CERN-PPE/92-122
CERN/LAA/MSL/92-012
20 July, 1992

**A χ^2 -TEST TO STUDY THE $\alpha_1\alpha_2\alpha_3$ CONVERGENCE FOR
HIGH PRECISION LEP DATA, HAVING IN MIND THE
SUSY THRESHOLD**

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Abstract

A χ^2 -test is proposed in order to establish in a physically sound way the meaning of the convergence of the $\alpha_1, \alpha_2, \alpha_3$ couplings, taking correctly into account the high precision LEP data. The naïve belief that SUSY breaking must be in the TeV range in order to obtain the "best" convergence of the couplings is shown not to be correct.

Submitted to "Il Nuovo Cimento"

1. INTRODUCTION

LEP allows to measure, with high precision, in the Z^0 -mass range, the couplings associated with all known gauge forces (excluding gravity). The purpose of this paper is to discuss how the high precision LEP data can be used to understand what happens with Supersymmetry. The fact that Supersymmetry was needed to unify $SU(3) \times SU(2) \times U(1)$ is known since 1979 [1]. The difficulty is in solving the many problems arising when the evolution of the couplings needs to be worked out over more than ten orders of magnitude in energy. The Renormalization Group Equations (RGEs) allow a rigorous extension of our knowledge over many orders of magnitude, provided we take into due account the physical effects connected with the evolution of the couplings (EC). These effects are the light threshold (ΔT_L), the heavy threshold (ΔT_H) and the evolution of masses (EGM), as we will see later.

Let us suppose that a Supergroup (for example SUSY [$SU(5)$]) containing the known gauge groups really exists. At some energy level this Supergroup will split into the three known gauge groups SUSY [$SU(3) \times SU(2) \times U(1)$], whose couplings $\alpha_1, \alpha_2, \alpha_3$ have the same value α_{GUT} , just before the splitting occurs.

Let us call E_{GUT} the energy where:

$$(1) \quad \alpha_1(E_{GUT}) = \alpha_2(E_{GUT}) = \alpha_3(E_{GUT}) = \alpha_{GUT}(E_{GUT})$$

Just below E_{GUT} , at some energy M_X , if we had a collider, we would discover the production of very heavy particles and could measure the "heavy threshold" (ΔT_H) with great accuracy. These threshold effects cannot be neglected if we want to know how and where the couplings $\alpha_1, \alpha_2, \alpha_3$ converge towards the same value, as indicated in formula (1). In all we have said so far we have assumed that Supersymmetry is valid at E_{GUT} and somewhere below it. Where? At low energy "our world" is not supersymmetric and even at LEP and Fermilab energies so far there is no trace of SUSY particles [2].

For those who believe in Supersymmetry, sooner or later, the first SUSY particle will show up. Let us call the energy level for the production of SUSY particles M_{SUSY} . If we had a collider working at this (unknown) energy, we could observe the detailed spectrum of all SUSY particles and measure the thresholds with great accuracy. These

threshold effects cannot be neglected if we want to know how and where the couplings $\alpha_1, \alpha_2, \alpha_3$ converge, as indicated in formula (1). In the difficult problem of studying the convergence of the couplings (CC) towards the condition (1) there are three energy levels:

M_Z : where high precision measurements are performed;

M_{SUSY} : where the SUSY threshold is;

M_X : where the heavy threshold is.

The detailed spectrum of the light and heavy thresholds is a further complex problem connected with our main task (CC).

The purpose of this paper is to study if high precision LEP data can allow to know where $\alpha_1, \alpha_2, \alpha_3$ unify and where the SUSY threshold could be.

The structure of the paper is as follows. In sect. 2 we use the $\alpha_1, \alpha_2, \alpha_3$ evolution equations with SUSY coefficients at the two loop level in order to study the convergence of the couplings at some E_{GUT} , taking into account the heavy threshold effects. In sect. 3 we define a χ^2 -test in order to compute the best M_X value and the best $\alpha_3(M_Z)$. In sect. 4 we study the light threshold effects as a cross-check of the results obtained in sect. 3. In sect. 5 we compare our χ^2 -test with other claims. The conclusions are in sect. 6.

2 THE EVOLUTIONS OF $\alpha_1, \alpha_2, \alpha_3$ USING THE TWO-LOOP APPROXIMATION

Among all possible SUSY-GUTs we choose minimal SUSY-SU(5) [3-6] whose coupling α_5 evolves above E_{GUT} following α_3 . The evolution of the couplings $\alpha_1, \alpha_2, \alpha_3$ is worked out starting from some very high energy, E_{MAX} , and going towards lower energies.

Here are the three evolution equations at the two-loop level:

$$(2) \quad \frac{1}{\alpha_1(\mu_1^2)} - \frac{1}{\alpha_1(\mu_0^2)} = - \frac{1}{24\pi} \times [39.6 \log(\mu_1^2/\mu_0^2) - 57.6 F^{(1)}(\mu_1^2, \mu_0^2, M_X^2)] + T_1 + F_1^{(2)}(\mu_1^2, \mu_0^2, M_X^2)$$

$$(3) \quad \frac{1}{\alpha_2(\mu_1^2)} - \frac{1}{\alpha_2(\mu_0^2)} = - \frac{1}{24\pi} \times [6 \log(\mu_1^2/\mu_0^2) -$$

$$(4) \quad \frac{1}{\alpha_3(\mu_1^2)} - \frac{1}{\alpha_3(\mu_0^2)} = \frac{18}{24\pi} \log(\mu_1^2/\mu_0^2) + T_3 - 24 F^{(1)}(\mu_1^2, \mu_0^2, M_X^2) + T_2 + F_2^{(2)}(\mu_1^2, \mu_0^2, M_X^2)$$

The T_i 's are defined as follows:

$$(5) \quad T_i = \sum_{j=1}^3 \frac{b_{ij}}{4\pi b_j} \log \frac{\alpha_j(\mu_1^2)}{\alpha_j(\mu_0^2)}$$

where the b_j and b_{ij} coefficients are given [5] by:

$$(6) \quad \{b_j\} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix}$$

$$(7) \quad \{b_{ij}\} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}$$

and the F-functions by (see appendix of ref. [7]):

$$(8) \quad F^{(1)}(\mu_1^2, \mu_0^2, M_X^2) = \log \left(\frac{\mu_1^2 + 4 M_X^2}{\mu_0^2 + 4 M_X^2} \right)$$

$$(9) \quad F_j^{(2)}(\mu_1^2, \mu_0^2, M_X^2) = (T_3 - T_j) \frac{F^{(1)}(\mu_1^2, M_Z^2 \cdot \mu_1^2/\mu_0^2, M_X^2)}{F^{(1)}(\mu_1^2, M_Z^2, M_X^2)}.$$

As mentioned in the introduction, the α_1 , α_2 and α_3 are the couplings of the gauge groups SU(3), SU(2), U(1), while μ_1 is the extreme energy value, $E^{\text{MAX}} = \mu_1$, and μ_0 is the low-energy point where input-output quantities can be experimentally measured [8]. Table I shows the high precision LEP data adopted. As the key condition is the convergence of the couplings (CC), expressed in (1), we start with only one input, $\alpha_3(M_Z)$:

$$\alpha_3(\mu_0) = \alpha_3(M_Z).$$

With the two-loop equations (2)-(4) we use an iteration procedure and evolve α_3 , from its value at the Z^0 mass (which, we repeat, is our unique input) up to an arbitrary high energy scale where the unification

condition (1) is assumed to have already occurred. Then we evolve α_1 , α_2 , α_3 down to the Z^0 mass (or μ_0) and get from the equations (2)-(4) the values for M_X , $\alpha_{em}(M_Z)$ and $\sin^2\theta(M_Z)$. These three results are worked out as a function of the single input, $\alpha_3(M_Z)$.

In fig. 1 we show the correlation between the values of $\sin^2\theta(M_Z)$ and $\alpha_3(M_Z)$. It is interesting to remark that, in order to approach the world average value for $\sin^2\theta(M_Z)$, it is necessary to have for $\alpha_3(M_Z)$ a value slightly above the world average, even if the world average $\alpha_3^{WA} = 0.118$ is certainly well fitted by the two-loop approximation. Figure 2 shows that $1/\alpha_{em}(M_Z)$ varies less with $\alpha_3(M_Z)$. However as for $\sin^2\theta(M_Z)$, slightly higher values than the world average are preferred for $\alpha_3(M_Z)$ if $\alpha_{em}(M_Z)$ has to be at its world average value^{*}).

3. THE χ^2 -TEST PROPOSED TO USE THE HIGH PRECISION LEP DATA

Having investigated the correlation between the three experimentally measured quantities, we now proceed to get the best value for M_X . For this we need to define the χ^2 . There are three experimentally measured quantities. The quantity $\alpha_3(M_Z)$ has been chosen as the input. The χ^2 can only be defined using the other two quantities, i.e.:

$$(10) \quad \chi^2 = \left(\frac{\sin^2\theta(M_Z) - \sin^2\theta(M_Z)^{exp}}{\sigma_{\sin^2\theta}} \right)^2 + \left(\frac{\alpha_{em}(M_Z) - \alpha_{em}(M_Z)^{exp}}{\sigma_{\alpha_{em}}} \right)^2$$

The results are shown in fig. 3 where the "best" value for M_X is

$$(11) \quad M_X = 2 \times 10^{16} \text{ GeV.}$$

The quantity M_X represents the mass of the superheavy gauge bosons which become massive as a result of the SUSY-SU(5) breaking. This threshold must exist if it is wanted that the unique value of α_5 splits into three values: α_1 , α_2 , α_3 , corresponding to SUSY $[SU(3) \times SU(2) \times U(1)]$.

^{*}) We recall that $\sin^2\theta = \alpha_1/(\alpha_1 + 5/3 \cdot \alpha_2)$ (where the 5/3 coefficient is the SU(5) normalization factor).

We assume for the superheavy bosons a degenerate spectrum: as if all superheavy particles, which become massive because of SUSY-SU(5) breaking, had the same mass. However we perform an exact treatment of the threshold.

If we call E_{GUT} the energy where the three β -functions can be considered as being equal, the mass value of the superheavy bosons M_X is about two times smaller than E_{GUT} . We show in fig. 4 the range of variation of M_X versus the input values of $\alpha_3(M_Z)$.

Figure 5 shows the smooth convergence of $\alpha_1, \alpha_2, \alpha_3$ towards E_{GUT} and the unified evolution of the couplings above E_{GUT} . Notice that smooth convergence towards E_{GUT} and unified evolution above E_{GUT} should be the key features for all attempts aimed at giving physical significance to the fact that three gauge couplings $\alpha_1, \alpha_2, \alpha_3$ unify at some E_{GUT} (CC).

Let us remark that the convergence of the couplings has as neighbour the heavy threshold, while the SUSY threshold is far away. Both threshold effects should be included in the evolution equations of the couplings. The heavy threshold is certainly closer to E_{GUT} than the SUSY threshold and therefore in no case can it be neglected when dealing with CC problems (1).

Now we want to use the information coming from the fact that the heavy threshold has been introduced into the evolution equations, to see what is the best value of $\alpha_3(M_Z)$. To this purpose we use the same χ^2 definition (10).

The next step is to apply the χ^2 -test to $\alpha_3(M_Z)$. We thus use different input values of $\alpha_3(M_Z)$ in our iterative process with the evolution equations (2)-(4). To each $\alpha_3(M_Z)$ will correspond a χ^2 minimum (10). The distribution of these minima is shown in fig. 6. It is interesting to note that the value of $\alpha_3(M_Z)$ which minimizes the distribution of these minimum values of χ^2 is

$$(12) \quad \alpha_3(M_Z) = 0.121$$

i.e. a value slightly above the world average (WA) (see table I). This value for $\alpha_3(M_Z)$ represents the best fit coming from the RGEs once the heavy threshold effect, to insure a smooth convergence towards E_{GUT} and a unified evolution above E_{GUT} , has been accounted for.

The striking feature of our analysis is that the evolution equations for

$\alpha_1, \alpha_2, \alpha_3$ show an excellent smooth convergence at E_{GUT} without any necessity to introduce SUSY breaking above M_Z . This means that, insofar as the wanted feature is CC (1), the SUSY threshold can be as low as the Z^0 mass. Therefore the light threshold ΔT_L is expected to be very far away from ΔT_H .

As a cross-check to complete our work we need to study the light threshold effects, using as input parameter the best fit (12) for $\alpha_3(M_Z)$.

4. THE LIGHT THRESHOLD

In a recent paper we have studied the evolution of the couplings (EC) taking into account a detailed SUSY spectrum (at one loop) and the evolution of the gaugino masses (EGM) at two loops [9]. Here are the basic equations:

$$(13) \quad \frac{1}{\alpha_1(M_Z)} = \frac{3}{5} \alpha_{\text{em}}^{-1}(1 - \sin^2 \theta) = \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[\frac{33}{5} \log\left(\frac{E_{\text{GUT}}}{M_Z}\right)^* - \right. \\ \left. - \frac{49}{60} \log\left(\frac{m_{\tilde{q}}}{M_Z}\right)^* - \frac{17}{30} \log\left(\frac{m_t}{M_Z}\right)^* - \frac{1}{60} \log\left(\frac{m_{\tilde{t}_L}}{M_Z}\right)^* - \right. \\ \left. - \frac{4}{15} \log\left(\frac{m_{\tilde{t}_R}}{M_Z}\right)^* - \frac{3}{10} \log\left(\frac{m_{\tilde{t}_L}}{M_Z}\right)^* - \frac{3}{5} \log\left(\frac{m_{\tilde{t}_R}}{M_Z}\right)^* - \right. \\ \left. - \frac{2}{5} \log\left(\frac{m_{\tilde{h}}}{M_Z}\right)^* - \frac{1}{10} \log\left(\frac{m_H}{M_Z}\right)^* \right] + \frac{C_1}{4\pi}$$

$$(14) \quad \frac{1}{\alpha_2(M_Z)} = \frac{\sin^2 \theta}{\alpha_{\text{em}}} = \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[\log\left(\frac{E_{\text{GUT}}}{M_Z}\right)^* - \log\left(\frac{m_{\tilde{q}}}{M_Z}\right)^* - \right. \\ \left. - \log\left(\frac{m_t}{M_Z}\right)^* - \frac{1}{2} \log\left(\frac{m_{\tilde{t}_L}}{M_Z}\right)^* - \frac{1}{2} \log\left(\frac{m_{\tilde{t}_R}}{M_Z}\right)^* - \right. \\ \left. - \frac{2}{3} \log\left(\frac{m_{\tilde{h}}}{M_Z}\right)^* - \frac{1}{6} \log\left(\frac{m_H}{M_Z}\right)^* - \frac{4}{3} \log\left(\frac{m_{\tilde{w}}}{M_Z}\right)^* \right] + \frac{C_2}{4\pi}$$

$$(15) \quad \frac{1}{\alpha_3(M_Z)} = \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{2\pi} \left[-3 \log\left(\frac{E_{\text{GUT}}}{M_Z}\right)^* - 2 \log\left(\frac{m_{\tilde{g}}}{M_Z}\right)^* - \right. \\ \left. - \frac{2}{3} \log\left(\frac{m_t}{M_Z}\right)^* - \frac{5}{3} \log\left(\frac{m_{\tilde{q}}}{M_Z}\right)^* - \frac{1}{6} \log\left(\frac{m_{\tilde{t}_L}}{M_Z}\right)^* - \right. \\ \left. - \frac{1}{6} \log\left(\frac{m_{\tilde{t}_R}}{M_Z}\right)^* \right] + \frac{C_3}{4\pi}$$

Where: $\log(x)^* \equiv \Theta(x-1) \cdot \log(x)$ and $\Theta(y) \equiv \begin{cases} 0 & \text{if } y < 0 \\ 1 & \text{if } y > 0 \end{cases}$. And the C_i stand for the following combinations of two-loop matrix coefficients:

$$(16) \quad C_i = \sum_j \frac{b_{ij}^s}{b_j^s} \log\left(\frac{\alpha_{\text{GUT}}}{\alpha_j(M_Z)}\right) + \left(\sum_j \frac{b_{ij}'}{b_j'} - \sum_j \frac{b_{ij}^s}{b_j^s} \right) \log\left(\frac{\alpha_j(m_{1/2})}{\alpha_j(M_Z)}\right)$$

for $i = 1, 2, 3$.

The b_{ij}^s and b_j^s are supersymmetric two-loop and one-loop coefficients of the β function, respectively. The b_{ij}' and b_j' are the corresponding non-supersymmetric ones.

To the equations (13)-(15), we apply our iterative procedure [10] using as input values for $\alpha_3(M_Z)$ the result of our best fit (12) and for the other inputs, $\sin^2\theta(M_Z)$ and $\alpha_{\text{em}}(M_Z)$ their world average values (table I).

The result is that M_{SUSY} can be as low as the Z^0 mass. This is an excellent cross-check. In fact our study, intended to work out a χ^2 -test for the three experimentally measured quantities $\sin^2\theta(M_Z)$, $\alpha_3(M_Z)$, $\alpha_{\text{em}}(M_Z)$ in terms of the $\alpha_1, \alpha_2, \alpha_3$ convergence (CC) at some E_{GUT} value, indicated that M_{SUSY} could be as low as the Z^0 mass.

5. COMPARISON BETWEEN OUR χ^2 -TEST AND OTHER CLAIMS

Some Authors [11] have claimed to be able to know where the SUSY breaking should be, by constructing a χ^2 -test based on the goodness of the $\alpha_1, \alpha_2, \alpha_3$ convergence.

5.1- The weak points of ref. [11]

Although some weak points have been discussed in the literature [8, 12-14] we would like to discuss them in more detail here, since they appear not to be generally known.

Here they are:

- i) Only one measurement of $\alpha_3(M_Z)$ has been taken as input, not the world average.

- ii) Only one (presumably not the most accurate) of the many possible solutions of the $\alpha_1, \alpha_2, \alpha_3$ coupled evolution equations has been considered.
- iii) The confidence level chosen to quote the uncertainties has been the lowest possible $\pm 1\sigma$.
- iv) A χ^2 -test deprived of any physical meaning to study the goodness of the $\alpha_1, \alpha_2, \alpha_3$ convergence at E_{GUT} has been adopted.
- v) The light threshold has been accounted for assuming mass degeneracy for all SUSY particles and using a step function.
- vi) The heavy threshold has been ignored, despite the emphasis on the search for $\alpha_1, \alpha_2, \alpha_3$ convergence at E_{GUT} .
- vii) The evolution of masses has been ignored.
- viii) The divergence of couplings above E_{GUT} has been ignored.

The first three weak points quoted above have already been discussed in our first paper [8], while points iv), vi) and viii) have been briefly mentioned in another paper of our group [7]. The problems raised by the light threshold (point v above) have been discussed by our group in two papers [9,10], while the evolution of masses (point vii) has been the subject of two other papers [10,15]. In the present paper we concentrate our study on the points iv), vi) and viii).

5.2- The χ^2 -test of ref. [11]

The Authors of ref. [11] emphasised that their goal was the goodness of the $\alpha_1, \alpha_2, \alpha_3$ convergence (CC) but ignored the heavy threshold needed to allow the breaking of the Super Group into SUSY [$\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$]. If the physical requirement is the “convergence” of the $\alpha_1, \alpha_2, \alpha_3$ couplings, the χ^2 -test should not be of “geometrical” nature. The fact that the two lines describing the evolution of two couplings have to meet is granted by Euclidean geometry.

What a Grand Unified Theory (GUT) wants is condition (1): this is not guaranteed by Euclidean geometry. To fulfil (1) the Authors of ref. [11] have defined a χ^2 -test where there is no reference to the three basic effects (heavy threshold, light threshold, evolution of masses) needed to describe the evolution of the couplings and therefore their convergence. Nevertheless the Authors claim to study how well the condition (1) can be satisfied using high precision LEP data. In the high energy range where the couplings should converge the χ^2 ignores the heavy threshold

effects on the convergence, of the couplings; the χ^2 is purely “geometrical”.

6. CONCLUSIONS

The $\alpha_1, \alpha_2, \alpha_3$ evolution equations (2)-(4) do guarantee two features:

1- A smooth convergence of $\alpha_1, \alpha_2, \alpha_3$ up to E_{GUT} . This is due to the treatment of the M_X threshold.

2- A fully unified evolution of $\alpha_1, \alpha_2, \alpha_3$, above E_{GUT} .

The detailed evolution for $\alpha_1, \alpha_2, \alpha_3$, showing the above quoted physically expected features, is reported in fig. 5, where the evolutions are calculated using as input value the world average for α_3 , i.e.:

$$\alpha_3^{\text{WA}}(M_Z) = 0.118$$

There is a crucial point to emphasise: the results presented herein show that the smooth convergence of the gauge couplings towards E_{GUT} and their unified evolution above E_{GUT} do not need any SUSY breaking all along the range from E_{GUT} down to $E = 10^2$ GeV. This is compatible with the threshold for SUSY breaking being in the 10^2 GeV range. The χ^2 -test from the E_{GUT} range on the high precision LEP data provides the “best” value for $\alpha_3(M_Z)$. This value is used for the detailed light thresholds study including EGM effects (at two loops) and confirms that the SUSY threshold can be as low as the Z^0 -mass range: a cross-check of great interest. The naïve belief that SUSY breaking is needed in the TeV range in order to obtain the “best” convergence [11] of the gauge couplings $\alpha_1, \alpha_2, \alpha_3$, is shown not to be correct. What is needed is a detailed study of the effects expected to exist below E_{GUT} and a set of correct equations which take into account the smooth convergence towards E_{GUT} . Once all this is done, the convergence of the couplings (CC) is “perfect” and M_{SUSY} can be as low as the energy range of LEP I, Tevatron, HERA and LEP II.

$\alpha_3(M_Z) = 0.118 \pm 0.008$
$\sin^2\theta(M_Z) = 0.2334 \pm 0.0008$
$1/\alpha_{\text{em}}(M_Z) = 127.9 \pm 0.2$

Table I
High precision LEP data.

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Figure Captions

- Figure 1 2-loop results for $\sin^2\theta_W(M_Z)$ as a function of the input $\alpha_3(M_Z)$ in the range (World Average $\pm 2\sigma$).
- Figure 2 2-loop results for $1/\alpha_{em}(M_Z)$ as a function of the input $\alpha_3(M_Z)$ in the range (World Average $\pm 2\sigma$).
- Figure 3 The χ^2 values versus M_X . The χ^2 depends on the values of $\sin^2\theta_W(M_Z)$ and $\alpha_{em}(M_Z)$ predicted by our coupled evolution equations, as explained in the text. Note that the only input of our model is $\alpha_3(M_Z)$ and the chosen values are in the range $\pm 2\sigma$ around the World Average value.
- Figure 4 2-loop results for the heavy mass M_X as a function of the input $\alpha_3(M_Z)$ in the range (World Average $\pm 2\sigma$).
- Figure 5 The smooth convergence of α_1 , α_2 , α_3 towards E_{GUT} and the unified evolution above E_{GUT} , following our model. We emphasise that our model has only one input: $\alpha_3(M_Z)$. The value of τ_p shown is computed using the “best” fit for M_X .
- Figure 6 The distribution of the χ^2 -minima versus $\alpha_3(M_Z)$ in the range (World Average $\pm 2\sigma$).

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6