix} \text{ MeV} ,$$

where the first error is the 1σ level and the second error gives the 2σ level⁷. Referring to the Review of Particle Properties[11], these conclusions do not look superseded by more recent results. Of course, what has to be noticed here is the large positive error on the ρ mass at the 2σ level. It is striking to see that the ρ mass is compatible with the ω mass slightly below the actual 2σ level.

⁴Generally, $g(m)$ is chosen[30] as m^{-1} or $[q_\pi^2(m) + q_\pi^2(m_\rho)]^{-1}$, or more rarely 1.

⁵Most of the considerations reported here have also been applied to charged ρ decay amplitudes[31].

⁶We refer the reader to ref.[31] for details. Let us point out that the discussion here is fully still valid, even if some considerations are somehow dated (mainly the discussion on CDD poles).

⁷These results are those considered as final by Pisut and Roos (relation (51) in reference [31]) ; the reason why the Particle Data Group has instead preferred to quote the result of the fit number 17 is not explicitly given in ref. [11].

Refining the analysis of an experiment on polarized nucleons, ref.[39] recently confirms previous claims[42][43] for a high value of the ρ^0 mass (777.4 ± 2.0 MeV) in agreement with ref.[31] at better than the 2σ level. Because this collaboration was able to extract the partial wave without hypotheses on the ρ^0 production mechanism, this claim can be assigned with a good confidence level. However, their fitted shape of the ρ decay amplitude cannot be excessively trusted for the reasons already quoted, even if the underlying hypotheses are weaker than in ref. [31].

To summarize, we can point out that the ρ^0 decay amplitude obtained in hadronic interaction cannot be considered as completely reliable since its precise shape depends on the production mechanism which is not well under control. Moreover, we have seen that the mass difference between the ρ^0 and ω mesons may not be as large as commonly believed[11].

2.3 The ρ^0 meson measured in e^+e^- experiments

The place where one can expect an accurate parametrization of the ρ decay amplitude is in $e^+e^- \rightarrow \pi^+\pi^-$ data. Indeed, in this case one is able to describe accurately the production process (see ref.[44] for a review of standard models) using the Vector Dominance Model (VDM) hypothesis. Let us notice that vector dominance has been proved to work quite well in particle physics[44][45]. At the beginning, when the amount of data collected was small, the data were well described[46] using a ρ^0 decay amplitude like that given by relation (10) with[30] $g(m) = m^{-1}$ or by means of the Gounaris-Sakurai model[47]. In this model, the leading left-hand singularities are described through a function which exhibits a branch point at $m^2 = 0$. As the statistics have increased it has been claimed that a simple description with only the ρ^0 and ω meson was unable to reproduce satisfactorily the data, even below the ϕ mass ; an opposite claim is made in ref. [50], however the fit of the pion form factor on which they rely is not especially convincing (see their figure (2)). One has been therefore led[48][49][51] to consider higher vector meson contributions such as the $\rho(1450)$ and $\rho(1700)$ [52], or even parametrize the influence of inelastic channels like $\omega\pi$. Let us notice the relatively wide range of ρ masses obtained in e^+e^- annihilation from 769 ± 3 MeV[11] to 778.5 ± 6.2 MeV[47], the latest result being[49] $775.9 \pm 0.8 \pm 0.8$ MeV. All these attempts to perform a fit rely on theoretically motivated models of the ρ decay amplitude and propagators. No attempt is reported to use phenomenologically motivated models as that given in ref.[31] or in ref.[39].

As far as we know, no attempt to fit the ρ decay amplitude has been performed on e^+e^- data ; no attempt is also reported to look for a non-resonant contribution[44][45] in the latest data[48][49]. Such attempts are interesting by themselves in order to describe the e^+e^- data ; they are also relevant since one could then be in a position to compare directly the mass and width of the ρ^0 with what is coming out from hadronic physics experiments. Let us stress that the parameters deduced using the Gounaris-Sakurai model[47] cannot be directly compared with the usual ones obtained by means of conventional Breit-Wigner shapes[30][31][39].

The meaning of a non-resonant contribution to the amplitude can easily be explained.

First of all, it cannot be excluded that such a term would simulate higher mass vector meson contributions ; we shall discuss this possibility later on. On the other hand, we cannot exclude that a non-VDM (hard) component to $F_\pi(s)$ could exist which would be the continuation in the non-perturbative region of the perturbative diagram given in figure (2). It has been computed using perturbative QCD[53][54], and found to behave like[54] :

$$F_\pi(Q^2) \underset{Q^2 \rightarrow \infty}{\sim} \frac{0.45 \text{ GeV}^2}{Q^2} .$$

If such a contribution has to be considered, its value and/or its shape below 1 GeV is unknown, possibly negligible. In what follows we shall work without and with it separately so as to determine its effect.

In appendix A, we display the model we shall use to fit the data published on the $e^+e^- \rightarrow \pi^+\pi^-$ cross section from the two-pion threshold up to 1.0 GeV. Its main aim is to fix notations and expressions consistent with the methods in current use in hadronic physics. We point out here that its main originality is simply to suggest as parametrization of the ρ decay amplitude :

$$\Gamma_\rho(m) = \Gamma_\rho(m_\rho) \left[\frac{q_\pi(m)}{q_\pi(m_\rho)} \right]^3 \left[\frac{m_\rho}{m} \right]^\lambda , \quad (11)$$

where λ is a parameter to be fitted on experimental data. The purpose of the last factor in rel. (11), is simply to allow for a phenomenologically accurate parametrization of the left-hand singularities of the $l = 1$ partial-wave $\pi\pi$ amplitude.

2.4 Two fits to $e^+e^- \rightarrow \pi^+\pi^-$ cross section

In accordance with what has been discussed above, we plan now to describe the fits which can be performed using the model described in appendix A. The data we shall use for this purpose are the cross section measurements of DM1, CMD and OLYA collaborations[48][49]. We have discarded the data set of ref.[46] which is rather unprecise and would have not influenced the fit. Since we are motivated by the η' decay, we have decided to fit the cross section data only below $m_{\pi\pi} = 1.0$ GeV.

In our first fit, we have removed the non-resonant amplitude by setting $A = 0$ (see appendix A). In this case, the best fit is shown in figure (3) ($\chi^2/\text{dof} = 90/78$) and the corresponding parameter values are :

$$\left\{ \begin{array}{l} m_\rho = 780.8 \text{ }^{+0.5}_{-0.4} \text{ MeV} \\ \Gamma_\rho = 153. \pm 2. \text{ MeV} \\ \lambda = 0.659 \pm 0.046 \\ \varphi = 94.3^\circ \pm 2.9^\circ \end{array} \right. \quad (12)$$

The values found for the ρ meson mass and width are now directly comparable with the corresponding numbers found in hadronic physics. They have been obtained using the minimization program MINUIT and the routine MINOS to find the limits of the contour on which $\chi^2 = \chi^2_{\min} + 1$. Using the Gounaris-Sakurai model, the corresponding values quoted in the

literature are[49] : $m_\rho = 775.9 \pm 0.8 \pm 0.8$ MeV and $\Gamma_\rho = 150.5 \pm 1.6 \pm 2.5$ MeV. We can therefore summarize the influence of this model. The ρ and ω are almost degenerate in mass while the width of the ρ is found to agree with the world average value[11]. Our value for the ρ mass is moreover in agreement at the 2σ level with ref.[31] and at the 1σ level with ref.[39]. Finally, using a standard formula[44], we can compute the value of φ expected from the fitted values of the ρ parameters and find :

$$\varphi = 90.9^\circ \pm 0.4^\circ ,$$

in pretty good agreement with the corresponding fitted value. No serious estimate of the systematic errors can follow from simply fitting the data ; however it is likely that those given in reference [49] apply to our results.

Before performing the (second) fit using the non-resonant amplitude, it is worthwhile to discuss the effect of such a term on a distribution. From the previous fit, it is clear that the ρ mass has to be shifted towards higher masses in order to agree with the data. This can be done in two ways : we can directly allow the ρ peak to move upwards and/or produce the requested shift by means of an interference with a non-resonant amplitude (the A term in relation (A10)). We can also combine both procedures. Therefore we are left with an uncomfortable freedom since such a contribution is not fixed in this region, neither in shape nor in magnitude. To limit somehow our freedom of choice, we have first decided to fix the ρ^0 mass at its world average value[11] (768.7 ± 0.7 MeV) ; the idea behind this is the assumption that the mass is possibly less sensitive to parametrizations than the width. Secondly, we have decided to assume that the non-resonant term A (see rel.(A10)) can be described in terms of an expansion of the form :

$$A(m) = -(c_0 + c_2 m^2 + c_4 m^4 + \dots) . \quad (13)$$

The minus sign in front is to connect with the diagram in figure (2), where the relevant charge sign is that of the u quark (*i.e.* opposite to the electron charge). We have finally decided to stop the expansion as soon as the effect of the next term is negligible ; we have thus found that terms of order higher than 2 in the expansion (13) are not necessary. In this case, our best fit is given in figure (4) and corresponds⁸ to $\chi^2/\text{dof} = 61/77$. The corresponding set of fitted parameters is :

$$\left\{ \begin{array}{ll} m_\rho &= 768.7 \pm 0.7 \text{ MeV} \quad (\text{input}) \\ \Gamma_\rho &= 142.8^{+1.5}_{-2.4} \text{ MeV} \\ \lambda &= 1.748 \pm 0.079 \\ \varphi &= 111.4^\circ \pm 3.2^\circ \\ c_0 &= 0.607 \pm 0.058 \\ c_2 &= -0.275 \pm 0.063 \text{ GeV}^{-2} \end{array} \right. \quad (14)$$

⁸Leaving free the ρ mass this second kind of model is able to provide a good fit to the cross section data ($\chi^2/\text{dof} = 80/96$) up to 1.2 GeV. If we omit the non-VDM contribution the fit is not acceptable ($\chi^2/\text{dof} = 202/97$) within the same range. Above 1.2 GeV, higher mass vector mesons surely contribute.

In this case, assuming the official ρ^0 mass, we are left with a ρ width significantly lower than the world average[11] 152.4 ± 1.5 MeV. Notice that the non-resonant cross section is tiny ($\simeq 6nb$) at the ρ mass ; it is the interference term which influences the details of the curve at the ρ peak.

As above, we can also compute the value expected for φ using the new set of ρ^0 parameters and find :

$$\varphi = 101.3^\circ \pm 0.6^\circ \quad .$$

The agreement with the fitted value is not as good as previously, but still acceptable. Finally, notice that the power λ which parametrizes the ρ decay amplitude (see rel. (11)) is sharply different in both cases.

Before discussing these results, let us allow for some remarks. It is expected that $F_\pi(0) = 1$. Continuing expression (13) to $m^2 = 0$ would give 0.607 which contributes to $F_\pi(0)$; we do not worry too much about the possible consequences of any such fixed value since we are using an expansion valid in the range $[2m_\pi, M_{\eta'}]$. Indeed, it is easy to see that a good fit can be performed with :

$$A(m) = \frac{1}{1 + \frac{m^2}{m_\rho^2}} \quad , \quad (15)$$

giving :

$$\left\{ \begin{array}{l} m_\rho = 773.2^{+0.5}_{-0.6} \text{ MeV} \\ \Gamma_\rho = 148.3^{+1.7}_{-1.6} \text{ MeV} \\ \lambda = 1.829 \pm 0.034 \\ \varphi = 102.4^\circ \pm 2.9^\circ \end{array} \right. \quad (16)$$

with a good quality of fit ($\chi^2/\text{dof} = 100/78$). In this case, $A(0)$ would contribute to $F_\pi(0)$ with the value 1 ; this only proves that values obtained using extrapolations cannot be seriously trusted. We shall discard this result – which provides more conventional ρ mass and width – because it has not a clear meaning to require such a shape on the mass interval we are interested in. Moreover, there is no reason to believe that the ρ mass (or another meson mass) would have some relevance in a non-VDM contribution to the electromagnetic pion form factor. One should however notice that the value of λ is compatible with that obtained with fitting using rel.(12). Keeping in mind that the production mechanism is much better under control in e^+e^- annihilation than in hadronic processes, this last fit illustrates the difficulty to define accurately the ρ mass.

Let us now turn to the discussion of the fits shown in figs. (3) and (4) and summarized by rels. (12) and (14). The main property of the second fit is that the non-resonant term is decreasing when m increases ; this is unlikely to simulate the contribution of a single higher mass vector meson since we would instead expect an increasing contribution. It can be possible to imagine a destructive interference between several such states. The natural candidates for

such an effect have been already mentioned as $\rho(1450)$ and $\rho(1700)$; another candidate is possibly located now at 1.302 GeV[55]. The first and last of these candidates have a small coupling to $\pi^+\pi^-$ [49][52][55], therefore the required interference looks rather unlikely. This conclusion is enforced by comparing the values of λ in rel. (12) and (14) : the largest effect of the non-resonant term is between the two pion threshold and the top of the ρ . It is unlikely that interferences among (or with) resonances far away would produce a large effect at such a location.

To summarize, the simple model described in appendix A is able to provide a fairly good description of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section. On statistical grounds, it is not possible to choose seriously among the two versions ($A = 0$ or $A \neq 0$). On the other hand, there are no theoretical grounds for assuming a significant non-VDM component to the pion electromagnetic form factor. Looking at the insets in figures (3) and (4) shows that the differences between the two fits are located on the top of the ρ while the fit of the wings provides indistinguishable curves. One cannot conclude from here that a non-VDM contribution to the electromagnetic form factor is preferred. Indeed, using a slightly more complicated form for the ρ decay amplitude :

$$\Gamma_\rho(m) = \Gamma_\rho(m_\rho) \left[\frac{q_\pi(m)}{q_\pi(m_\rho)} \right]^3 \left[\frac{m_\rho}{m} \right]^\lambda \left[\frac{1 + R^2 q_\pi^2(m)}{1 + R^2 q_\pi^2(m_\rho)} \right] ,$$

allows a better description of the top region of the cross section ($\chi^2/\text{dof} = 75/77$) without the help of a non-VDM amplitude, but the mass and width remain as in rel. (12). It leads to the same physics as rel. (11) with parameters given in rel. (12).

In summary, from our analysis of the e^+e^- data we can conclude that either the ρ^0 mass is nearly degenerate with the ω mass, or there is strong evidence for a non-VDM contribution to the pion electromagnetic form factor in the non-perturbative timelike region.

3 Chiral anomalies and the electric charge of quarks

We recall briefly here the properties of chiral anomalies and their connection with the value of the electric charge carried by quarks. Notice only that the anomaly phenomenon is expected to be general in particle physics, even if the single identified one is the triangle anomaly. We shall closely follow the lines proposed by Chanowitz[3].

3.1 The Chanowitz relations

The triangle anomaly[5] depicted in figure (1a) is considered as responsible for the π^0 , η , η' decays into $\gamma\gamma$. The most general amplitude which describes these decays can be written[1][3] :

$$M(X \rightarrow \gamma\gamma) = B_X(k_1 \cdot k_2) \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon^\rho(\gamma_1) \epsilon^\sigma(\gamma_2) \quad , \quad X = \eta, \eta'. \quad (17)$$

In this expression k_i and $\epsilon^\delta(\gamma_i)$ ($i = 1, 2$) are the 4-momenta and polarization vectors of both photons. At low energy, i.e. at the unphysical point where all 4-momenta vanish, it has been proved that[1][3] :

$$\begin{cases} B_\eta(0) = -\frac{\alpha_{em}}{\pi\sqrt{3}} \left[\frac{\cos \theta_{PS}}{f_8} - 2\sqrt{2} \xi \frac{\sin \theta_{PS}}{f_1} \right] \\ B_{\eta'}(0) = -\frac{\alpha_{em}}{\pi\sqrt{3}} \left[\frac{\sin \theta_{PS}}{f_8} + 2\sqrt{2} \xi \frac{\cos \theta_{PS}}{f_1} \right] \end{cases} \quad (18)$$

The parameter ξ which enters rels.(18) is connected with the electric charge carried by quarks[1-3]. Because there is only one possible (colour singlet) current in QCD, we have $\xi = 1$. If quarks carry integral charges[57], another (colour octet) current also exists. In two-photon decays, products of two currents occur and their singlet component contributes to the decay amplitudes ; if QCD holds there is only one singlet component in this product, but in integral charge quark theories the singlet component of the product of two octet currents also arises. The possible values of ξ follow from here. In some realizations of the integral charge quark model, it is however possible to have $\xi = 1$ [58]. Therefore finding experimentally $\xi = 1$ does not actually prove that QCD is the theory underlying the strong interactions, but finding $\xi = 2$ would exclude QCD in favour of an integral charge quark model.

It is not expected that rels.(18) are spoiled at physical energies[3], therefore from :

$$\Gamma(X \rightarrow \gamma\gamma) = \frac{m_X^3}{64\pi} |B_X(0)|^2, \quad X = \eta, \eta' \quad (19)$$

one can get the absolute value of both matrix elements (18). Using additional theoretical information (the value of the ratio f_π/f_8 deduced from the Wess-Zumino lagrangian[6][7] and assuming $\xi = 1$), one can obtain f_π/f_1 and θ_{PS} . This has been the single procedure used[8][22] and the latest quoted results are[56] :

$$\theta_{PS} = -22.4^\circ \pm 1.5^\circ, \quad f_\pi/f_1 = 0.955 \pm 0.025.$$

On the other hand, it is expected that the box anomaly depicted in figure (1b) contributes to the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays. The most general matrix elements in this case are[1][3] :

$$M(X \rightarrow \pi^+\pi^-\gamma) = E_X(p_+ \cdot k, p_- \cdot k) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu(\gamma) k^\nu p_+^\rho p_-^\sigma, \quad X = \eta, \eta', \quad (20)$$

where $p_\pm$ are the 4-momenta of the outgoing $\pi^\pm$, k and $\epsilon(\gamma)$ are respectively the photon 4-momentum and its polarization vector. At low energies, the functions E_X are given by [3] :

$$\begin{cases} E_\eta(0) = -\frac{e}{4\pi^2\sqrt{3}} \frac{1}{f_\pi^2} \left[\frac{\cos \theta_{PS}}{f_8} - \sqrt{2} \frac{\sin \theta_{PS}}{f_1} \right] \\ E_{\eta'}(0) = -\frac{e}{4\pi^2\sqrt{3}} \frac{1}{f_\pi^2} \left[\frac{\sin \theta_{PS}}{f_8} + \sqrt{2} \frac{\cos \theta_{PS}}{f_1} \right] \end{cases} \quad (21)$$

where $e = \sqrt{4\pi\alpha_{em}}$ and $f_\pi = 93$ MeV is the pion decay constant[11]. For these decays which involve a single photon, the octet component of the current does not produce any effect and the corresponding matrix elements are free of any dependence upon ξ . Assuming that moving to physical energies does not affect seriously expressions (21), they provide a prediction for a non-resonant component in $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays.

3.2 The ρ^0 component in η/η' decays

As already mentioned, the η and η' two pion spectra are dominated by the ρ^0 meson. The question now arises how to account for this contribution, indeed the largest one.

In the case of η decay, it has been suggested[3] to multiply the ρ propagator, suitably normalized by the corresponding matrix element (21). The prediction obtained[3] is far from agreeing with experimental data[8]. Relying on this result, even the idea[1][3] to use the η decays has been abandoned[8][56].

Moreover, it has also been suggested[3] that the dominance of the ρ^0 could well render impossible the use of the η' decay. To reach this conclusion it has been argued that in the case of η' , we have to extrapolate across the ρ pole. Because the effective location of the ρ pole in the complex plane is relatively far from the physical region, it is far from being sure that intrinsically the ρ pole causes any problem. This argument could possibly not apply to states like the ω meson.

In our opinion, the problem is threefold : i/ are expressions(18) and (21) actually valid at physical energies ? ii/ should the amplitudes describing the η and η' mesons be structurally different ? iii/ are the anomalous contributions to η and η' decays actually independent of the ρ contribution?

Answering the first question is surely a phenomenological issue : whichever is the value of ξ , one can estimate some values we can expect, assuming SU(3) symmetry is not too badly broken. f_π/f_8 is expected to be $\simeq 0.8$ with a renormalization mass $\mu = 1.0$ GeV[6], it can be slightly different due to possible higher order corrections and/or higher renormalization mass ; then we can roughly expect this ratio to be $0.5 \div 1.5$. From mass formulae, we expect the mixing angle θ_{PS} to be negative and lying between 0° and $\simeq -(25^\circ \div 30^\circ)$. Any reasonable method giving results in agreement with these educated guesses can be considered. On the other hand, apart from questions related with the ρ meson, if rels. (18) are valid at physical energies – this underlies the relevance of all results[8][22][56] on the pseudoscalar parameters – rels. (21) would have also to be such.

The second point is whether there can be some differences between the treatment of the ρ in η and η' decays. From first principles, the answer would be negative. The single possible difference should come from the available phase space : it is cut $\simeq 200$ MeV below the ρ peak for η decay, while it is cut $\simeq 200$ MeV above this peak for η' decay. This influences the decay rates through known phase space factors, but not the amplitudes.

The question of the ρ itself is more difficult to answer. The main problem is whether the anomalous contribution to the decay amplitude (see fig. (1b)) is independent of that of the ρ depicted in figure (5). The suggested prescription to treat the η in ref.[3] corresponds actually to a negative answer. This leads to inconsistencies between chiral predictions and experimental data[8]. No attempt is reported in the literature to treat the anomalous and the resonance contributions as two distinct elementary processes⁹. It is indeed what we plan to perform. As explained with some details in section 1, we are motivated by the nearly unanimous claim that the $\eta' \rightarrow \pi^+\pi^-\gamma$ data cannot be described with simply a ρ contribution.

⁹As mentioned in the introduction, ref.[23] actually tried implicitly such an attempt to describe the $\eta' \rightarrow \pi^+\pi^-\gamma$ decay. The way it is done does not allow however a simple connection with the box anomaly.

3.3 Expected magnitudes of the anomalies

For further purposes, it is useful to have in mind the order of magnitude of the anomalies. It is, of course, not enough to find a non-resonant component beneath the ρ contribution to claim that it is the box anomaly. Indeed allowing interference between a resonance contribution to the amplitude and a non-resonant contribution can easily simulate other neglected effects.

Assuming the following likely bounds :

$$\begin{cases} 0.5 \leq f_\pi/f_8 \leq 1.5 \\ -30^\circ \leq \theta_{PS} \leq 0^\circ \end{cases} \quad (22)$$

using rels. (18) and (21), the experimental values of the two photon decay widths and their errors[11], one can find the approximate bounds :

$$\begin{cases} 4 \leq |E_\eta| \leq 7 & (\xi = 1) \\ 5 \leq |E_{\eta'}| \leq 6 \end{cases} \quad (23)$$

and also :

$$\begin{cases} 2 \leq |E_\eta| \leq 6 & (\xi = 2) \\ 1.6 \leq |E_{\eta'}| \leq 2.6 \end{cases} \quad (24)$$

in units of GeV^{-3} (In the following E_η and $E_{\eta'}$ will always be expressed using in this unit), by means of Monte Carlo methods.

From now on, we shall always remove the arguments of the B and E functions, which must be understood as 0. From predictions[1][3] it is known that these parameters carry a minus sign.

Rel. (23) and (24) illustrate that deciding whether $\xi = 1$ or $\xi = 2$ can only follow from η' data. This could be expected since the anomaly phenomenon is more sensitive in the $\text{SU}(3)$ singlet and therefore more in η' than in η decay. To assert anything firmly about the relevance of the box anomaly, it is however better to have a cross check using the η decay, even if the corresponding data are scarcer than η' data.

Of course, if nothing else than the anomalous and resonance contributions are present in the data, we expect :

$$E_\eta, \quad E_{\eta'} \equiv 0,$$

if actually the box anomaly is an irrelevant degree of freedom.

4 Dealing with the data on $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays

All the available experimental data on $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays have been quoted in the introduction. In the following, we shall describe how these data have to be handled, which ones have to be discarded and why.

In all cases unfortunately the data are published as histograms of event numbers per bin of $\pi^+\pi^-$ invariant mass, or per bin of photon energy in the η rest frame. We therefore have had

to read the numbers out of magnified figures. Moreover these spectra are generally uncorrected for acceptance and efficiency effects. Instead these corrections are applied on the model used to describe the data. In all cases the informations on the model are complete and one can get the acceptance/efficiency function by dividing up the published theoretical prediction by the model function. In this way we get for each bin the integrated value of the acceptance/efficiency function. To describe these data in a new way, we have only performed bin by bin the product of the (new) model function and of this smeared acceptance/efficiency function. Even if this is not the optimal procedure, it is the only possible way to use all published experimental data. Indeed we have had a more direct access only to the informations published in references [24], [26] and [28].

We have performed all the possible checks by refitting the data with the original models¹⁰ and comparing the corresponding results with the published conclusions. At this stage, we have been forced to abandon the data of ref. [9] ; the reason is that the informations contained in their figure (4) coming from fitting their spectrum with a phase space term and with a ρ^0 Breit-Wigner are conflicting.

We have also decided to use only data where the background is not too large ; this second condition leads us to discard the data of MARK II collaboration in references [22] and [25] which are indeed the same. Another data set [19] of this collaboration has been studied, but not taken into account when computing mean values. The reason is that the background subtraction seems to have been performed through a global fit. Indeed, we prefer to use spectra which have surely been subtracted conventionally by counting the number of η' in the $\pi^+\pi^-\gamma$ invariant mass spectrum, for given $\pi^+\pi^-$ mass intervals. This is the most precise way to get the $\pi^+\pi^-$ invariant mass distribution in the η' decay, and it avoids some systematic uncertainties.

The data of ARGUS collaboration contained in ref.[21] have been discarded because they are superseded by the corresponding data of ref.[27]. Moreover, following the author of this thesis, we have restricted the fit to the top part of the $\pi^+\pi^-$ spectrum since the tails undergo contamination from states other than η' .

Because in the tails of the $\pi^+\pi^-$ mass spectra statistics are always low and/or the acceptance/efficiency function is either small or sharply varying, we have checked when fitting that the quality of the fit is not spoiled by these tails ; in this way, we have also been led to exclude the tails when fitting the data set of ref.[20].

These remarks on the data sets of refs.[20] and [27] mean that some background remains and/or systematic effects are not well under control. The largest statistics experiment¹¹ ($\simeq 2000$ events) is that providing the data of ref.[23] ; they have also removed carefully all identified background effects and sources of systematics.

The only set of subtracted data published for the first time in this paper is that of the WA76 collaboration ; these data have been collected in an experiment on double pomeron collision and are already published but only as unsubtracted spectra[28]. To deal with these data, we have first fitted the η' mass spectrum with a Gaussian. When fitting the η' spectrum for each $\pi^+\pi^-$ mass interval, we have kept fixed the η' parameters as determined on the full sample. Ref.[14] used the same procedure but relaxed this last condition.

¹⁰It must be noticed that there are various differences in what is used as Breit-Wigner amplitudes in all these papers. In dealing with the data, this peculiarity has been carefully taken into account.

¹¹A comparable statistics data is reported for ARGUS in ref.[27], otherwise all other data sets deal with 200÷500 events only.

Thus the data used for the η' decay are those of references [14][18][20][23][24][26–28] ; this leaves us with 8 independent sets of data and allows us to remove systematic effects specific of each experiment. Most of the data sets we shall use have been collected in experiments where the photon is generally of very small energy (≤ 200 MeV) ; therefore the requested energy corrections are large and can introduce some systematic errors which are hard to account for. Two experiments[23][28] only do not suffer too much from this problem since the observed photon energy is generally well above 1 GeV.

With data on η decay, we are much less lucky since we are left with only one usable set of data (that of ref.[10]). We therefore have to be cautious when dealing with η data.

5 Study of the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays

In order to perform a new analysis of all available experimental data, we shall use the model described in appendix B which assumes an anomalous contribution besides the resonance contribution. The single free parameter in this model is precisely the box anomaly constant. It is shown in appendix B that the sign of this constant cannot follow from fitting. Therefore, as for the triangle anomaly, the box anomaly is only fixed in absolute magnitude.

In order to extract a nonleading non-resonant contribution with its correct magnitude when analyzing spectra highly dominated by the ρ meson, the main question is which ρ amplitude has to be introduced. The discussion in section 2 is motivated by this matter and our conclusion is that the most likely ρ distributions are those determined from e^+e^- data. We have shown that there are here ambiguities related with the magnitude of a possible non-VDM contribution to the pion electromagnetic form factor. Therefore, we have to discuss the influence of this ambiguity.

5.1 Former fits to the data

We have already noticed that nearly all attempts to fit $\eta' \rightarrow \pi^+\pi^-\gamma$ with only the ρ contribution failed[18][20][24][26][27]. A more complete analysis has been attempted in ref.[23] with the same conclusion and finding moreover that a non-resonant term could solve the puzzle. We have also tried fitting the η' data using the two ρ models described by rels. (12) and (14) and models described in refs. [31] and [39] ; in this case, only two data sets[14][19] can accomodate with simply a ρ contribution. Therefore, we can conclude that the ρ alone is unable to explain the whole set of η' data.

To describe the $\eta \rightarrow \pi^+\pi^-\gamma$, refs.[9][10] used a crude approximation of the ρ amplitude¹² to get a more satisfactory result ($\chi^2/\text{dof} \simeq 11.5/13$ and $\simeq 25/14$ respectively) than using a phase space distribution ($\chi^2/\text{dof} \simeq 145/13$ and $\simeq 113/14$ respectively). We have redone the analysis of the data in ref.[10] using conventional Breit–Wigner ρ amplitudes[30][31][39]. Even if it remains much better than phase space, the result obtained is not so good ($\chi^2/\text{dof} \simeq 40/14$).

Therefore, using conventional shapes for the ρ distribution, the data analysis have not been able to provide a satisfactory description of most data on both η and η' decays.

¹²The amplitude used is a fixed-width Breit–Wigner.

5.2 The two possible fits to η/η' decays

We therefore allow for an additional non-resonant term and look for its connection with the magnitudes predicted from the box anomaly. If the box anomaly is not required by the data, the corresponding parameters E_η and $E_{\eta'}$ have to be *simultaneously* consistent with zero. We shall use in this section only the ρ distributions determined in e^+e^- collisions (see section 2.4). We shall call M2 the model presented in appendix B using the ρ parameters and shape obtained in e^+e^- data when assuming no non-VDM term in the pion form factor (see rels. (12)), and M1 the same model but with the ρ parameters and shape deduced assuming such a term in e^+e^- data (see rels. (14)) ; therefore the two models differ by the ρ shape and parameters only.

Concerning the $\eta' \rightarrow \pi^+\pi^-\gamma$ data[10], we have fitted the parameter E_η and obtained :

$$\begin{cases} E_\eta = -4.481 \pm_{-0.686}^{+0.687} & (\text{model M2}) \\ E_\eta = -5.042 \pm_{-1.527}^{+1.541} & (\text{model M1}) \end{cases} \quad (25)$$

where the errors are those obtained using the MINOS routine. The fits and spectra are presented in fig. (6) and corresponds to $\chi^2/\text{dof} = 13/14$ using model M1 (see fig. (6a)), and to $\chi^2/\text{dof} = 8/14$ using model M2 (see fig. (6b)). Therefore the effect is significant at more than 6σ for model M2, and at more than 3σ for model M1. One should notice the sensitivity of the error on E_η to the model. In both cases, we get results significantly different from zero and consistent with the bounds quoted in section 3.3.

	Model M2		Model M1	
	χ^2/dof	S-factor	χ^2/dof	S-factor
PLUTO	15.3/17	1.06	15.4/17	1.09
TASSO	6.6/11	1.05	6.3/11	1.08
MARK II	14.7/13	1.18	13.7/13	1.34
TPC/2 γ	7.0/5	1.07	6.7/5	1.09
CELLO	18.2/20	1.09	17.8/20	1.13
ARGUS	5.2/7	1.17	4.7/7	1.07
Grigorian	8.2/5	1.09	7.3/5	1.15
Lepton F	31.5/19	1.20	32.0/19	1.37
WA76	6.7/11	1.05	6.1/11	1.08

Table 1 : χ^2/dof when fitting the dipion mass spectra in $\eta' \rightarrow \pi^+\pi^-\gamma$ decays. S-factor used to weigh each experiment in the global fit (see text).

For comparison, we have attempted to describes the η data assuming $E_\eta \equiv 0$ using our model M1 and M2. Using model M1, the fit corresponds to¹³ $\chi^2/\text{dof} = 24/15$ (instead of 13/14

¹³In these cases, the fits are without any free parameter.

with $E_\eta \neq 0$), using model M2 the description of the data corresponds to $\chi^2/\text{dof} = 43/15$ (instead of $8/14$, with $E_\eta \neq 0$). The corresponding histograms are shown in figure (6).

Concerning the $\eta' \rightarrow \pi^+\pi^-\gamma$ decay, we have separately fitted each of the above-mentioned $\pi^+\pi^-$ data sets to get the parameter $E_{\eta'}$. The results of these fits together with the experimental histograms are given in figure (7) in the case of model M1 ; the corresponding fits using model M2 give almost the same curves with almost the same values of χ^2/dof (see table (1)). The description obtained is always acceptable and of much better quality than the attempts already quoted¹⁴. The fitted values of $E_{\eta'}$ for each experiment are given in figs. (8) and (9) for models M2 and M1 respectively. One first notices the relatively large spread of values which clearly points towards the existence of non-negligible systematic errors.

We have attempted to perform a weighted mean on the η' results. It cannot be obtained using standard algebraic methods since systematic errors cannot be neglected and they are not estimated by any experiment. We have preferred performing a global fit of all the data sets ; however, it is unrealistic to give each experiment the same weight. Therefore we have chosen to determine some kind of S-factor[11] to weight each data set when fitting. We have studied for this purpose the sensitivity of each data set to varying the parameter λ (see rel.(11)) by $\pm\sigma(\lambda)$. This modification produces a shift $\delta_\lambda(E_{\eta'})$ of $E_{\eta'}$ and, for each experiment we have defined a weighting factor $1/S$ using the ratio of this shift to the (MINOS) error $\sigma(E_{\eta'})$ obtained when fitting each experiment separately :

$$S = 1 + \frac{|\delta_\lambda(E_{\eta'})|}{\sigma(E_{\eta'})} .$$

These numbers can be found in table (1). Our motivation in using this procedure is that systematic errors would mainly skew the ρ shape and this tends to modify the effective value of the parameter λ ; a possible effect on the location of the ρ peak can be expected to be small. Indeed, all experiments reporting a shift with respect to the expected ρ mass give values close to each other. Proceeding this way, we obtain :

$$\begin{cases} E_{\eta'} = -2.167 \begin{smallmatrix} +0.504 \\ -0.507 \end{smallmatrix} & (\text{model M2}) \\ E_{\eta'} = -5.110 \begin{smallmatrix} +0.707 \\ -0.713 \end{smallmatrix} & (\text{model M1}) \end{cases} \quad (26)$$

from a good fit ($\chi^2/\text{dof} = 122/102$ and $\chi^2/\text{dof} = 110/102$ respectively for models M2 and M1). These values have also been plotted in figure (8) and (9).

The model depends also on another parameter F_X fixed by E_X (see appendix B). To be complete, the values of this parameter connected with rels. (25) and (26) are :

$$\begin{cases} F_{\eta'} = 0.443 \pm 0.005 , & F_\eta = -0.291 \pm 0.028 & (\text{model M2}) \\ F_{\eta'} = 0.420 \pm 0.004 , & F_\eta = 0.474 \begin{smallmatrix} +0.043 \\ -0.069 \end{smallmatrix} & (\text{model M1}) \end{cases} \quad (27)$$

¹⁴Except ref.[23] with which we agree since they give a fit adding a non-resonant term to the standard resonance contribution.

The errors quoted here have been obtained by varying E_η and $E_{\eta'}$ of $\pm\sigma$ around their central values. The parameters in rels. (27) are in units of GeV^{-1} .

5.3 Comments on the model and fit

It is now interesting to compare rels. (25) and (26) with the approximate bounds given in rels. (23) and (24). We first remark that the additive method we have used leads to values for the box anomaly parameters which are consistent with the expectations. We conclude from here that the box anomaly and the ρ contribution can be considered as independent.

It must also be noticed that the values obtained for the box anomaly constants are not sharply dependent on the ρ mass and width in rels. (12) and (14). The main difference between models M1 and M2 in η/η' decays is due to the large difference between the corresponding values of the λ parameter, which exhibit sharply different $\pi\pi \rightarrow \pi\pi$ left-hand singularities.

In former attempts, instead of using the ρ distributions deduced from e^+e^- data, we tried using the ρ decay amplitudes determined in hadronic experiments[31][39]. The numerical results obtained tend to be comparable with model M1 above for η' data. For η data however the results were unacceptable (E_η ranging from 20 to 100 GeV^{-3}). It is worth noting that the ρ distributions giving consistent results in both the η and η' sectors are those coming from e^+e^- data.

The results obtained strongly rely on the fact that the vertex functions F_X ($X = \eta, \eta'$) are slowly varying functions of m^2 and can be approximated by constants. If instead one accepts sharp variations, the box anomaly can be cancelled (or amplified). A simple way to cancel the effect in η' data is to allow a mean behaviour :

$$|F_{\eta'}|^2 \simeq k_\gamma^{-1}$$

(k_γ is the photon momentum in the η' rest frame) with a good quality of fit ; another way is to allow :

$$|F_{\eta'}|^2 \simeq \left[\frac{m}{m_\rho} \right]^6 ,$$

with worse (but still acceptable) χ^2 . In both cases, this tends simply to shift the ρ peak to higher masses. Of course, by choosing suitably the behaviour of $F_{\eta'}$, instead of cancelling the effect, we can easily amplify it.

We consider presently these kinds of hypotheses as rather artificial and find more reliable to assume $F_{\eta'}$ constant over the range of relevance.

There remains however an ambiguity due to the ρ distributions. This ambiguity can only be solved from theory and e^+e^- physics.

6 Consistency checks of anomaly constants in η/η' decays

Chanowitz equations (18) and (21) allow for a consistency check among the values found for $E_{\eta'}$ and E_{η} . Setting :

$$\lambda = \frac{\alpha_{em}}{\pi\sqrt{3}} \frac{1}{f_{\pi}} , \quad \mu = \frac{e}{4\pi^2\sqrt{3}} \frac{1}{f_{\pi}^3} ,$$

equations (18) give :

$$\begin{cases} B_{\eta} \cos \theta_{PS} + B_{\eta'} \sin \theta_{PS} = -\lambda \frac{f_{\pi}}{f_8} \\ B_{\eta} \sin \theta_{PS} - B_{\eta'} \cos \theta_{PS} = 2\sqrt{2}\xi\lambda \frac{f_{\pi}}{f_1} \end{cases} \quad (28)$$

These relations allow to rewrite equations (21) as

$$\begin{cases} \left[\frac{\lambda}{\mu} E_{\eta'} - B_{\eta'} \right] t^2 - B_{\eta} \left[1 - \frac{1}{2\xi} \right] t + \left[\frac{\lambda}{\mu} E_{\eta'} - \frac{1}{2\xi} B_{\eta'} \right] = 0 \\ \left[\frac{\lambda}{\mu} E_{\eta} - \frac{1}{2\xi} B_{\eta} \right] t^2 - B_{\eta'} \left[1 - \frac{1}{2\xi} \right] t + \left[\frac{\lambda}{\mu} E_{\eta} - B_{\eta'} \right] = 0 \end{cases} \quad (29)$$

where $t = \tan \theta_{PS}$.

Using the masses and widths of the η and η' mesons[11], we can obtain the experimental values of B_{η} and $B_{\eta'}$ by means of rel. (19) :

$$\begin{aligned} B_{\eta} &= [-23.7 \pm 3.8]10^{-3} \\ B_{\eta'} &= [-32.1 \pm 0.9]10^{-3} \end{aligned} \quad (30)$$

in units of GeV^{-1} .

Using these values and our fitted values to $E_{\eta'}$, we can compute t from the first equation (29). Then, this value of t allows us to compute the value of E_{η} expected from B_{η} , $B_{\eta'}$ and $E_{\eta'}$. Actually, equations (29) being second degree, we have *a priori* two possible solutions for t associated with a given value of $E_{\eta'}$; it is only by using both equations (29) that a single t can be selected. The method thus sketched has been applied by generating B_{η} , $B_{\eta'}$ and $E_{\eta'}$ with gaussian distributions having for means their central values and for standard deviations the corresponding errors. We have also allowed the 8 possible sign combinations ; the single combination which gives results consistent with expectations is that for which the 3 parameters are negative. This could be expected from the signs in the Chanowitz equations.

The distribution has been computed using models M2 and M1, both with $\xi = 1$ and $\xi = 2$. Using the model M2 result in rels. (26), we have obtained¹⁵ :

$$\begin{aligned} E_{\eta} &= -7.853 \pm 0.931 \quad (\text{solution } (+)) \\ E_{\eta} &= -3.504 \pm 0.841 \quad (\text{solution } (-)) \end{aligned} \quad (31)$$

¹⁵Solutions (+) and (-) refer to the sign of the discriminant term used when solving the first equation (29).

assuming $\xi = 2$ and no solution if $\xi = 1$ (for real t). Solution (+) is at 2.9σ from the model M2 result (-4.481) in rels. (25), while solution (-) agrees within 0.9σ . Therefore solution (-) provides a positive cross check in the case $\xi = 2$ and cannot accomodate $\xi = 1$. Solution (+) can be discarded.

Using the model M1 result in rels. (26) with $\xi = 2$, we obtain :

$$\begin{aligned} E_\eta &= -10.670 \pm 0.572 \quad (\text{solution } (+)) \\ E_\eta &= -0.599 \pm 0.428 \quad (\text{solution } (-)) \end{aligned} \tag{32}$$

In this case, solution (+) is at 3.4σ from the model M1 result (-5.042) in rel (25), while solution (-) is at 2.8σ from the same result. We can conclude from here that model M1 is inconsistent assuming $\xi = 2$.

Instead, the model M1 result in rels. (26) with $\xi = 1$ leads to :

$$\begin{aligned} E_\eta &= -8.500 \pm 0.825 \quad (\text{solution } (+)) \\ E_\eta &= -5.112 \pm 0.774 \quad (\text{solution } (-)) \end{aligned} \tag{33}$$

In this case, solution (+) is at 2σ of the fitted value (-5.042), while solution (-) nearly coincides with it. We can therefore conclude that model M1 provides a positive cross check for the fitted box anomaly constants under the hypothesis $\xi = 1$. Comparing the accuracy of both solutions with respect to the fitted value, it is clear that solution (-) is preferred, even if solution (+) cannot be completely ruled out.

Therefore we can conclude that model M2, favouring $\xi = 2$ is inconsistent with QCD, while model M1 which favours $\xi = 1$ is consistent with QCD predictions. Let us recall that model M1 requires the existence of a non-VDM component of the pion electromagnetic form factor, while M2 completely excludes this hypothesis. Therefore, in order to be compatible with QCD, we need to include a non-VDM term to describe the $e^+e^- \rightarrow \pi^+\pi^-$ cross section.

Thus, we have shown that the box anomaly constants E_η and $E_{\eta'}$ obtained by our fit are consistent with each other when using the data provided by $\eta/\eta' \rightarrow \gamma\gamma$ decay widths and the Chanowitz equations. This result allows two conclusions : i/ our procedure to get the mean value and error of $E_{\eta'}$ actually avoids the influence of the systematic errors in each experiment performed on $\eta' \rightarrow \pi^+\pi^-\gamma$ decay, ii/ the influence of the systematic errors in the data set of ref.[10] on $\eta \rightarrow \pi^+\pi^-\gamma$ decay is actually small.

Then, taking into account the difficulties to get a satisfactory fit to η' data with simply a ρ contribution, comparing results (25) and (26) with expectations (23) and (24) as well as the analysis above gives strong evidence for the box anomaly in both η and η' decays.

7 Geometrical properties of the solutions

Relations (28) and (29) can be considered as defining f_π/f_8 and f_π/f_1 as functions of $(B_\eta, B_{\eta'}, E_{\eta'})$ or of $(B_\eta, B_{\eta'}, E_\eta)$. They are written as parametric functions depending on θ_{PS} . From rels. (28), f_π/f_8 and f_π/f_1 are sinusoidal functions of θ_{PS} with period 2π . From rels. (29), E_η and $E_{\eta'}$ are also sinusoidal functions but with period π . Therefore the curves $(f_\pi/f_8, E_{\eta'})$, $(f_\pi/f_1, E_{\eta'})$, $(f_\pi/f_8, E_\eta)$, $(f_\pi/f_1, E_\eta)$ are Lissajous curves of degree 4 with a double point

located at $f_\pi/f_8 = 0$ or at $f_\pi/f_1 = 0$, as it can be seen in figures (10) for both models M2 and M1. In these figures, the curves take into account the uncertainties on B_η and $B_{\eta'}$; $\pm\sigma$ intervals around the central values of parameters E_η and $E_{\eta'}$ are shown in each figure (10) on top and bottom.

Figures (10) clearly show that the experimental intervals of $E_{\eta'}$ contain the points $f_\pi/f_8 = 0$; this explains why it is not suitable to sample f_8 but its inverse.

For instance, at any fixed value of $E_{\eta'}$, we have 0 or 4 solutions and it happens that experimental data choose only the solutions close to $\theta = 0^\circ$. In this case, the previous solution (+) corresponds to the branch with larger values of f_π/f_8 and f_π/f_1 , while solution (-) corresponds to the branch with lower values. It must also be noticed that the “physical regions” for $E_{\eta'}$ and E_η are frequently located close to a boundary where it is possible that no real solution exists.

Finally, it must be remarked that the spread of all curves in figures (10) is mainly determined by the experimental errors¹⁶ on $\Gamma(\eta \rightarrow \gamma\gamma)$ and $\Gamma(\eta' \rightarrow \gamma\gamma)$. The effect they produce on the accuracy of f_1 , f_8 and θ_{PS} happens to be less important than the errors on $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ spectra; however it is clearly not negligible, mainly for f_8 .

8 The electric charge of quarks

In the previous section we have shown that the two box anomaly constants determined from experimental data can be considered with a high confidence level. Therefore, B_η , $B_{\eta'}$, E_η and $E_{\eta'}$ can be considered on an equal footing. Using the four Chanowitz equations, it is a simple matter to get :

$$\xi = \frac{1}{2} \frac{B_\eta E_{\eta'} + B_{\eta'} E_\eta - \frac{1}{\beta} B_\eta B_{\eta'}}{B_\eta E_{\eta'} + B_{\eta'} E_\eta - \beta E_\eta E_{\eta'}} \quad (34)$$

where $\beta = \sqrt{4\pi\alpha_{em}}f_\pi^2$. We have studied this distribution by considering once more the four anomaly parameters as gaussian random variables having as means their measured central values and as standard deviations their measured errors. The parameters of the ξ distribution are :

$$\begin{cases} \bar{\xi} = 2.659, & \sigma(\xi) = 0.564, & (\text{modele M2}) \\ \bar{\xi} = 1.155, & \sigma(\xi) = 0.185, & (\text{modele M1}) \end{cases} \quad (35)$$

These results give a direct confirmation of what could be already inferred from the previous sections : model M2 favours an integral charge quark model and is inconsistent with QCD at the 3σ level while model M1 agrees with QCD predictions. It is a surprising situation that the ambiguity generated by these two models does not produce an ambiguity between a likely value of ξ and an exotic one, but between the two possible values of ξ .

To our knowledge, integral charge quark models are not yet ruled out since they deliver results indistinguishable from QCD for almost all processes. Important exceptions are the η and η' decays we just studied and some $\gamma\gamma$ scattering processes. Actually, naive integral charge

¹⁶Errors on M_η and $M_{\eta'}$ play also a role, however it is negligible.

quark models[57] could recently be ruled out from photon-photon to two jets reaction[61]. Extensions[62][63] of such models (gauge integer charge quark models) introduce additional free parameters like gluon masses. The PLUTO experiment could only restrict the parameter range but not rule out the model[61].

Deciding between model M2 and model M1 cannot follow from more detailed analysis of η or η' data. Theoretically, the answer requires a reliable estimate of the $\rho^0 - \omega$ mass difference and/or a reliable estimate of a non-VDM component of the pion electromagnetic form factor in the non-perturbative timelike region. Let us also remark that the existence of such a term is controversial. Some time ago it was considered[44][45] ; however recently the general tendency was that no such term exists, but there also exist effective models where it has been introduced[59][60]. We have shown in section 2.4 that it is unlikely that the non-VDM term could be explained by contributions of high mass vector mesons. Actually it seems presently believed that the high-energy behaviour of the pion form factor follows entirely from contributions of vector resonances at low energy through a kind of duality principle. Nevertheless, such a term is needed to reconcile QCD with the experimental data on η and η' decays by allowing $\xi = 1$.

9 Comparison of $\eta \rightarrow \rho^0 \gamma$ and $\rho^0 \rightarrow \eta \gamma$ decays

The parameter F_η given in rels. (27) comes from the part of the $\eta \rightarrow \pi^+ \pi^- \gamma$ amplitude which describes the decay $\eta \rightarrow \rho^0 \gamma$. This amplitude (see appendix B for notations and properties) is :

$$T_X^{ij} = F_\eta \epsilon_{\mu\nu\rho\sigma} \epsilon_i^\mu(\gamma) \epsilon_j^\nu(\rho) p^\rho k^\sigma . \quad (36)$$

Let us consider now the decay $\rho^0 \rightarrow \eta \gamma$. Using the same notations as above, the most general amplitude which describes it can be written :

$$T_X^{ij} = \mathcal{F}_\eta \epsilon_{\mu\nu\rho\sigma} \epsilon_i^\mu(\gamma) \epsilon_j^{\nu*}(\rho) p^\rho k^\sigma . \quad (37)$$

If the coefficients in rels. (36) and (37) are real, we expect :

$$F_\eta = \mathcal{F}_\eta .$$

It is easy to compute the decay width of $\rho^0 \rightarrow \eta \gamma$:

$$\Gamma(\rho^0 \rightarrow \eta \gamma) = \frac{[m_\rho^2 - m_\eta^2]^2}{48\pi m_\rho} |\mathcal{F}_\eta|^2 . \quad (38)$$

Using the experimental data, rel. (38) allows for an estimate of $\mathcal{F}_\eta$, assuming it is constant and using for m_ρ the central ρ mass. There is however an ambiguity since experiments[64][65] provide two solutions, one with a destructive $\rho - \omega$ interference (DI), one with a constructive interference (CI) ; the latter solution is considered as favoured by the quark model and is quoted as reference value[11]. Using the data of refs. [64] and [65], we have computed both averages and by means of rel. (38), we have deduced the absolute values :

$$\begin{cases} |\mathcal{F}_\eta| = 0.282 \pm 0.027 \text{ GeV}^{-1} , & \text{(solution CI)} \\ |\mathcal{F}_\eta| = 0.356 \pm 0.046 \text{ GeV}^{-1} , & \text{(solution DI)} \end{cases} \quad (39)$$

The good agreement between rels. (39) and (27) is interesting at various levels. It first proves that approximating $F_\eta(m^2)$ by a constant is correct since rels. (27) suppose some kind of averaging between $2m_\pi$ and m_η while rels. (39) suppose an averaging over the whole ρ spectrum. Secondly, this agreement proves that using our procedure of appendix B provides not only consistency between E_η and $E_{\eta'}$ (see section 6 above), but also consistency between our estimate of $F_\eta(m^2)$ in $\eta \rightarrow \pi^+\pi^-\gamma$ decay and the estimate deduced from $\rho^0 \rightarrow \eta\gamma$ decay in a wider kinematical region of m^2 .

Finally, a finer comparison of these relations also shows that our model M2 (*i.e.* $\xi = 2$) is in fairly good agreement with the value favoured by the quark model (CI), while our model M1 (*i.e.* $\xi = 1$) would prefer solution (DI) (the distance of both means is 1.5σ), presently considered as disfavoured by the quark model. Notice moreover that our rels. (27) give a prediction on the sign of $\mathcal{F}_\eta$ which depends on which model is valid, M1 or M2.

10 Experimental values of the pseudoscalar parameters

Using the Chanowitz equations and the experimental values of the four anomaly parameters, we can for the first time achieve a determination of f_1 , f_8 and θ_{PS} on purely experimental grounds.

Because we know that ξ can only be 1 or 2, the optimal procedure is to solve the Chanowitz equations by fixing ξ . This allows for a constrained fit. Proceeding this way we can perform the minimization procedure using a χ^2 and we can obtain in addition the probability of the fit.

Let us start with model M2 data (see rels. (25), (26)). Assuming $\xi = 2$ in Chanowitz equations, we thus get :

$$\left\{ \begin{array}{l} \frac{f_\pi}{f_1} = 0.478^{+0.015}_{-0.016} \\ \frac{f_\pi}{f_8} = 0.582^{+0.165}_{-0.164} \\ \theta_{PS} = -24.16^\circ \quad {}^{+4.23^\circ}_{-4.15^\circ} \end{array} \right. \quad (\text{model M2 , } \xi = 2) \quad (40)$$

with a perfect fit ($\chi^2/\text{dof} = 0.03/1$). Trying to fit model M2 data assuming $\xi = 1$ provides instead $\chi^2/\text{dof} = 42./1$.

The model M1 data from rels. (25) and (26), assuming $\xi = 1$, give :

$$\left\{ \begin{array}{l} \frac{f_\pi}{f_1} = 0.961^{+0.031}_{-0.046} \\ \frac{f_\pi}{f_8} = 0.464^{+0.429}_{-0.380} \\ \theta_{PS} = -26.75^\circ \quad {}^{+9.62^\circ}_{-8.37^\circ} \end{array} \right. \quad (\text{model M1 , } \xi = 1) \quad (41)$$

with a good quality of fit ($\chi^2/\text{dof} = 0.54/1$). Trying to fit model M1 data assuming $\xi = 2$ gives unacceptable results ($\chi^2/\text{dof} = 12./1$). To work out these last computations, we have assumed the value[11] $f_\pi = 93 \text{ MeV}$.

One can see from rels. (40) and (41) that the value found for θ_{PS} in both cases is in agreement with what is commonly believed[8][22][56], which clearly means that it is not sharply sensitive

to the value of f_π/f_8 [6][7]. Since there is no prediction for f_π/f_1 our results cannot be compared to theory ; let us remark that the quantity $\xi f_\pi/f_1$ looks alike in both models M2 and M1. In case of model M1, the value we find agrees also with what is believed[8][22][56].

It is also worth noting that the expected value $f_\pi/f_8 \simeq 0.8$ is at 1σ from the experimental value whichever is the value of ξ . However, the central values deduced from fit are relatively far from the predictions[6][7].

One can worry about the magnitude of the error for f_π/f_8 when $\xi = 1$; this is entirely due to the error on E_η in the case of model M1 (see second relation(26)), hence to the lack of data on $\eta \rightarrow \pi^+\pi^-\gamma$ decay.

Let us stress that the results given by rels. (40) and (41) are the most precise which can be obtained on fully experimental grounds from the presently available data. No serious improvement can be expected before having at our disposal higher statistics spectra and a better control of the systematic errors. Moreover choosing between solution (40) or (41) (*i.e.* whether $\xi = 2$ or $\xi = 1$) proceeds from e^+e^- physics and from ρ meson spectroscopic properties.

In section 6, we have seen that there is algebraically two solutions to our problem (called solutions (+) and (-) respectively). A careful analysis shows that solution (-) is naturally chosen by the minimization procedure. We have also seen that solution (+) is rejected at the 3σ level for model M2, but only at the 2σ level for model M1. Hence, it cannot be excluded that improving the quality of the data would not allow a switch from solution (-) to solution (+). In this case, it is worth noting that the results would become different. For instance, the central value of the mixing angle would switch to $\theta_{PS} \simeq -(5 \div 8)^\circ$ in both models. It is striking that the value of θ_{PS} chosen by the fit, and the solution which is disfavoured correspond to the two values of θ_{PS} provided by the SU(3) mass formulae.

11 Analysis using Vector Dominance Hypothesis

Relying on VDM hypothesis, Chanowitz proposed in refs [2] and [3] a relation to compute ξ (see appendix C, rel. (C1)) ; let us recall it :

$$\xi^2 = \left[\frac{3f_\rho}{4e} \right]^2 \frac{|B_{\eta'}|^2 + |B_\eta|^2 - \frac{1}{3}|B_{\pi^0}|^2}{|F_{\eta'}|^2 + |F_\eta|^2 - \frac{1}{3}|C_\omega|^2} . \quad (42)$$

Using our results and the experimental data[11] for $\pi^0/\eta/\eta' \rightarrow \gamma\gamma$ and $\omega \rightarrow \pi^0\gamma$ decays, it is an easy exercise to compute ξ this way :

$$\begin{cases} \xi = 1.121 \pm 0.086 & \text{(Model M2)} \\ \xi = 0.939 \pm 0.091 & \text{(Model M1)} \end{cases} \quad (43)$$

From here, we can conclude that model M1 fulfills the VDM hypothesis while model M2 does not (a factor of 2 is missing). If we refer to the concluding remarks in ref. [2], this situation is not completely unexpected. Indeed, to get rel. (42) when $\xi = 2$ one needs to assume some properties to gluonic components of mesons which are not necessarily fulfilled in this case[2].

In appendix C, we have established the following relation using VDM :

$$|F_\eta|^2 + |F_{\eta'}|^2 = \frac{3\alpha_{em}}{4\pi^2} \frac{f_\rho^2}{4\pi} \left[\frac{1}{f_8^2} + \frac{2}{f_1^2} \right] . \quad (44)$$

This relation has some nice features : it connects the ρ^0 part of the amplitudes for $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ to the full anomaly physics. It should also be noticed that the dependence upon ξ is only implicit. Using rels. (25), (26) and (27), we can check to which extent rel. (44) is fulfilled by experimental data. This provides another consistency check (however subject to the applicability of VDM). The result can be written :

$$\begin{cases} 0.282 \pm 0.017 = 0.127 \pm 0.044 & (\text{Model M2}) \\ 0.384 \pm 0.051 = 0.423 \pm 0.094 & (\text{Model M1}) \end{cases} \quad (45)$$

correspondingly to equation (44). Here also, we see that model M1 fulfills the condition, while a factor 2 is missing in case of model M2, exactly as above.

From these comparisons, we see that model M1 provides a positive cross check of our results, while model M2 does not. It is not sure whether this result should rule out model M2, because, as stressed by Chanowitz in ref. [2], relation (42) – and, hence relation (44) – is on less secure grounds when $\xi = 2$ than when $\xi = 1$.

12 Conclusion

From the present study, the first important conclusion is that the box anomaly, as predicted by PCAC or the Wess–Zumino lagrangian, occurs in $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays at the level expected from Chanowitz equations.

Since the quality of the available data is not impressive, it has been of concern to find cross checks of the fitted values we get, as well as checks with other experimental results. We have thus shown that the equations of Chanowitz and the $\eta/\eta' \rightarrow \gamma\gamma$ decays allow for a positive consistency cross check of the numerical values of the box parameters found in η and η' decays. We have also shown that our results on $\eta \rightarrow \pi^+\pi^-\gamma$ decay are consistent with the available experimental data on $\rho \rightarrow \eta\gamma$ decay. From these results we concluded that we can rely on the final results we get. Therefore our study confirms the expected generality of the anomaly phenomenon in particle physics.

Our conclusion has been reached only by assuming that the ρ^0 contribution and the anomalous contribution are actually two distinct elementary processes (this is achieved with the phenomenological model presented in appendix B).

We have proved that Chanowitz equations provide the tool to achieve a fully experimental determination of the pseudoscalar nonet parameters. Two sets of parameters are possible depending on which is the correct ρ^0 distribution. They are given in rels. (40) and (41). The quoted accuracy of f_1 , f_8 and θ_{PS} cannot be seriously improved using only the presently available data. The lack of data on $\eta \rightarrow \pi^+\pi^-\gamma$ is especially addressed ; new data on $\eta' \rightarrow \pi^+\pi^-\gamma$ can however be expected from the WA91 collaboration within a couple of years.

We have shown that consistency checks do not allow for a definite choice among these two solutions, one which is in agreement with QCD ($\xi = 1$) and one which is in clear disagreement

with it ($\xi = 2$). Among all cross checks we did, the only one which favours $\xi = 1$ with respect with $\xi = 2$ strongly relies on the applicability of VDM when $\xi = 2$, which is not obvious. Therefore, if the model $\xi = 2$ has to be discarded, it cannot be for the physics we have examined but because of other tests.

To establish our numerical results, we have also been led to use the most reliable ρ distributions. We have argued that reliability can only be achieved using e^+e^- data, because the production mechanism here is much more under control than in hadronic physics.

We have shown that there is ambiguity coming from e^+e^- physics. This can be summarized by a question : does a non-VDM contribution to the pion electromagnetic form factor exist ? Present theoretical understanding of this physics seems to favour a negative answer, even if recently some tendency rises to restore it. If this feeling would turn to a certainty, then the relevance of QCD would have to be seriously reconsidered. From a phenomenological point of view the situation is unclear. Indeed, a high mass value ($m_\rho \simeq m_\omega$) is far from being excluded by the data and then no extra term is required. Moreover, no firm conclusion can be drawn from comparing with $\rho^0 \rightarrow \eta\gamma$ or even using VDM.

This is an unfortunate situation since a definite answer to these questions is the single possibility to distinguish between $\xi = 1$ (requested by QCD) and $\xi = 2$.

Within the framework we have developped which gives consistent results, if QCD is the correct theory, then we must have $\xi = 1$ and hence a non-VDM coupling $\gamma\pi^+\pi^-$ should exist. Otherwise, we have $\xi = 2$ and it is hard to reconcile data and QCD. A possible way to determine if such a non-VDM term exists is to analyze the ρ shape in a high statistics sample of $\tau^\pm \rightarrow \rho^\pm \nu$ decays. Hadronic experiments on polarized targets, if confirming the high mass ρ of Chabaud *al.*, can also contribute to the answer.

Let us finally conclude by remarking that the $(f_1, f_8, \theta_{PS}, \xi)$ problem happens to be one of the most intriguing problem in particle physics.

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Appendix A : A model for $e^+e^- \rightarrow \pi^+\pi^-$

The model we shall describe now is not original to a large extent. Indeed the single original feature is our way to parametrize the ρ meson width. However, since this kind of Radio Days¹⁷ Physics is now far from most physicist minds, it is not completely useless to recall it briefly.

The amplitude to describe the process $e^+e^- \rightarrow \pi^+\pi^-$ can be written :

$$T = \bar{v}(p_1)[-ie\gamma_\mu]u(p_2)\frac{ie}{s}J^\mu = \frac{e^2}{s}\bar{v}(p_1)\gamma_\mu u(p_2)J^\mu \quad (\text{A.1})$$

where $s = (p_1 + p_2)^2 = m^2$, $e^2 = 4\pi\alpha_{em}$ and J^μ is the matrix element of the electromagnetic current between vacuum and the hadronic final state. Neglecting contributions from the ϕ meson or from high mass vector mesons like[49] the $\rho(1450)$ or $\rho(1700)$, J^μ can be written :

$$J^\mu = \left[A \sum_i \varepsilon_\mu^i(\gamma)\varepsilon_\nu^{i*}(\gamma) + g_{\rho\gamma}\frac{1}{D_\rho(m)}G_\rho \sum_i \varepsilon_\mu^i(\rho)\varepsilon_\nu^{i*}(\rho) + g_{\omega\gamma}\frac{e^{i\varphi}}{D_\omega(m)}G_\omega \sum_i \varepsilon_\mu^i(\omega)\varepsilon_\nu^{i*}(\omega) \right] r^\nu \quad (\text{A.2})$$

where $r = p_1 - p_2$, $\varepsilon_\mu^i(\gamma)$, $\varepsilon_\mu^i(\rho)$, $\varepsilon_\mu^i(\omega)$ are the polarization vectors of respectively the virtual photon, the ρ^0 and ω mesons, with helicity $i = 0, \pm 1$ in the e^+e^- center of mass system. The first term in this expression describes a (possible) non resonant contribution to the pion electromagnetic form factor $F_\pi(s)$ [44][45] ; the parameter A is *a priori* an arbitrary function of m . Introducing this term allows for a non-VDM component to $F_\pi(s)$ and has been considered in the early stages of e^+e^- physics[44][45]. The two D functions in rel. (A2) are the (Breit-Wigner) inverse propagators :

$$\begin{cases} D_\rho(m) = (m_\rho^2 - m^2) - im_\rho\Gamma_\rho(m) \\ D_\omega(m) = (m_\omega^2 - m^2) - im_\omega\Gamma_\omega(m_\omega) \end{cases} \quad (\text{A.3})$$

The phase factor φ (expected to be $\simeq 100^\circ$) describes the isospin 1 component of the actual ω meson[44][52]. Because the ω is a sharp resonance and its decay is dominated by three body channels, we approximate its total width by its constant tabulated value[11] $\Gamma_\omega(m_\omega)$. The situation is quite different for the ρ meson, then we choose to parametrize its decay amplitude by :

$$\Gamma_\rho(m) = \Gamma_\rho(m_\rho) \left[\frac{q_\pi(m)}{q_\pi(m_\rho)} \right]^3 \left[\frac{m_\rho}{m} \right]^\lambda, \quad (\text{A.4})$$

where λ is a parameter to be determined from fits to experimental data ; this is the simplest way to account for the influence of the left-hand singularities on the physical region $s \geq 4m_\pi^2$. Other ways exist such as that in refs.[31] or [39] which are not actually more meaningful, or less. In relativistic physics the notion of penetration factor is indeed as *ad hoc* as the one we propose.

The parameters $g_{\rho\gamma}$ and $g_{\omega\gamma}$ are related with the leptonic decay widths of the vector mesons[44][45] :

¹⁷Woody Allen.

$$\Gamma_V(e^+e^-) = \frac{4\pi\alpha_{em}^2}{3m_V^3} g_{V\gamma}^2, \quad V = \rho, \omega. \quad (\text{A.5})$$

Finally, the quantities G_ρ and G_ω are defined according relation (7) (see text) by :

$$G_V = \frac{1}{q_\pi^{3/2}(m)} \sqrt{6\pi m_V m \Gamma_V(\pi^+\pi^-)}, \quad V = \rho, \omega. \quad (\text{A.6})$$

In the case of the ρ meson, this relation together with relation (A4) completely determines G_ρ ; in the case of the ω meson, we use :

$$\Gamma_\omega(m) = Br_\omega(\pi^+\pi^-) \Gamma_\omega(m_\omega) \left[\frac{q_\pi(m)}{q_\pi(m_\omega)} \right]^3 \frac{m_\omega}{m}, \quad (\text{A.7})$$

where $Br_\omega(\pi^+\pi^-)$ is the $\pi^+\pi^-$ branching fraction in the ω decay. Neglecting the electron mass and using standard rules[40], it is easy to compute the cross section. After integration over angles, we find :

$$\sigma(m) = \frac{8\pi\alpha_{em}^2}{3} \frac{q_\pi^3(m)}{m^5} \left| A + \frac{g_{\rho\gamma}}{D_\rho(m)} G_\rho + \frac{g_{\omega\gamma}}{D_\omega(m)} e^{i\varphi} G_\omega \right|^2. \quad (\text{A.8})$$

Setting $A = 0$ and $g_{\omega\gamma} = 0$, this relation becomes :

$$\sigma(m) = 16\pi^2 \alpha_{em}^2 \frac{g_{\rho\gamma}^2}{m^4} \frac{m_\rho \Gamma_\rho(m)}{(m_\rho^2 - m^2)^2 + m_\rho^2 \Gamma_\rho^2(m)}, \quad (\text{A.9})$$

which coincides with the corresponding equation in ref.[44] (see last equation on page 46) ; this proves that the convention (A6) allows us to recover the usual expression in e^+e^- physics, as it does in hadronic physics (see text).

Putting all the results together leads to the model cross section :

$$\sigma(m) = \frac{12\pi}{m^4} \left| A \frac{\alpha_{em}}{3} \left[\frac{2q_\pi^3(m)}{m} \right]^{1/2} + \frac{m_\rho^2 \sqrt{\Gamma_\rho(e^+e^-) \Gamma_\rho(m)}}{D_\rho(m)} + e^{i\varphi} \frac{m_\omega^2 \sqrt{\Gamma_\omega(e^+e^-) \Gamma_\omega(m)}}{D_\omega(m)} \right|^2. \quad (\text{A.10})$$

Let us remark that this model is in some sense minimal. Extending it to include the ϕ meson is straightforward following the lines of the ω meson. Reliably including higher vector mesons is another matter, mainly because it is difficult to model their decay amplitudes.

In fitting the e^+e^- data, we shall fix the ω parameters at their tabulated values[11] as well as both leptonic decay widths. On the other hand, we shall follow two lines : first by fitting the ρ parameters (and the phase factor φ) assuming $A = 0$, second by leaving A free but with the ρ mass fixed at the world average value[11].

Let us also point out that expression (A10) gives the absolute cross section without any normalization factor.

Appendix B : A model for $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays

We consider that the decay $X \rightarrow \pi^+\pi^-\gamma$, X being either of η , η' , is described by adding the amplitudes associated with the elementary processes sketched in figures (1b) and (5). We shall discuss later on possible additional contributions.

B1 The model

Let us first fix some notations. We denote p_1 and p_2 the 4-momenta of the two outgoing pions ; we shall show later on that it does not matter which one is associated with π^+ or with π^- . We call P the 4-momentum of the decaying meson X . We define the photon 4-momentum k and we call $\varepsilon_j(\rho)$, $\varepsilon_i(\gamma)$ respectively the polarization vectors of the outgoing ρ and γ particles ; i, j denote the helicity indices in the X rest frame. Finally, we shall use also $r = p_1 - p_2$, $p = p_1 + p_2$, $p^2 = m^2$ and $P^2 = M^2$ as parameters.

The most general amplitude which describes the decay of a pseudoscalar particle into a photon and a vector meson can be written :

$$T_X^{ij} = F_X \epsilon_{\mu\nu\rho\sigma} \varepsilon_i^\mu(\gamma) \varepsilon_j^\nu(\rho) p^\rho k^\sigma , \quad (\text{B.1})$$

where the vertex function F_X is *a priori* an arbitrary function of m^2 only since we shall approximate M by its constant central value.

The most general amplitude describing the decay of a ρ meson into a $\pi^+\pi^-$ final state has already been given in the main text :

$$T_j = G_\rho \varepsilon_\mu^{j*} r^\mu , \quad (\text{B.2})$$

where G_ρ is also an arbitrary function of m^2 .

Therefore the most general amplitude describing the decay ($X \rightarrow \rho^0\gamma$, $\rho^0 \rightarrow \pi^+\pi^-$) is simply obtained by multiplying amplitudes (B1) and (B2) together with the ρ^0 propagator and summing up the helicity index j of the intermediate ρ^0 meson. Denoting by $D_\rho(m)$ the inverse ρ^0 propagator, it is easy to find :

$$T_i = \frac{G_\rho F_X}{D_\rho(m)} \epsilon_{\mu\nu\rho\sigma} \varepsilon_i^\mu(\gamma) r^\nu p^\rho k^\sigma . \quad (\text{B.3})$$

The helicity index i occurring here is that of the photon. It is easy to check that expressions (B1) and (B3) have all the requested properties : transversity of the ρ meson as well as the correct number of helicity degrees of freedom. For simplicity, it is better to rewrite expression (B3) using the pion 4-momenta :

$$T_i = \frac{2G_\rho F_X}{D_\rho(m)} \epsilon_{\mu\nu\rho\sigma} \varepsilon_i^\mu(\gamma) p_1^\nu p_2^\rho k^\sigma . \quad (\text{B.4})$$

The possible anomalous contribution to the X decay (see rel.(20) in the main text) is :

$$T'_i = E_X \epsilon_{\mu\nu\rho\sigma} \varepsilon_i^\mu(\gamma) p_1^\nu p_2^\rho k^\sigma , \quad (\text{B.5})$$

where E_X is [3] an arbitrary function of two independent invariants ; they can be chosen as $p^2 = m^2$ and $r \cdot k$ for instance.

Because, we have no diagrammatic rule to connect the order of the pion momenta in relations (B4) and (B5), it is clear that the sign obtained when fitting will have no physical meaning. This is moreover true if we keep in mind that G_ρ is known only in absolute value (see rel.(2)) and that F_X is unknown.

Finally, for each photon helicity state, the amplitude describing the process $X \rightarrow \pi^+\pi^-\gamma$ is the sum of expressions (B4) and (B5).

Let us now define the photon momentum in the X rest frame by $k_\gamma = (M^2 - m^2)/2M$ and the pion momentum in the dipion rest frame $q_\pi = 1/2\sqrt{m^2 - 4m_\pi^2}$. Using standard rules[40], it is a simple matter to compute the decay rate :

$$\frac{d\Gamma}{dm} = \frac{1}{48\pi^3} \left| \frac{2G_\rho F_X}{D_\rho(m)} + E_X \right|^2 k_\gamma^3 q_\pi^3 . \quad (\text{B.6})$$

Up to the E term, this expression coincides with some reported models[19][20]26][27] ; possible variants only deal with the definition of G_ρ . The way ref.[23] introduces the non-resonant term is slightly different from ours ; it does not allow a straightforward connection with the box anomaly.

B2 Fixing some model parameters

We choose the function G_ρ using :

$$G_\rho = \sqrt{6\pi \frac{mm_\rho}{q_\pi^2} \Gamma_\rho(m)} , \quad (\text{B.7})$$

the ρ^0 decay amplitude being fitted from experimental data (see section 2.4). We assume that G_ρ is a real function. Finally, we use the standard ρ^0 inverse propagator :

$$D(m) = (m_\rho^2 - m^2) - im_\rho \Gamma_\rho(m) . \quad (\text{B.8})$$

To connect with the predictions of Chanowitz[1][3], we also assume the parameters E_η and $E_{\eta'}$ to be constant. This is of course a strong hypothesis. Finally, we are left with two arbitrary functions $F_X(m)$. The simplest guess is that they are slowly varying functions all over the range where they play a role ($2m_\pi$, M_X). Practically, the simplest hypothesis is to assume them constant.

Assuming the parameter functions F_X constant, we can use a further constraint to fix them, each as a function of the corresponding E_X . Indeed, integrating relation (B6) over m , its left-hand side becomes simply the partial widths $\Gamma(X \rightarrow \pi^+\pi^-\gamma)$ which have been measured with a good accuracy[11]. One should notice that the corresponding equation is of the form :

$$A_X = A_1 F_X^2 + A_2 F_X E_X + A_3 E_X^2 ,$$

therefore, even the relative sign of E_η and $E_{\eta'}$ is not fixed since there is no reason why F_η and $F_{\eta'}$ would have like signs.

To conclude, the model just developed provides a one parameter fit to the data¹⁸, since even the absolute normalization of each spectrum is fixed by the ratio of the integrated model distribution to the experimental one.

B3 Possible additional contributions

Further contributions can be added. The first one coming to mind is the ω contribution in the η' decay. We have discarded it after checking that its branching ratio to $\pi^+\pi^-$ ($\simeq 2\%$), combined with the small branching ratio[11] $\eta' \rightarrow \omega\gamma$ ($\simeq 3\%$) was unable to allow a visible effect with the present available statistics ; this conclusion had already been reached by ref.[23].

Other contributions can be tested in $e^+e^- \rightarrow \pi^+\pi^-$. Because of the existence of the triangle anomaly, contributions like that given in figure (11) can occur. They happen to be fixed by VDM and the $\gamma\gamma$ decays of η and η' and amount to the following changes :

$$\left\{ \begin{array}{l} E_X \Rightarrow E_X + \frac{2AB_X}{m^2} \\ \frac{2G_\rho F_X}{D_\rho(m)} \Rightarrow \frac{2G_\rho}{D_\rho(m)} \left[F_X + \frac{g_{\rho\gamma} B_X}{m^2} \right] \end{array} \right. \quad (\text{B.9})$$

in relation (B6), where A represents the magnitude of the $\gamma\pi^+\pi^-$ coupling (see rel.(A2)), $g_{\rho\gamma}$ is the VDM coupling (see rels. (A2), (A5)) and B_X is related with the two-photon decay widths (see rel.(19)).

These additional contributions are non-leading since they are suppressed by two powers of the dipion invariant mass m ; we have furthermore checked that they are also numerically suppressed because of the smallness of the two-photon decay widths. Therefore it is natural to remove them ; this is a fortunate situation since this avoids problems with doubly counting some contributions.

¹⁸In the early stages of this work, we allowed a constant phase for F_X , but it never significantly improved the fit. Until more accurate data on η' are available, it is useless to keep this freedom of choice.

Appendix C : Consequences of Vector Dominance Model (VDM)

In this appendix, we recall an expression of ξ given by Chanowitz[2][3] using the vector dominance hypothesis. We also derive a relation between the couplings $\eta\rho\gamma$, $\eta'\rho\gamma$ and the box and triangle anomaly constants.

C1 The value of ξ from VDM

In ref. [2] and [3], Chanowitz also proposed to test the value of ξ using vector meson dominance. The relation of relevance (see rel.(49) in ref. [3]) can be written :

$$\xi^2 = \left[\frac{3f_\rho}{4e} \right]^2 \frac{|B_{\eta'}|^2 + |B_\eta|^2 - \frac{1}{3}|B_{\pi^0}|^2}{|F_{\eta'}|^2 + |F_\eta|^2 - \frac{1}{3}|C_\omega|^2} \quad (\text{C.1})$$

where B_η , $B_{\eta'}$, F_η , $F_{\eta'}$ have been already defined (see rels (17) and (B1)), $e = \sqrt{4\pi\alpha_{em}}$ and [3] $f_\rho/4\pi = 2.96$. B_{π^0} is defined by the $\pi^0 \rightarrow \gamma\gamma$ coupling[6] :

$$A(\pi^0 \rightarrow \gamma\gamma) = B_{\pi^0} \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu(\gamma_1) k_1^\nu \varepsilon^\rho(\gamma_2) k_2^\sigma, \quad B_{\pi^0} = \frac{\alpha_{em}}{2\pi f_\pi}, \quad (\text{C.2})$$

where $\varepsilon(\gamma_i)$ and k_i are respectively the polarization vectors and 4-momenta of both photons ($i=1,2$). Finally C_ω is related to the decay $\omega \rightarrow \pi^0\gamma$, the amplitude of which being :

$$A(\omega \rightarrow \pi^0\gamma) = C_\omega \epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu \varepsilon^{\rho*}(\omega) \varepsilon^\sigma(\gamma), \quad (\text{C.3})$$

where p and q are the 4-momenta of respectively the ω and γ and $\varepsilon(\omega)$ and $\varepsilon(\gamma)$ are the corresponding polarization vectors.

Indeed, it is easy to see that our F_η , $F_{\eta'}$ are nothing but $\mathcal{M}(\rho \rightarrow \eta\gamma)$ and $\mathcal{M}(\eta' \rightarrow \rho\gamma)$ of ref. [3].

The parameter C_ω can be deduced from experimental data[11] using :

$$\Gamma(\omega \rightarrow \pi^0\gamma) = \frac{[m_\omega^2 - m_{\pi^0}^2]^2}{48\pi m_\omega} |C_\omega|^2. \quad (\text{C.4})$$

C2 A new relation using VDM

Let us define the quantities B_{η_1} and B_{η_8} by :

$$\begin{cases} B_{\eta_1} = B_{\eta'} \cos \theta_{PS} - B_\eta \sin \theta_{PS} \\ B_{\eta_8} = B_{\eta'} \sin \theta_{PS} + B_\eta \cos \theta_{PS} \end{cases} \quad (\text{C.5})$$

and correspondingly :

$$\begin{cases} F_{\eta_1} = F_{\eta'} \cos \theta_{PS} - F_\eta \sin \theta_{PS} \\ F_{\eta_8} = F_{\eta'} \sin \theta_{PS} + F_\eta \cos \theta_{PS} \end{cases} \quad (\text{C.6})$$

Using SU(3) symmetry and vector meson dominance, Chanowitz[2][3], has proved a relation (see rel. (7) in ref. [2] or rel. (43) in ref. [3]) which can be rewritten using our notations :

$$|B_{\eta}|^2 = \frac{16}{9} \frac{e^2}{f_\rho^2} \xi^2 |F_{\eta}|^2 \quad . \quad (C.7)$$

On the other hand from SU(3) flavour symmetry (see rel. (47) and (48) in ref. [3]) and the Gell Mann–Sharpe–Wagner relation (see rel. (51) in ref. [3]), it is easy to show that we have also :

$$|B_{\eta_8}|^2 = \frac{4}{9} \frac{e^2}{f_\rho^2} |F_{\eta_8}|^2 \quad . \quad (C.8)$$

From rels. (C5)–(C8), it is easy to find :

$$|F_{\eta_1}|^2 + |F_{\eta_8}|^2 = \frac{4}{9} \frac{e^2}{f_\rho^2} \left[|B_{\eta_8}|^2 + \frac{1}{4\xi^2} |B_{\eta}|^2 \right] \quad (C.9)$$

Using rels. (C5) , (C6) and (17), this relation can be rewritten :

$$|F_{\eta}|^2 + |F_{\eta'}|^2 = \frac{3\alpha_{em}}{4\pi^2} \frac{f_\rho^2}{4\pi} \left[\frac{1}{f_8^2} + \frac{2}{f_1^2} \right] \quad , \quad (C.10)$$

where $f_\rho^2/4\pi = 2.96$.

This expression has the nice feature to be independent of ξ and connect the $\eta\rho^0\gamma$ and $\eta'\rho^0\gamma$ couplings to the anomalies. We expect the conditions of validity[2] of rel. (C10) to be analogous to that of (C1) ; *i.e.* they are surely valid if $\xi = 1$ but more questionable if $\xi = 2$.

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Figure Captions

- Figure 1 : (a) The triangle anomaly responsible of the π^0 , η , η' decays into $\gamma\gamma$, (b) The box anomaly responsible of the non-resonant contribution to the η and η' decays into $\pi^+\pi^-\gamma$.
- Figure 2 : One of the four contributions to the pion electromagnetic form factor at large photon invariant mass.
- Figure 3 : $e^+e^- \rightarrow \pi^+\pi^-$ cross section. Data are coming from refs.[48][49]. The curve superimposed is the result of the fit assuming only contributions of the ρ and ω mesons.
- Figure 4 : $e^+e^- \rightarrow \pi^+\pi^-$ cross section. Data are coming from refs.[48][49]. The curve superimposed is the result of the fit assuming contributions of the ρ and ω mesons and also a non-VDM contribution.
- Figure 5 : Contribution of the ρ meson to the η/η' decay amplitude. This contribution is considered to add up to that of figure (1b).
- Figure 6 : Photon momentum spectrum and fit in $\eta \rightarrow \pi^+\pi^-\gamma$ data. In (a) the histograms correspond to model M1 with E_η free (dashed histogram) or with $E_\eta \equiv 0$ (dotted histogram). In (b) the corresponding histograms obtained with model M2 are plotted.
- Figure 7 : Dipion mass spectra and fits in $\eta' \rightarrow \pi^+\pi^-\gamma$ data. The model used to fit is M1.
- Figure 8 : The box anomaly parameter $E_{\eta'}$ in all experiments ; the model used is model M2 ($e^+e^- \rightarrow \pi^+\pi^-$ data fitted without a non-VDM contribution).
- Figure 9 : The box anomaly parameter $E_{\eta'}$ in all experiments ; the model used is model M1 ($e^+e^- \rightarrow \pi^+\pi^-$ data fitted with a non-VDM contribution).

Figure 10 : f_π/f_1 (a,b), f_π/f_8 (c,d) as functions of E_η and $E_{\eta'}$ in the case $\xi = 1$. The corresponding curves for the case $\xi = 2$ are displayed in (e) to (h). The experimental intervals E_η and $E_{\eta'}$ are represented. The drawing takes into account the measurement errors due to $\eta/\eta' \rightarrow \gamma\gamma$ decay widths. Values of θ_{PS} (which parametrizes the curves) are located by $\star$ symbols.

Figure 11 : Contributions to the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays associated with the triangle anomaly. These contributions are non-leading.

(a)

(b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

a

b

c

d

Fig. 10

e

f

g

h

Fig. 10 (cont.)

Fig.11