Quantum chromodynamics at finite energy with baryon number and strangeness conservation

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We formulate quantum chromodynamics (QCD) in a medium of finite energy and conserved charges using the microcanonical distribution in the real-time formalism. The generating functional and Feynman rules are given and used to calculate perturbatively the entropy of the quark-gluon plasma at finite energy and fixed baryon number. We discuss the relation to the grand canonical formulation and study the implications of our results on the confinement-deconfinement phase transition.

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I. INTRODUCTION

In this paper we discuss quantum chromodynamics at finite energy and conserved quantities, such as the baryon number, using the microcanonical distribution in the path integral formalism. We derive the Feynman rules and also show the relations to quantities in the statistical dynamics. As an application, we discuss the entropy of a strongly interacting system at finite baryon number, and its implications in the confinement-deconfinement phase transition, using the bag model arguments.

In a previous paper, we formulated the quantum field theory at finite energy using the microcanonical distribution in the path integral formulation [1]. In this paper we apply this formulation to quantum chromodynamics (QCD). Our physical motivation in the present work is to investigate the early Universe and the hot hadronic matter created in the high-energy ion collision experiments [2,3]. In the application to the early Universe, the baryon number density is not large. In the late stage of the evolution of the Universe, the canonical description is justified; however, at the early stage of its evolution, the microcanonical description is preferable. In the application to ion collision experiments, the baryon number density may be very large, and hence finite-energy and finite-volume effects are important. Thus this problem is suitable for the microcanonical formulation.

An important issue in the behavior of strongly interacting matter at high energy and/or temperature is its

phase transition from the hadron (confinement) phase to the quark-gluon (deconfinement) phase. An enormous amount of work has already been done on this issue (see, e.g., [4,5]). Among the obtained results, the most interesting ones are given by lattice gauge theory. So far, such investigations have been done in the finite-temperature formulation, namely, in the canonical ensemble formalism. The Monte Carlo simulations of finite-temperature lattice QCD give us an important picture of the phase structure of strongly interacting matter. The subtle problem in lattice simulations is the validity of the thermodynamical limit. By definition, the canonical ensemble implies an infinite volume and energy, but the lattice simulations are performed on a finite-size lattice. Detailed studies are usually needed to check finite-size effects. On the other hand, for a microcanonical ensemble the finite volume does not introduce any difficulties, and thus the microcanonical method is suitable for finite-size lattice simulations. At the present stage, it is not obvious how our formulation of QCD in terms of the microcanonical ensemble can be applied to lattice studies, but knowing the structure of the formulation is helpful for further applications.

In principle, with the microcanonical method we can consider finite systems. However, there exists the technical difficulty how to treat the finite volume generally, when the boundary conditions and hence the eigenfunctions should be specified. Thus we replace the summation over the momenta by a momentum integral as an approximation. In the case of a periodic box, the replacement of $k_i \rightarrow 2\pi n_i/V^{1/3}$, $n_i = \text{integer}$, for the three components of momentum, $i = 1, 2, 3$, can be made in the subsequent formulas. We have numerically checked that such a replacement indeed amounts to an accuracy of better than 5% in all the physical cases treated

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throughout the present paper. Thus, from now on, in all the expressions we formally write down the momenta as if being continuous. Regarding the finite energy and conserved charges such as baryon number and strangeness, however, our formulas treat them exactly.

Our formulation is given in terms of the path integral, which plays an essential role to quantize the gauge theory. Also we will formulate it in such a way that it can be readily applied to the situation with conserved quantities, such as the baryon number and the strangeness.

The paper is organized as follows. In Sec. II, we will present the formulation of QCD at finite energy using the microcanonical ensemble, and the Feynman rules taking the conservation of charges into account. Section III is used for the discussion on the entropy. There, the definition of entropy and its relation to statistical-

dynamical quantities are shown. As an example, we give the results for the entropy at finite baryon number, where its evaluation is done in two ways, saddle-point approximation and exact numerical computation. Section IV is devoted to a discussion of the present formulation and to its perspectives; we also discuss the confinement-deconfinement phase transition using the bag model arguments.

II. FEYNMAN RULES

The following formulation of QCD at finite energy using the microcanonical ensemble is a natural extension of the one in Ref. [1]. Let us start by defining $Z_{E,V,\{Q\}}[J]$, which represents the number density of states at energy E , the volume V and conserved charges $\{Q_j\}$ in the presence of the external source J :

$$\begin{aligned} Z_{E,V,\{Q\}}[J] &= \sum_{\{\Phi\}} \left\langle \Phi, t \left| \delta(\hat{H} - E) \prod_j \delta(\hat{Q}_j - Q_j) \mathcal{T} \left[\exp \left(i \int d^4x J \Phi \right) \right] \right| \Phi, t \right\rangle \\ &= \int \frac{d\alpha}{2\pi} e^{i\alpha E} \prod_j \int \frac{d\sigma_j}{2\pi} e^{i\sigma_j Q_j} \sum_{\{\Phi\}} \left\langle \left[\prod_j e^{i\sigma_j \hat{Q}_j} \Phi \right], t + \alpha \left| \mathcal{T} \left[\exp \left(i \int d^4x J \Phi \right) \right] \right| \Phi, t \right\rangle, \end{aligned} \quad (2.1)$$

where the trace in $\{\Phi\}$ is meant over the physical states at a certain time t ; $\hat{H}$ is the Hamiltonian and $\hat{Q}$ are the operators of the conserved charges. Equations (2.1) can be rewritten in the path integral form

$$Z_{E,V,\{Q\}}[J] = \int \frac{d\alpha}{2\pi} e^{i\alpha E} \prod_j \int \frac{d\sigma_j}{2\pi} e^{i\sigma_j Q_j} \mathcal{N}_{\alpha,\{\sigma_j\}} \int_{\text{phys}} D\Phi \exp \left[i \int d^4x (\mathcal{L} + J\Phi) \right], \quad (2.2)$$

where phys means that the integration is over the physical states. Because of the definition of the trace in Eq. (2.1), the temporal integration of the action is done over the time path C , connecting

$$C: t \rightarrow t + \alpha,$$

and the fields are under the constraint of the periodic boundary condition

$$\Phi(t + \alpha, x) = \pm \exp \left[i \sum_j \sigma_j Q_j \right] \Phi(t, x), \quad (2.3)$$

where the plus sign on the right-hand side is for bosonic fields and the minus sign for fermionic fields [6–8].

In the integration of the non-Abelian gauge field $\Phi = A_\mu^a$, the procedure to integrate over physical spectrum is well known [9]. After some algebra, it is given with first-class constraints and gauge fixing for them as

$$\begin{aligned} \int_{\text{phys}} \mathcal{D}A \mathcal{D}\Pi \rightarrow \int \mathcal{D}A \mathcal{D}\Pi \prod_{a,x} \delta(\Pi_0^a(x)) \delta(D_i \Pi_i^a(x)) \\ \times \prod_{a,x} [\delta(A_0^a(x)) \delta(\partial_i A_i^a(x))] \text{Det}(M), \end{aligned} \quad (2.4)$$

$$M^{ab}(x, y) = \partial_i (\delta^{ab} \partial_i - g f^{abc} A_i^c(x)) \delta(x - y). \quad (2.5)$$

The procedure to go to the covariant gauge is the same as usual. The ghost fields are introduced to exponentiate the determinant of M in Eq. (2.4). Since this determinant is defined in the functional space of the gauge field, the ghost field thus introduced has the same boundary condition as the gauge field [10]. The explicit check is shown below.

Using the definition in Eq. (2.1), the generating functional of the Green function, which is the statistical average of field operators in the microcanonical ensemble, is given by

$$\begin{aligned} Z_{E,\{Q\}}[J] &= Z_{E,\{Q\}}[J] / Z_{E,\{Q\}}[0] \\ &= \int_0^E dE' \prod_j \int dQ'_j \rho_{E-E',\{Q_j-Q'_j\}} Z_{E',\{Q'\}}^c[J], \end{aligned} \quad (2.6)$$

where Z^c is the generating functional of the diagrams connected to the external source:

$$Z_{E',\{Q'\}}^c[J] = \int \frac{d\alpha}{2\pi} e^{i\alpha E'} \prod_j \int \frac{d\sigma_j}{2\pi} e^{i\sigma_j Q'_j} \exp \left[i \int d^4x \mathcal{L}_I \right] \exp \left[i \int d^4x d^4x' J(x) D_{\alpha,\sigma}(x - x') J(x') \right] \Big|_{\text{CD}}, \quad (2.7)$$

where $|_{\text{CD}}$ stands for the connected diagrams and $D_{\alpha,\sigma}(x-x')$ is the propagator. The factor $\rho_{E-E',\{Q_j-Q'_j\}}$ in Eq. (2.6) gives the statistical weight of the connected diagram with energy E' and the charge $\{Q_j-Q'_j\}$, which is given by

$$\rho_{E-E',\{Q_j-Q'_j\}} = Z_{E-E',\{Q_j-Q'_j\}}[0] / Z_{E,\{Q\}}[0].$$

For the detailed derivation of Eq. (2.6), see Ref. [1].

The Lagrangian of QCD, which is used in the following, is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{k=1}^{N_f} \bar{\psi}_k [i\gamma^\mu(\partial_\mu - ig A_\mu^a T^a) - m_k] \psi_k,$$

where N_f is the number of flavors and m_k is the quark mass. The restriction to choose the time path C is that it must connect a certain time t to $t+\alpha$. In the same way as in Ref. [1], we choose it as the one shown in Fig. 1. This choice of the time path consists of three parts: $C_1: -T \rightarrow T$, $C_2: T \rightarrow -T$ and $C_3: -T \rightarrow -T+\alpha$. This choice is convenient for the real-time formulation in which we can evaluate the Green function of time-

FIG. 1. The time path C used to formulate the finite-energy quantum field theory.

dependent operators. In order to guarantee the convergence of the path integral, we introduced an infinitesimal negative imaginary part in α , namely $\text{Im}\alpha = \epsilon \rightarrow 0$ at the end of the computations. An important point in this choice of the path is that the $C_1 + C_2$ path decouples from the C_3 path in the limit $T \rightarrow \infty$, which is the same situation as in the real-time formulation of finite-temperature field theory [6,7].

Because of the decoupling of the $C_1 + C_2$ path from the C_3 path, the propagators for the gauge boson become

$$\begin{aligned} \exp \left\{ -\frac{i}{2} \int_c d^4x d^4x' J^{a\mu}(x) K_{\alpha,\mu\nu}^{ab}(x-x') J^{b\nu}(x') \right\} &= \exp \left\{ -\frac{i}{2} \sum_{r,s=1,2} \int d^4x d^4x' J_r^{a\mu}(x) K_{\alpha,\mu\nu}^{rs,ab}(x-x') J_s^{b\nu}(x') \right\} \\ &\times \exp \left\{ -\frac{i}{2} \int_{C_3} d^4x d^4x' J_3^{a\mu}(x) K_{\alpha,\mu\nu}^{33,ab}(x-x') J_3^{b\nu}(x') \right\}, \end{aligned} \quad (2.8)$$

where $J_r^{a\mu}(x)$ is the external source coupling to the gauge boson, a and μ are gauge-group and space-time indices, and the index r represents that this source has a support on the path C_r . The propagator satisfies the differential equation

$$\left[g^{\mu\nu} \partial_x^2 - \left(1 - \frac{1}{\xi} \right) \partial_x^\mu \partial_x^\nu \right] K_{\alpha,\nu\lambda}^{rs,ab}(x-y) = \delta_\lambda^\mu \delta^{ab} \delta^4(x-y). \quad (2.9)$$

The Fourier component of the solution to this equation is given as

$$iK_{\alpha,\nu\lambda}^{rs,ab}(k) = - \left[g_{\mu\nu} - (1-\xi) \frac{k_\mu k_\nu}{k^2} \right] \delta_\lambda^\mu \delta^{ab} iD_\alpha^{rs}(k), \quad (2.10)$$

where $D_\alpha^{rs}(k)$ is the propagator of the scalar boson connecting the fields on the paths C_r and C_s . In deriving Eq. (2.10), we have made the approximation of plane wave and continuum momentum (see the Introduction for details). The explicit expressions are collected in Appendix A.

Similarly the part including the ghost fields are written as

$$\begin{aligned} \exp \left\{ i \int_c d^4x d^4x' \bar{J}_c^a(x) K_{\alpha,c\bar{c}}^{ab}(x-x') J_{\bar{c}}^b(x') \right\} &= \exp \left\{ i \sum_{r,s} \int d^4x d^4x' \bar{J}_{c,r}^a(x) K_{\alpha,c\bar{c}}^{rs,ab}(x-x') J_{\bar{c},s}^b(x') \right\} \\ &\times \exp \left\{ i \int_{C_3} d^4x d^4x' \bar{J}_{c,3}^a(x) K_{\alpha,c\bar{c}}^{33,ab}(x-x') J_{\bar{c},3}^b(x') \right\}, \end{aligned}$$

where $\bar{J}_c$ and $J_{\bar{c}}$ are the external sources coupled to the ghost fields c and $\bar{c}$. The propagator is given in terms of the scalar boson's propagator:

$$K_{\alpha,c\bar{c}}^{rs,ab}(k) = i\delta^{ab} D_\alpha^{rs}(k). \quad (2.11)$$

From Eqs. (2.10) and (2.11), it follows that the propagators of the gauge boson and the ghost field have the same

boundary condition, which is implied by the boundary condition of fields in Eq. (2.3) with plus sign.

The quark fields are under the boundary condition of Eq. (2.3) with minus sign and phases according to their charges. The part of the generating functional including fermion propagators is written in a way similar to that of the bosons:

$$\exp \left\{ -i \sum_{r,s} \int d^4x \, d^4x' \, \bar{\eta}_r^i(x) S_{\alpha}^{rs,ij}(x-x') \eta_s^j(x') \right\} \\ \times \exp \left\{ -i \int_{C_3} d^4x \, d^4x' \, \bar{\eta}_3^i(x) S_{\alpha}^{33,ij}(x-x') \eta_3^j(x') \right\},$$

where the Grassmann external sources are represented by η , $\bar{\eta}$, and i, j are the group indices of fermions. The propagator of the fields on the path C_1 and C_2 is given by a 2×2 matrix form with respect to indices $r, s = 1, 2$:

$$S_{\alpha, \sigma}^{rs,ij}(p) = \delta^{ij}(\not{p} + m) \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where

$$S_{\alpha, \sigma}^{33,ij}(x-y) = -i \delta^{ij} \theta(\alpha) \{ \theta(x_0 - y_0) \langle \psi(x) \bar{\psi}(y) \rangle - \theta(y_0 - x_0) \langle \bar{\psi}(y) \psi(x) \rangle \} \\ - i \delta^{ij} \theta(-\alpha) \{ -\theta(x_0 - y_0) \langle \bar{\psi}(y) \psi(x) \rangle + \theta(y_0 - x_0) \langle \psi(x) \bar{\psi}(y) \rangle \}, \quad (2.14)$$

where the Fourier components of $\langle \psi(x) \bar{\psi}(y) \rangle$ and $\langle \bar{\psi}(y) \psi(x) \rangle$ are given by

$$\langle \psi(x) \bar{\psi}(y) \rangle|_p = 2\pi\epsilon(p_0) \delta(p^2 - m^2) (\not{p} + m) [1 - g_Q(p_0)], \\ \langle \bar{\psi}(y) \psi(x) \rangle|_p = 2\pi\epsilon(p_0) \delta(p^2 - m^2) (\not{p} + m) g_Q(p_0).$$

Using Eqs. (2.12) and (2.14), we can check the boundary condition

$$S_{\alpha, \sigma}(x-y)|_{x_0 = -T + \alpha} \\ = -\exp \left[i \sum_j \sigma_j Q_j \right] S_{\alpha, \sigma}(x-y)|_{x_0 = -T}.$$

The Feynman rules for vertices are collected in Appendix A.

In the thermodynamical limit, one can easily show that the above propagators become the well-known finite-temperature propagators for the corresponding time path [1]. As can be seen in the definition of the generating functional in Eq. (2.6), in order to compute Green functions at finite energy, we need an explicit expression of the state density with vanishing external source $Z[0]$, which is related to the entropy by $\delta E Z_E[0] = \exp(S_E)$, where δE is the energy width, which is not distinguished quantum mechanically. This is the main reason which makes the computation in the microcanonical method difficult.

There are lots of interesting problems as applications of the present formulation. For example, one of them is the collective effect in a medium of finite energy such as the plasmon effect [11]. Another interesting problem is

$$A = \frac{1}{p^2 - m^2 + i\epsilon} \\ + 2\pi i \delta(p^2 - m^2) [\theta(p_0) g_Q(\omega_p) + \theta(-p_0) g_{\bar{Q}}(\omega_p)], \\ B = -i 2\pi\epsilon(p_0) \delta(p^2 - m^2) g_Q(p_0), \\ C = i 2\pi\epsilon(p_0) \delta(p^2 - m^2) [1 - g_Q(p_0)], \quad (2.12)$$

$$D = \frac{-1}{p^2 - m^2 - i\epsilon} \\ + 2\pi i \delta(p^2 - m^2) [\theta(p_0) g_Q(\omega_p) + \theta(-p_0) g_{\bar{Q}}(\omega_p)],$$

and the $g_Q(\omega_p)$ is given by

$$g_Q(\omega_p) = \frac{1}{\exp(i\alpha\omega_p + i \sum_j \sigma_j Q_j) + 1} \quad (2.13)$$

and $g_{\bar{Q}}(\omega_p)$ is given by replacing the charges by their conjugates. Because of the small negative imaginary part of α , the expansion of Eq. (2.13) is well defined. The propagator between fields on the C_3 path is

the finite-energy effect to the effective coupling constants [12], where we choose the energy-density-dependent renormalization point. In any case, we need the expression of the entropy for these computations. As we mentioned above, it is difficult to obtain a simple analytical expression for the entropy in general, and the numerical analysis is sometimes required. In a special case, such as the one where the thermodynamical limit is a good approximation, the analysis is performed analytically and the finite-energy corrections are easily obtained.

III. ENTROPY AT FINITE BARYON NUMBER

This section contains an evaluation of the entropy at finite baryon number, which is used in the next section to discuss its implication on the phase transition. As a simplest approximation, we use a one-loop approximation, which is considered to be good at high-energy density because of the renormalization group arguments. The extension to include higher-order corrections perturbatively is straightforward.

As we have mentioned in the previous section, the computation of the entropy is the starting point in the microcanonical analysis of the quantum field theory [cf. Eq. (2.6)]. This is a situation very different from the finite-temperature field theory in which a simple expression of a statistical weight is readily given by the Boltzmann factor.

In order to define the entropy, we first define the number of the states in the energy interval $E \sim E + \delta E$, which is not distinguished quantum mechanically:

$$W_{E,\delta E,V,Q}[J] = \int_E^{E+\delta E} dE' Z_{E',V,Q}[J] \sim \delta E Z_{E,V,Q}[J],$$

where Z_E is defined in Eq. (2.1). Physically the interval δE is given by $\delta E \sim V^{-1/3}$, where V is the volume of the system. Then the entropy at the zero external source is defined by

$$S_{E,\delta E,V,Q}[0] = \ln W_{E,\delta E,V,Q}[0].$$

The variation of the entropy is given by

$$\begin{aligned} e^{S_{E,N_b}} = & \delta E \int_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\alpha}{2\pi} \int_{-3\pi}^{3\pi} \frac{d\sigma}{6\pi} e^{i\alpha E + i\sigma N_b} \\ & \times \exp \left\{ \frac{V}{(2\pi)^2} 2(N_c^2 - 1) \int d^3\mathbf{k} \sum_{n=1}^{\infty} \frac{1}{n} e^{-i\alpha n|\mathbf{k}|} \right. \\ & \left. + 4N_c V \sum_{j=1}^{N_f} \sum_{n_j=1}^{\infty} \frac{m_j^2}{2\pi^2 i\alpha} \frac{(-)^{n_j+1}}{n_j^2} \cos \left[\frac{n_j \sigma}{3} \right] K_2(n_j m_j i\alpha) \right\}, \end{aligned} \quad (3.2)$$

where K_2 is the modified Bessel function. The parameters are N_b is the baryon number, N_c the number of colors, namely, $SU(N_c)$, and N_f the number of flavors. The σ integration is done between -3π and 3π because the baryon number of a quark is $1/3$. In Eq. (3.2) the summation over discrete momentum is approximated by integration (see the Introduction for details), and as it can be seen the expression in (3.2) presents the Fourier transform of the finite-temperature partition function with the imaginary temperature and chemical potential [13].

We will evaluate Eq. (3.2) in the approximation of neglecting the mass of quarks. As we discussed above, the perturbative computation is a good approximation when the energy density is sufficiently high compared with the QCD energy scale. In such a situation, the masses of light quarks are neglected with a good approximation. Although the condition for the saddle-point method to be a good approximation is restrictive, nevertheless it provides us with analytic results that are convenient to analyze. On the other hand, in the numerical computation there is less restriction theoretically, and it can be used in a more general situation. However, in the latter method there is a new restriction, which is the limitation of the computational capacity. In addition, the analysis is much more time consuming than the saddle-point method.

The difficulty in evaluating Eq. (3.2) comes from the oscillating property of the integrand with respect to α and σ . Because of this oscillation, a subtle cancellation occurs when one integrates it. This property of oscillation has been discussed in the context of the finite-temperature quantum field theory in Ref. [13]. A similar difficulty is found in the lattice simulation of QCD at nonzero chemical potential, when one integrates over field configurations [14].

We will discuss the saddle-point method first. Using the identity

$$dS_{E,\delta E,V,Q}[0] = \beta dE + \beta P dV + \beta \mu_{Q_j} dQ_j. \quad (3.1)$$

The coefficients of the variation on the right-hand side are related to the thermal quantities in the thermodynamical limit, namely, β to the inverse temperature, P to the pressure, and μ to the chemical potential.

In the case of conserved baryon number, the charge operator $\hat{Q}$ is replaced by the baryon number operator. In the one-loop approximation, the entropy is obtained as

$$\begin{aligned} \sum_{n=1}^{\infty} (-)^{n+1} \frac{\cos nx}{n^4} & \equiv \frac{1}{48} \left[\frac{7\pi^4}{15} - 2\pi^2 x^2 + x^4 \right], \\ x & \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right], \end{aligned}$$

Eq. (3.2) is written as

$$\begin{aligned} e^{S_{E,N_b}} = & \delta E \int_{-\pi}^{\pi} \frac{dt}{2\pi} e^{3iN_b t} \int_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\alpha}{2\pi} e^{i\alpha E} \\ & \times \exp \left\{ i \frac{V}{\alpha^3} \left[Q + K(t^4 - 2\pi^2 t^2) \right] \right\}, \end{aligned} \quad (3.3)$$

where

$$Q = 2(N_c^2 - 1) \frac{\pi^2}{90} + 4N_c N_f \frac{7\pi^2}{720}, \quad (3.4)$$

and

$$K = 4N_c N_f \frac{1}{48\pi^2}.$$

We integrate Eq. (3.3) using the saddle-point method, first with respect to α and then with respect to t .

In order to do the α integration, one has to be careful about the sign of $Q + K(t^4 - 2\pi^2 t^2)$ in the exponent in Eq. (3.3). After a straightforward but lengthy computation, we obtain

$$\begin{aligned} e_{E,N_b}^S \simeq & \text{Re} \delta E \int_{-\pi}^{\pi} \frac{dt}{2\pi} e^{i3N_b t} \frac{1}{2} \left[\frac{1}{2\pi} \right]^{1/2} \left[\frac{3Vf(t)}{E^5} \right]^{1/8} \\ & \times \exp \left\{ \frac{4}{3} [3E^3 V f(t)]^{1/4} \right\}, \end{aligned} \quad (3.5)$$

where Re stands for the real part and $f(t)$ is defined by

$$f(t) \equiv Q + K(t^4 - 2\pi^2 t^2).$$

The condition that the saddle-point approximation to ob-

tain Eq. (3.5) is a good one is that E and V are large, with E/V finite. Details to derive Eq. (3.5) are given in Appendix B.

Since the saddle point of t is determined by the exponential factors, Eq. (3.5) is approximated as

$$e^{S_{E,N_b}} \sim \delta E \frac{1}{2} \left[\frac{1}{2\pi} \right]^{1/2} \text{Re} \left[\frac{3Vf(\tilde{t})}{E^5} \right]^{1/8} \int_{-\pi}^{\pi} \frac{dt}{2\pi} e^{iF(t)}, \quad (3.6)$$

where

$$F(t) = 3N_b t - i \frac{4}{3} \{ 3E^3 V [Q - K(2\pi^2 t^2 - t^4)] \}^{1/4},$$

and $\tilde{t}$ on the right-hand side of Eq. (3.6) is the saddle point defined by $F'(\tilde{t}) = 0$, namely,

$$N_b + i \frac{\frac{4}{3} K (\pi^2 \tilde{t} - \tilde{t}^3) (E^3 V)^{1/4}}{\{ 3(Q - K[2\pi^2 \tilde{t}^2 - \tilde{t}^4]) \}^{3/4}} = 0. \quad (3.7)$$

Thus the next step is to find a solution to Eq. (3.7). Since this is not simple in general, we will show two special cases in which we could easily find approximate solutions.

Case (1): $N_b/V \ll (E/V)^{3/4}$. In this case, the solution to Eq. (3.7) is obtained as

$$\tilde{t} = i \frac{3(3Q)^{3/4}}{4K\pi^2} \frac{N_b/V}{(E/V)^{3/4}}.$$

The entropy is

$$S_{E,N_b} \simeq \frac{4}{3} (3E^3 V Q)^{1/4} - \frac{9}{8} \frac{(3Q)^{3/4}}{K\pi^2} \frac{N_b^2/V}{(E/V)^{3/4}}, \quad (3.8)$$

with Q and K given by Eq. (3.4). This result is consistent with the one usually obtained in the canonical method in the small baryon number density limit.

Case (2): $[K/27(E/V)^3]^{1/4} \gtrsim 3/4 |N_b|/V$. The solution to Eq. (3.7) in this case is given by

$$\tilde{t} = \pm i \left[\frac{\pi^2/2(K/27E^3V)^{1/4}}{(K/27E^3V)^{1/4} - \frac{3}{4}|N_b|} \right]^{1/2}, \quad (3.9)$$

where plus and minus signs are for positive and negative baryon number, respectively. Thus the leading contribution to the entropy is

$$S_{E,N_b} \sim \frac{4}{3} \left\{ \frac{\pi^2}{2} (3KE^3V)^{1/4} \left[(3KE^3V)^{1/4} - \frac{9}{4} |N_b| \right] \right\}^{1/2}. \quad (3.10)$$

This expression is a good approximation when $|\tilde{t}| \gg 1$, and thus gives the condition given above. We notice that this is a limit which is not usually considered in the canonical method.

The property that Eqs. (3.8) and (3.10) have in common is that they are invariant under the baryon number reflection, $N_b \rightarrow -N_b$. This is a natural consequence of the baryon-number symmetry in QCD; it can also be checked in Eq. (3.2).

We have looked for other solutions to Eq. (3.7), especially in the case of large baryon number. However, as we will discuss below in the numerical computation, the saddle-point method is not a good approximation when the baryon number is large. Therefore, the results in Eqs. (3.8) and (3.10) are our only ones in the saddle-point method.

For the numerical evaluation of the entropy, we start with Eq. (3.2) by setting $m_j \rightarrow 0$. The method we use is to expand the exponentials and integrate each term with respect to α and σ . This method has already been used in the context of the scalar field theory in Refs. [1] and [15].

The entropy in Eq. (3.2) is written as a convolution of the contributions of the gauge boson and the quarks,

$$e^{S_{E,N_b}} = \delta E \int_0^E dE' Z_g(E-E') Z_{f,N_b}(E'), \quad (3.11)$$

where Z_g and Z_{f,N_b} represent the contributions of the gauge boson and the quarks with the baryon number N_b , respectively. The state density of the gauge boson at the energy E in the one-loop approximation is

$$Z_g(E) = \frac{1}{E} \sum_{n=0}^{\infty} \frac{1}{n!(3n-1)!} \left[2(N_c^2-1) \frac{\pi^2}{90} VE^3 \right]^n. \quad (3.12)$$

Similarly, the state density of N_f light quarks is given by

$$Z_{f,N_b}(E) = \frac{1}{E} \sum_{n=0}^{\infty} \frac{1}{n!(3n-1)!} \times \left[4N_c N_f \frac{1}{48\pi^2} VE^3 \right]^n F_{N_b,n}, \quad (3.13)$$

where the factor $F_{N_b,n}$ is

$$F_{N_b,n} = (24)^n \sum_{m_1=1}^{\infty} \dots \sum_{m_n=1}^{\infty} \frac{(-)^{m_1+1} (-)^{m_2+1} \dots (-)^{m_n+1}}{m_1^4 m_2^4 \dots m_n^4} \sum_{\{\pm\}^n} \delta_{3N_b \pm m_1 \pm m_2 \dots \pm m_n, 0} \\ = \int_0^{\pi} \frac{dt}{\pi} \cos(3N_b t) \left(\frac{7\pi^4}{15} + t^4 - 2\pi^2 t^2 \right)^n. \quad (3.14)$$

In the first line of Eq. (3.14), the sum of $\{\pm\}^n$ means the one over all possible combinations of $\pm$ signs.

The procedure to evaluate the entropy is obvious. That is computing $Z_g(E-E')$ and $Z_{f,N_b}(E')$ and in-

tegrating them as shown in Eq. (3.11). The difficulty of this computation occurs in Z_{f,N_b} . As one sees in Eq. (3.14), the factor $F_{N_b,n}$ is not positive definite. Actually it changes sign successively as one increases n , and there

are subtle cancellations among the terms in Eq. (3.13). Especially for large baryon number, such cancellations become so severe that the numerical accuracy to compute $F_{N_b, n}$ in Eq. (3.14) becomes insufficient, which sets an upper limit on the value of the baryon number we can reach. One can imagine that Stirling's formula is used to approximate the factorials in Eq. (3.13), but because of the subtle cancellations discussed above, the computation using Stirling's formula gives the wrong answer for large baryon number. It appears to be difficult to find an efficient and accurate approximation to evaluate Eq. (3.13) for large baryon number.

We have done computations of the entropy using parameters $N_c=3$ and $N_f=2$, which mean SU(3) and two light flavors. The entropy is a function of a dimensionless variable defined by

$$XE \equiv (VE^3)^{1/3}. \quad (3.15)$$

Figure 2 shows the entropy as a function of the baryon number for fixed $XE=2784$, 2320, and 1856. The behavior of the entropy shown in Fig. 2 is well described by the result of the saddle-point method in Eq. (3.8). Figure 3 shows the entropy as a function of XE for fixed baryon numbers $N_b=0$, 10, and 20.

For example, if we fix the energy of the system to $E=100$ GeV, its volume is given as a function of XE , $V \equiv (XE \times 0.2 \times 10^{-2})^3 \text{ fm}^3$. Then three curves in Fig. 2 are considered to be the results for three different volumes $V \approx 173$, 100, and 51 fm^3 , i.e., the energy density $E/V=0.58$, 1, and 2 GeV/fm^3 . Figure 3 is also read in the same manner.

Using Eq. (3.1), we can obtain coefficients of the variation of the entropy. For example, β in Eq. (3.1) is given as a first derivative of the entropy with respect to XE :

$$\beta = V^{1/3} \frac{\partial S}{\partial (XE)}. \quad (3.16)$$

Figure 4 shows the dimensionless quantity $100 \times \beta V^{-1/3}$ as a function of the baryon number for fixed $XE=2784$,

FIG. 2. The entropy as a function of baryon number for fixed XE : $XE=1856$ for the dotted line, 2320 for the solid line, and 2784 for the dashed line.

FIG. 3. The entropy as a function of XE for fixed baryon number: $N_b=0$ for the solid line, 10 for the dotted line, and 20 for the dashed line.

2320, and 1856. In this figure, β and V are in units of GeV^{-1} and fm^3 , respectively. Similarly, Fig. 5 shows $100 \times \beta V^{-1/3}$ as a function of XE for fixed baryon numbers $N_b=0$, 10, and 20.

Let us set $E=100$ GeV and concentrate on the solid curve in Fig. 4. Then the volume of the system in this case is $V=100 \text{ fm}^3$. From Fig. 4, we can read $\beta=5.80 \text{ GeV}^{-1}$ for $N_b=0$, $\beta=5.94 \text{ GeV}^{-1}$ for $N_b=20$, and so on. If the system is large enough to be described by thermodynamical quantities, these values of β correspond to temperatures $T=0.172$ and 0.168 GeV , respectively.

The coefficient P in Eq. (3.1) is given by $P=\beta^{-1} \partial S / \partial V$, which is the pressure in the thermodynamical limit. Using Eq. (3.16), it is shown that the equation of the state derived from the entropy in Eq. (3.2) is the one for an ideal gas, $PV=\frac{1}{3}E$. This is a natural consequence of our approximation. If one wants to study a deviation from the ideal gas, one has to include higher-

FIG. 4. $100 \times \beta V^{-1/3}$ defined in Eq. (3.16) as a function of baryon number. The values of XE for the lines are the same as in Fig. 2.

FIG. 5. $100 \times \beta V^{-1/3}$ as a function of XE . Baryon numbers of the lines are the same as in Fig. 3.

order perturbative contributions using the method in Sec. II.

IV. DISCUSSION

In Sec. II we formulated the gauge theory at finite energy using the microcanonical distribution. The entropy at finite baryon number is computed in Sec. III. Although we did not show any computations of correlation functions of time-dependent operators, the procedure for such computations is clear. However, as we have discussed at the beginning, the evaluation of correlation functions is not an easy task. One has to know about the weight factor $\rho_{E,Q}$ in Eq. (2.6). In the case of the conserved baryon number, the weight factor ρ_{E,N_b} is derived by the entropy computed in Sec. III.

We now discuss the implication of the entropy computed in Sec. III in the problem of confinement-deconfinement phase transition. We have no way of investigating this issue in a satisfactory manner yet; nevertheless we can obtain a rough description of this phase transition using the simple argument of the bag model [16,17]. Assuming that the results of lattice simulations of QCD give us a correct picture of the phase transition, we consider that the (de)confinement and chiral phase transitions occur at the same point. According to the bag model, we introduce the bag constant B , which is a vacuum energy density [5]. First, let us base our discussion on the results of the saddle-point method for the small baryon number in Sec. III. The entropy of the quark-gluon plasma (QGP) is given by [cf. Eq. (3.8)]

$$S_{E,N_b}^{\text{QGP}} \simeq \frac{4}{3} [3Q(E-BV)^3 V]^{1/4} - \frac{9}{8} \frac{(3Q)^{3/4}}{K\pi^2} \frac{N_b^2/V}{[(E-BV)/V]^{3/4}}, \quad (4.1)$$

where Q and K are given in Eq. (3.4). This entropy is a good approximation when $N_b/V \ll [(E-BV)/V]^{3/4}$. When the baryon number is zero, the coefficient P in the variation of the entropy S^{QGP} becomes

$$P = \frac{1}{3} \left[\frac{E}{V} - 4B \right].$$

Since P has the physical meaning of pressure in the thermodynamical limit, this is a desired relation between the pressure and the energy density in the presence of the bag constant B . It is interesting to compare Eq. (4.1) with the effective action of the symmetry-broken theory derived in the microcanonical formalism [1]. The way the bag constant enters into the entropy in Eq. (4.1) is the same as the one the potential at nonzero classical field enters into the effective action.

In the same manner as in Sec. III, we can obtain the entropy of the system of hadrons.¹ For simplicity, we consider that the system consists of pions, protons, and neutrons in the approximation of neglecting their masses. Similarly to Eq. (3.8), the entropy of this system with small baryon number is given by

$$S_{E,N_b}^H \simeq \frac{4}{3} (3Q_H E^3 V)^{1/4} - \frac{1}{8} \frac{(3Q_H)^{3/4}}{K_H \pi^2} \frac{N_b^2/V}{(E/V)^{3/4}}, \quad (4.2)$$

where Q_H and K_H are defined by

$$Q_H \equiv \frac{\pi^2}{30} + \frac{7\pi^2}{90}, \quad K_H \equiv \frac{1}{6\pi^2}. \quad (4.3)$$

These states, i.e., the quark-gluon plasma and the hadrons, are two different states of the strongly interacting matter. The nature of these two states depends on the order of the phase transition: If it is first order, these two are associated with two isolated maxima of the entropy in the configuration space of the gauge field. Physically the state which gives a larger entropy is realized. Thus, roughly speaking, the transition point is obtained by setting $S_{E,N_b}^{\text{QGP}} = S_{E,N_b}^H$, which gives the critical energy density

$$\frac{E}{V} \Big|_{\text{cr}} \simeq \epsilon_0 + \frac{9 \frac{(3Q)^{3/4}}{K} (\epsilon_0 - B)^{3/4} - \frac{(3Q_H)^{3/4}}{K_H} \epsilon_0^{-3/4}}{8\pi^2 \left[\left(\frac{3Q}{\epsilon_0 - B} \right)^{1/4} - \left(\frac{3Q_H}{\epsilon} \right)^{1/4} \right]} \left(\frac{N_b}{V} \right)^2, \quad (4.4)$$

$$\epsilon_0 = \frac{(Q/Q_H)^{1/3}}{(Q/Q_H)^{1/3} - 1} B,$$

¹Recently J. Cleymans *et al.* [18] discussed the thermodynamics of hadrons at fixed baryon number and strangeness, using the Boltzmann statistics.

where the approximation $N_b/V \ll \epsilon_0^{3/4}$ is used and cr stands for critical. Below this critical-energy density the state of hadrons is realized, and above it the state of the quark-gluon plasma occurs. One can see from Eq. (4.4) that the increase of the baryon number increases the critical energy. For example, if we choose $N_c=3$, $N_f=2$, and $B^{1/4}=0.15$ GeV, Eq. (4.4) gives

$$\left. \frac{E}{V} \right|_{cr} \simeq (0.179 + 5.28n_b^2) \text{ GeV/fm}^3,$$

where n_b is the baryon number per fm^3 . At low energies, the approximation of massless nucleons is not good, and there is room for improvement of our discussion above. However, our simple argument gives a reasonable estimate of the critical-energy density of the strongly interacting matter.

We also have performed the numerical computations for the entropy of the quark-gluon plasma and the hadrons (pions, protons, neutrons) in the massless approximation, but without assuming large E, V , which was needed for the saddle-point method. The procedure to do numerical computations is the same as the one explained in Sec. III. Figure 6 shows the entropy for the quark-gluon system and for hadrons when the baryon number is $N_b=0$ or 10, as a function of XE defined in Eq. (3.15). The parameters used in this computation are $N_c=3$, $N_f=2$, $B^{1/4}=0.15$ GeV and $V=4 \text{ fm}^3$. One can read from Fig. 6 the tendency of the baryon number to become larger, the higher the critical-energy density. For example, when $N_b=0$ the energy density is about 0.16 GeV/fm^3 at the critical point. If the actual mass of nucleons is used in this computation, the value of the entropy is found to be smaller and the critical energy density becomes smaller.

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FIG. 6. The entropy of the hadron gas and the quark-gluon plasma. The solid line is the hadron gas of $N_b=0$. The long-short dashed line is for the hadron gas of $N_b=10$. The dotted line and the dashed line are for the quark-gluon gas of $N_b=0$ and 10, respectively. The difference of the hadron gas between $N_b=0$ and 10 is difficult to see here.

APPENDIX A: FEYNMAN RULES

This appendix contains the notations of propagators and Feynman rules. As we have discussed in Sec. II, we choose the time path as in Fig. 1 and, in the limit $T \rightarrow \infty$, the $C_1 + C_2$ path decouples from the path C_3 . Thus it is convenient to consider the $C_1 + C_2$ path and the C_3 path separately.

The path $C_1 + C_2$ is used to compute the correlation functions of time-dependent operators. The generating functional is given in Eq. (2.6), and the Feynman rule is needed to compute the connected diagrams. The propagators have a 2×2 matrix form, of which the (r, s) matrix element means the propagator connecting fields on the C_r path and the C_s path. The propagators of the gauge field and the ghost field are written in terms of the scalar boson's propagator $D_\alpha^r(x-y)$, cf. Eqs. (2.10) and (2.11). The Fourier component of $D_\alpha^r(x-y)$, $r, s=1, 2$, is given by [1]

$$iD_\alpha^r(k) = \begin{pmatrix} \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi\delta(k^2 - m^2)f_\alpha(\omega_k) & -2\pi\epsilon(k_0)\delta(k^2 - m^2)f_\alpha(k_0) \\ -2\pi\epsilon(k_0)\delta(k^2 - m^2)[1 + f_\alpha(k_0)] & \frac{-i}{k^2 - m^2 - i\epsilon} + 2\pi\delta(k^2 - m^2)f_\alpha(\omega_k) \end{pmatrix}, \quad (\text{A1})$$

where $\omega_k = \sqrt{\mathbf{k}^2 + m^2}$ and f_α is defined by

$$f_\alpha(u) = \frac{1}{e^{iau} - 1}, \quad (\text{A2})$$

and the approximation of continuum momentum is made (for details see paragraph 4 in the Introduction). To insert Eq. (A1) into Eqs. (2.10) and (2.11), the mass m should be put equal to zero. The propagator of the quark field is given in Eq. (2.12).

The Feynman rule for the $C_1 + C_2$ path is derived in the usual manner. For the internal lines of the gauge boson, the

boson's propagator, as Eqs. (2.10) and (2.11). On the C_3 path, the propagator of the scalar boson $D_\alpha^{33}(x-y)$ is given by

$$iD_\alpha^{33}(x-y) = \theta(\alpha) iD_\alpha^A(x-y) + \theta(-\alpha) iD_\alpha^B(x-y) , \quad (\text{A3})$$

where D_α^A is used when α is positive and D_α^B when α is negative; $D_\alpha^{A,B}$ is given by

$$\begin{aligned} iD_\alpha^{A,B}(x-y) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{|\alpha|} \sum_{n \in \mathbb{Z}} \exp \left[-i \frac{2\pi n}{\alpha} (x_0 - y_0) + i \mathbf{k} \cdot (\mathbf{x} - \mathbf{y}) \right] iD_{\alpha,n,k}^{A,B} , \\ iD_{\alpha,n,k}^A &= \frac{i}{\left[\frac{2\pi n}{\alpha} \right]^2 - \omega_k^2 + i\epsilon} + 2\pi\delta \left[\left[\frac{2\pi n}{\alpha} \right]^2 - \omega_k^2 \right] f_\alpha(\omega_k) , \\ iD_{\alpha,n,k}^B &= \frac{i}{\left[\frac{2\pi n}{\alpha} \right]^2 - \omega_k^2 - i\epsilon} + 2\pi\delta \left[\left[\frac{2\pi n}{\alpha} \right]^2 - \omega_k^2 \right] f_\alpha(\omega_k) . \end{aligned} \quad (\text{A4})$$

The factor $f_\alpha(\omega_k)$ is defined in Eq. (A2). The first term on the right-hand side of $iD_{\alpha,n,k}^{A,B}$ has a form similar to that of the Matsubara-Green function [19].

The propagator of the quark field on the path C_3 also has parts for the positive and negative α [cf. Eq. (2.14)]:

$$S_{\alpha,\sigma}^{33,ij}(x-y) = \theta(\alpha) S_{\alpha,\sigma}^{A,ij}(x-y) + \theta(-\alpha) S_{\alpha,\sigma}^{B,ij}(x-y) . \quad (\text{A5})$$

The Fourier expansion of them is given by

$$S_{\alpha,\sigma}^{A,B,ij}(x-y) = \frac{1}{|\alpha|} \sum_{n \in \mathbb{Z} + 1/2} \int \frac{d^3p}{(2\pi)^3} \exp \left[-i \frac{1}{\alpha} [2\pi n - \sigma Q] (x_0 - y_0) + i \mathbf{p} \cdot (\mathbf{x} - \mathbf{y}) \right] S_{\alpha,\sigma,n,p}^{A,B,ij} , \quad (\text{A6})$$

where σQ stands for $\sum_j \sigma_j Q_j$. The Fourier components are

$$\begin{aligned} S_{\alpha,\sigma,n,p}^{A,B,ij} &= \delta^{ij} \left[\gamma_0 \left[\frac{2\pi n - \sigma Q}{\alpha} \right] - \mathbf{p} \cdot \boldsymbol{\gamma} + m \right] \\ &\times \left[\frac{\pm 1}{\left[\frac{2\pi n - \sigma Q}{\alpha} \right]^2 - \omega_p^2 \pm i\epsilon} + \frac{2\pi i}{2\omega_p} \left[\delta \left[\omega_p - \frac{2\pi n - \sigma Q}{\alpha} \right] g_Q(\omega_p) + \delta \left[\omega_p + \frac{2\pi n + \sigma Q}{\alpha} \right] g_{\bar{Q}}(\omega_p) \right] \right] , \end{aligned} \quad (\text{A7})$$

where the sign $\pm$ on the right-hand side means that plus for S^A and minus for S^B is used. Then the Feynman rule for the C_3 path, which is used to compute time-independent quantities such as the entropy, is the same as listed above, except for the replacement

$$(2\pi)^4 \delta^4(k_1 + k_2 + \dots) \Rightarrow (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \dots) \alpha \delta_{n_1 + n_2 + \dots, 0} ,$$

where the conservation of continuous energy is replaced by that of a discrete one.

APPENDIX B: DERIVATION OF EQ. (3.5)

In this appendix we show the derivation of Eq. (3.5) from Eq. (3.3). We evaluate the α integration by the saddle-point method in Eq. (3.3),

$$e^{S_{E,N_b}} = \delta E \int_{-\pi}^{\pi} \frac{dt}{2\pi} e^{3iN_b t} \int_{-\infty - i\epsilon}^{\infty - i\epsilon} \frac{d\alpha}{2\pi} \exp \left[i\alpha E + i \frac{V}{\alpha^3} f(t) \right] , \quad (\text{B1})$$

where $f(t)$ is defined by

$$f(t) \equiv Q + K(t^4 - 2\pi^2 t^2) .$$

This function $f(t)$ has a zero point between $t=0$ and $t=\pi$ for $\text{SU}(3)$ color and $N_f \geq 2$. Let us define this zero point of $f(t)$ by $\bar{t}$:

$$f(\bar{t}) = 0 , \quad \bar{t} \in [0, \pi] .$$

For $|t| \leq \bar{t}$, $f(t)$ is positive and for $\bar{t} \leq |t| \leq \pi$ it is negative. Equation (B1) is then separated into two parts:

$$e^S = \delta E \int_{-\bar{\tau}}^{\bar{\tau}} \frac{dt}{2\pi} e^{3iN_b t} \int \frac{d\alpha}{2\pi} \exp \left[i\alpha E + i \frac{V}{\alpha^3} f(t) \right] + \delta E \left[\int_{-\pi}^{-\bar{\tau}} + \int_{\bar{\tau}}^{\pi} \right] \frac{dt}{2\pi} e^{3iN_b t} \int \frac{d\alpha}{2\pi} \exp \left[i\alpha E - i \frac{V}{\alpha^3} |f(t)| \right]. \quad (\text{B2})$$

The α integration in each term is evaluated by the saddle-point method when E, V are large and E/V is finite. In finding saddle points, one should notice that the imaginary part of α must be negative, because of the requirement of the convergence of the path integral.

The α integral of the first term in Eq. (B2) has a saddle point at

$$\alpha = -i \left[\frac{3Vf(t)}{E} \right]^{1/4},$$

the second term has two saddle points at

$$\alpha = e^{\pi i 5/4} \left[\frac{3V|f(t)|}{E} \right]^{1/4} \quad \text{and} \quad e^{\pi i 7/4} \left[\frac{3V|f(t)|}{E} \right]^{1/4}.$$

Then the saddle-point approximation gives

$$\begin{aligned} e^{S_{E,N_b}} \simeq & \delta E \int_{-\bar{\tau}}^{\bar{\tau}} \frac{dt}{2\pi} e^{i3N_b t} \frac{1}{2} \left[\frac{1}{2\pi} \right]^{1/2} \left[\frac{3Vf(t)}{E^5} \right]^{1/8} \exp \left[\frac{4}{3} [3E^3 V f(t)]^{1/4} \right] + \delta E \left[\int_{-\pi}^{-\bar{\tau}} + \int_{\bar{\tau}}^{\pi} \right] \frac{dt}{2\pi} e^{i3N_b t} \\ & \times 2 \operatorname{Re} \left\{ \frac{1}{2} \left[\frac{1}{2\pi} \right]^{1/2} e^{-\pi i/8} \left[\frac{3V|f(t)|}{E^5} \right]^{1/8} \exp \left[\frac{4}{3} e^{-\pi i/4} (3E^3 V |f(t)|)^{1/4} \right] \right\}. \end{aligned} \quad (\text{B3})$$

By neglecting the factor 2 in the second term, Eq. (B3) is approximated by Eq. (3.5).

APPENDIX C: DERIVATION OF THE PROPAGATORS ON THE C_3 PATH

This appendix is used to illustrate the derivation of the propagators on the C_3 path in Eqs. (A4) and (A7). First, let us examine the periodicity of $D_{\alpha}^{A,B}(x-y)$ in Eq. (A3). The propagators $D_{\alpha}^{A,B}$ are written [1] as

$$\begin{aligned} iD_{\alpha}^A(x-y) &= \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi\delta(k^2 - m^2) f_{\alpha}(\omega_k) \right\}, \\ iD_{\alpha}^B(x-y) &= \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \frac{-i}{k^2 - m^2 - i\epsilon} + 2\pi\delta(k^2 - m^2) f_{\alpha}(\omega_k) \right\}. \end{aligned} \quad (\text{C1})$$

Using Eq. (C1), one can show their periodicity:

$$iD_{\alpha}^{A,B}(x-y) \Big|_{x_0=-T} = iD_{\alpha}^{A,B}(x-y) \Big|_{x_0=-T+\alpha}.$$

This is consistent with the boundary condition of fields in Eq. (2.3). Because of this boundary condition, $D_{\alpha}^{A,B}$ is expanded in a series like the first line of Eq. (A4). For instance, the component $iD_{\alpha,n,k}^A$ is given by

$$iD_{\alpha,n,k}^A = \int_{-T}^{-T+\alpha} dx_0 \int d^3 x \exp \left[i \frac{2\pi n}{\alpha} (x_0 - y_0) - i \mathbf{k} \cdot (\mathbf{x} - \mathbf{y}) \right] iD_{\alpha}^A(x-y).$$

Inserting Eq. (C1) and performing $d^4 k$ and $d^3 x$ integrations, one obtains

$$\begin{aligned} iD_{\alpha,n,k}^A &= \int_{-T}^{-T+\alpha} dx_0 \exp \left[i \frac{2\pi n}{\alpha} (x_0 - y_0) \right] \\ &\times \left\{ \theta(x_0 - y_0) \left[\frac{1}{2\omega_k - i\epsilon} e^{-i(\omega_k - i\epsilon)(x_0 - y_0)} (1 + f_{\alpha}(\omega_k)) + \frac{1}{2\omega_k + i\epsilon} e^{i(\omega_k + i\epsilon)(x_0 - y_0)} f_{\alpha}(\omega_k) \right] \right. \\ &\quad \left. + \theta(y_0 - x_0) \left[\frac{1}{2\omega_k - i\epsilon} e^{i(\omega_k - i\epsilon)(x_0 - y_0)} (1 + f_{\alpha}(\omega_k)) + \frac{1}{2\omega_k + i\epsilon} e^{-i(\omega_k + i\epsilon)(x_0 - y_0)} f_{\alpha}(\omega_k) \right] \right\}. \end{aligned} \quad (\text{C2})$$

Then integration over x_0 gives

$$\begin{aligned}
iD_{\alpha,n,k}^A = & (1+f_\alpha) \frac{1}{2\omega_k - i\epsilon} \frac{1}{i(2\pi n/\alpha - \omega_k + i\epsilon)} \left[\exp \left\{ i(2\pi n/\alpha - \omega_k + i\epsilon)(-T + \alpha - y_0) \right\} - 1 \right] \\
& + f_\alpha \frac{1}{2\omega_k + i\epsilon} \frac{1}{i(2\pi n/\alpha + \omega_k + i\epsilon)} \left[\exp \left\{ i(2\pi n/\alpha + \omega_k + i\epsilon)(-T + \alpha - y_0) \right\} - 1 \right] \\
& + (1+f_\alpha) \frac{1}{2\omega_k - i\epsilon} \frac{1}{i(2\pi n/\alpha + \omega_k - i\epsilon)} \left[1 - \exp \left\{ i(2\pi n/\alpha + \omega_k - i\epsilon)(-T - y_0) \right\} \right] \\
& + f_\alpha \frac{1}{2\omega_k + i\epsilon} \frac{1}{i(2\pi n/\alpha - \omega_k - i\epsilon)} \left[1 - \exp \left\{ i(2\pi n/\alpha - \omega_k - i\epsilon)(-T - y_0) \right\} \right], \quad (C3)
\end{aligned}$$

where f_α stands for $f_\alpha(\omega_k)$. One should be careful about the order in which to take limits $T \rightarrow \infty$ and $\epsilon \rightarrow 0$. The result depends on the order in which these two limits are taken. First, we take the limit $T \rightarrow \infty$, keeping ϵ finite. Then T -dependent terms vanish when they are integrated over $d^3\mathbf{k}$, because of the Riemann-Lebesgue theorem. It is important to keep ϵ finite, otherwise the above theorem is not valid because the integrand in the momentum integration has singularities. Next we taken the limit $\epsilon \rightarrow 0$ and obtain $D_{\alpha,n,k}^A$ as in Eq. (A4).

One can obtain Eq. (A7) for the quark field in a similar way. The explicit form of fermion's propagator in Eq. (A5) is given by

$$S_{\alpha\sigma}^{A,B,ij}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} (\not{p} + m) \left\{ \frac{\pm 1}{p^2 - m^2 \pm i\epsilon} + 2\pi i \delta(p^2 - m^2) [\theta(p_0) g_Q(\omega_p) + \theta(-p_0) g_{\bar{Q}}(\omega_p)] \right\}, \quad (C4)$$

where plus stands for S^A and minus for S^B . Using Eq. (C4), one obtains the periodicity property

$$\begin{aligned}
S_{\alpha,\sigma}^{A,B,ij}(x-y) \Big|_{x_0 = -T+\alpha} &= -e^{i\sigma Q} S_{\alpha,\sigma}^{A,B,ij}(x-y) \Big|_{x_0 = -T}, \\
S_{\alpha,\sigma}^{A,B,ij}(x-y) \Big|_{y_0 = -T+\alpha} &= -e^{-i\sigma Q} S_{\alpha,\sigma}^{A,B,ij}(x-y) \Big|_{y_0 = -T},
\end{aligned}$$

where σQ means $\sum_j \sigma_j Q_j$. These periodicities are consistent with Eq. (2.3). By a similar procedure in deriving Eq. (A4), one obtains Eqs. (A6) and (A7).

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