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ON GAUGE INVARIANCE OF THE RENORMALIZED Z^0 REST MASS*

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The properly renormalized Z^0 rest mass is shown to be gauge invariant in the rigorous, complete on-shell renormalization scheme. Thus, the mass measured at LEP/SLC is indeed a physical observable and can be used as a basic parameter of the $SU_{2L} \times U_1$ theory, as indeed it has so been.

1. INTRODUCTION

Recently, the question of the gauge invariance of the renormalized Z^0 rest mass in the on-shell renormalization scheme has come under considerable discussion.^{1,2} For, this mass is currently one of the best measured parameters of the $SU_{2L} \times U_1$ theory³ and, hence, one would like to be sure that it is indeed a physical observable that is being measured. Thus, in this paper we prove that this mass is indeed a physical observable, i.e., that it is indeed gauge invariant.

First, let us emphasize that 'tHooft, 'tHooft and Veltman, and Lee and Zinn-Justin⁴ have proved that the renormalized mass parameters of any spontaneously broken gauge theory for physical particles are gauge invariant by proving that S-matrix elements are gauge invariant, since these latter are the residues of Green functions at squared 4-momenta equal

to just these masses squared, i.e., they have shown that the renormalized Lagrangian possesses a gauge invariance which leaves the renormalized masses of all physical particles invariant under the respective gauge transformations. Operationally, it is enough to note that the value of each physical particle's mass determines the positions of a series of thresholds in the S-matrix so that such masses must be gauge invariant if the S-matrix is gauge invariant itself.

The proofs in Ref. 4, however, are formulated for zero width physical particles, that is to say, for physical particles whose propagators have poles on the real axis, like that of the electron. The heavy vector bosons Z^0 and $W^\pm$ as well as the GSW physical Higgs field H in fact have non-trivial widths in general in comparison to their "rest masses." Thus, their propagator poles lie an observable

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distance from the real axis and the entire discussion from Refs. 4 has to be extended to include this complex pole case. In particular, even the issue of what the "rest mass" is can be raised, as it has by some.^{1,2}

Concerning this latter issue, Gounaris and Sakurai⁵ (GS), in their discussion of the ρ sometime ago, have already shown that, near the resonance itself, the inverse propagator should take the form, suppressing an overall phase and trivial kinematic tensor factors,

$$s - m_V^2 + i\sqrt{s}\Gamma_V(s) \quad (1)$$

where $\Gamma_V(s)$ is the so-called s -dependent width. Already in (1), we see that $\Gamma_V \neq 0$ implies that the real part of the solution, sp , of the equation

$$s - m_V^2 + i\sqrt{s}\Gamma_V(s) = 0 \quad (2)$$

is not exactly m_V^2 : there is a shift of Res_p from m_V^2 of size $\sim \Gamma_V^2$ if $\Gamma_V^2/m_V^2 \ll 1$. Such a shift, in a high precision environment like Z^0 physics at LEP/SLC, is observable and, hence, one may ask what should one call the mass of the ρ (or Z^0 , etc.), is the parameter m_V^2 actually gauge invariant and, if it is, is it related to the conventional on-shell value of m_V^2 as derived in renormalized quantum field theory, etc. In other words "Have LEP/SLC experimentalists been comparing their data to the 'right' quantity when they say they are measuring M_{Z^0} ?"

Concerning what should be called M_{Z^0} , we are fortunate to have the method of the operator field to answer this question. For, the

$SU_{2L} \times U_1$ Lagrangian density is well-known:

$$\mathcal{L}_U = -\frac{1}{4}F_{Z^0}^2 + \frac{1}{2}M_{Z^0}^2 Z^0^2 + \dots \quad (3)$$

where... represents the remaining terms⁶ in $\mathcal{L}_U$, the unrenormalized $SU_{2L} \times U_1$ Lagrangian density. The specific form of these remaining terms will not be necessary for our current analysis. Thus, in (3), $F_{Z^0}^{\mu\nu}$ is the unrenormalized Z^0 field strength tensor, Z_μ^0 is the unrenormalized Z^0 field, and M_{Z^0} is the unrenormalized Z^0 mass: we get, therefore, the answer that the unrenormalized Lagrangian Z^0 rest mass squared is the coefficient of $\frac{1}{2} Z_\mu^0^2$ in $\mathcal{L}_U$. This is unambiguous.

For the renormalized theory, the value of what we call the renormalized mass parameter will depend on our renormalization scheme. In any given scheme, on-shell, $\overline{MS}$, etc., however, this renormalized Z^0 mass parameter, M_{Z^0} , will have a unique value. For our analysis, we will use the on-shell scheme in which, for zero width particles, the value of the renormalized mass parameter is just the observed rest mass of the respective particle (All other renormalized mass parameter values arising from different schemes are related to our on-shell parameter for a given particle via standard renormalization group methods). Thus, here, for the renormalized theory, we introduce on-shell counter-terms as usual (if we were using the $\overline{MS}$ scheme, we would introduce its counter-terms here and everything we do henceforward would still be

valid for the corresponding MS parameter values) through

$$\mathcal{L}_U = \mathcal{L}_R + \Delta\mathcal{L} \quad , \quad (4)$$

$$\mathcal{L}_R = -\frac{1}{4}F_{Z^\circ}^2 + \frac{1}{2}M_{Z^\circ}^2 Z_R^{\circ 2} + \dots \quad (5)$$

and the counter-term Lagrangian given by

$$\begin{aligned} \Delta\mathcal{L} = & -\frac{(Z_3-1)}{4}F_{Z^\circ}^2 + \frac{(Z_3-1)}{2}M_{Z^\circ}^2 Z_R^{\circ 2} \\ & + \frac{1}{2}\Delta M^2 Z_3 Z_R^{\circ 2} + \dots \end{aligned} \quad (6)$$

where... now represents respectively the rest of the renormalized Lagrangian $\mathcal{L}_R$, and the rest of $\Delta\mathcal{L}$, Z_R° is the renormalized Z° field and $M_{Z^\circ}^2$ is the squared renormalized Lagrangian Z° rest mass: $M_{Z^\circ}^2$ is the coefficient of $\frac{1}{2}Z_R^{\circ 2}$ in $\mathcal{L}_R$. Here, Z_3 is the Z° field renormalization constant and ΔM^2 is the Z° mass counter-term so that

$$\begin{aligned} Z_u^\circ &= Z_3^{1/2} Z_R^\circ \\ M_{Z^\circ, u}^2 &= M_{Z^\circ}^2 + \Delta M^2 \end{aligned} \quad (7)$$

Thus, we see that $M_{Z^\circ}^2$ is well defined in the renormalized $SU_{2L} \times U_1$ Lagrangian density. Since $\Gamma_{Z^\circ} \neq 0$, to show that $M_{Z^\circ}^2$ is gauge invariant we now must extend the arguments in Ref. 4 to cover the case that the pole of the renormalized Z° propagator is not on the real axis in the complex s plane. There are many ways to do this. The simplest in SLC/LEP physics is to observe that $s_p = M_{Z^\circ}^2 - iM_{Z^\circ} \Gamma_{Z^\circ}$, the position of the pole in the complex s plane of the Z° in the S-matrix element for $e^+e^- \rightarrow \mu^+\mu^-$, must be gauge invariant. Hence, we can show

that $M_{Z^\circ}^2$ is gauge invariant by expressing it in terms of $M_{Z^\circ}^2$ and Γ_{Z° , since they are gauge invariant. As a by-product, we shall also express $\Gamma_{Z^\circ}(M_{Z^\circ})$, the decay rate for the Z° at $P^2 = M_{Z^\circ}^2$ to all final states, in terms of Γ_{Z° and M_{Z° .

Specifically, we write the inverse Z° propagation function as

$$\begin{aligned} \bar{D}_{F_{\mu\nu}}^{-1} = & g_{\mu\nu}(s - M_{Z^\circ}^2 - \Delta M^2 Z_3 + \text{Re} \sum_{n=1}^{\infty} \pi^{(n)}(M_{Z^\circ}^2) \\ & + (s - M_{Z^\circ}^2) \sum_{n=1}^{\infty} \text{Re} \pi^{(n)'}(M_{Z^\circ}^2) + \dots + i \text{Im} \pi(s) \\ & + (Z_3 - 1)(s - M_{Z^\circ}^2) + q_\mu q_\nu - \text{terms} \end{aligned} \quad (8)$$

where...represent the remaining terms in the Taylor series of $\text{Re} \pi(s)$ about $s = M_{Z^\circ}^2$, prime denotes differentiation, and $A^{(n)}$ is always the n -loop contribution to the quantity A . Here

$$\pi = \pi_{Z^\circ Z^\circ} + \pi_{\gamma Z^\circ} \pi_{Z^\circ \gamma} / (s + \pi_{\gamma\gamma}) \quad (9)$$

where $\pi_{Z^\circ Z^\circ}(\gamma\gamma)$ is the transverse 1PI $Z^\circ - Z^\circ$ ($\gamma - \gamma$) two point vertex function and $\pi_{\gamma Z^\circ}$ is the 1PI $\gamma - Z^\circ$ two point vertex function.

Henceforth, we continue to suppress the explicit form of the gauge dependent $q_\mu q_\nu$ terms but we work in an arbitrary gauge, nonetheless. (Here, we define the 1PI B-C two point function as $i\pi_{BC}^{\mu\nu}(q) = -i\pi_{BC}(q^2)g^{\mu\nu} + i\pi_{LBC}(q^2)q^\mu q^\nu / q^2, B, C = Z^\circ, \dots$)

The standard⁷ complete on-shell renormalization conditions then allow us, as $s \rightarrow M_{Z^\circ}^2$, to identify

$$Z_3 = 1 - R\theta \sum_n \pi^{(n)'}(M_Z^2) \quad (10)$$

$$\Delta M_Z^2 = (R\theta \sum_{n=1}^{\infty} \pi^{(n)}(M_Z^2)) / Z_3$$

Note the factor of Z_3^{-1} in ΔM_Z^2 . It is this factor which makes M_Z^2 gauge invariant. Hence, we may now write

$$\tilde{D}_{F_{\mu\nu}}^{-1}(q) = g_{\mu\nu} D(s) + q_\mu q_\nu - \text{terms} \quad (11)$$

where

$$D(s) = s - M_Z^2 + i \text{Im} \pi(s) + \sum_{n=1}^{\infty} \sum_{k=2}^{\infty} (s - M_Z^2)^k \frac{1}{k!} \frac{d^k}{ds^k} R\theta \pi^{(n)}(s) \Big|_{s=M_Z^2} \quad (12)$$

We want to prove that M_Z^2 in (12) is gauge invariant.

Indeed, let us compute s_p , the complex pole associated with (12) in the $e^+e^- \rightarrow \mu^+\mu^-$ S-matrix element, in terms of M_Z^2 , the rigorous on-shell Lagrangian Z^0 rest mass. This means that only that part of (12) which is left after all order-by-order cancellations with contributions from other pieces of the S-matrix element have been effected should be used to compute s_p . Specifically, in $\text{Im}\pi(s)$, we must omit from our s_p computation all contributions from unphysical vector boson polarization states, unphysical Higgs scalars, and Feynman-Faddeev-Popov ghosts, since 't Hooft⁴ has shown that, in the S-matrix, the sum of all such imaginary part contributions from all of its proper vertex components vanishes. Hence, let

$\text{Im}\tilde{\pi}^{(n)}(s)$ be that part of $\text{Im}\pi^{(n)}$ which remains after these 't Hooft cancellations in our S-matrix element. (There is an implied associated real part $\text{Re}\tilde{\pi}^{(n)}(s)$ which is gauge invariant and is determined from $\tilde{\pi}^{(n)}$ via dispersion relations; we do not need it here because we only work through order α^2 .) Then, working through order α^2 for definiteness, we have

$$0 = s_p - M_Z^2 + i \text{Im} \tilde{\pi}^{(1)}(M_Z^2) + (s_p - M_Z^2) i \text{Im} \tilde{\pi}^{(1)'}(M_Z^2) + i \text{Im} \tilde{\pi}^{(2)}(M_Z^2) + O(\alpha^3) \quad (13)$$

This gives

$$s_p = M_Z^2 - \frac{i \text{Im} \tilde{\pi}^{(1)}(M_Z^2) + i \text{Im} \tilde{\pi}^{(2)}(M_Z^2)}{1 + i \text{Im} \tilde{\pi}^{(1)'}(M_Z^2)} + O(\alpha^3) \quad (14)$$

We conclude that M_Z^2 is the rigorous order α^0 part of Res_p . This proves that it is gauge invariant. Further, we recover the result of Bardin et al.⁸ that $\text{Res}_p^{1/2}$ is shifted by ~ 34 MeV relative to M_Z^2 :

$$\begin{aligned} \bar{M}_Z^2 = \text{Res}_p &= M_Z^2 - i \text{Im} \tilde{\pi}^{(1)}(M_Z^2) \\ &\quad i \text{Im} \tilde{\pi}^{(1)'}(M_Z^2) + O(\alpha^3) \\ &= M_Z^2 - \Gamma_Z^{(1)}(M_Z^2) + O(\alpha^3) \end{aligned} \quad (15)$$

We also may note from (14)

$$\bar{M}_Z \cdot \bar{\Gamma}_Z = M_Z \cdot (\Gamma_Z^{(1)} + \Gamma_Z^{(2)}) + O(\alpha^3) \quad (16)$$

This proves that $\Gamma_Z(M_Z)$ is also gauge invariant to order α^2 . Actually, by construction it follows $\Gamma_Z(M_Z)$ is gauge invariant to all orders in α . For, since we want s_p to be a

pole in the S-matrix, we follow 't Hooft⁴ and omit all unphysical states from the $SU_{2L} \times U_1$ unitarity equations.

In summary, the complete rigorous on-shell renormalized Lagrangian Z^0 rest mass, defined as the square root of the order α^0 part of Res_p or as the coefficient of $Z_R^{\alpha\mu} Z_{R\mu}^0/2$ in the renormalized $SU_{2L} \times U_1$ Lagrangian defined by

$$\mathcal{L}_U = \mathcal{L}_R + \Delta\mathcal{L}$$

where $\mathcal{L}_U$ is the complete unrenormalized Lagrangian and $\Delta\mathcal{L}$ is complete counterterm Lagrangian, is a manifestly gauge invariant quantity. This result clarifies recent discussion in the literature.^{1,2} Of course, from the standpoint of S-matrix theory, one may also use the pole rest mass squared Res_p as the fundamental mass squared parameter of the Z^0 . Here, we only want to emphasize that both M_Z^2 and Res_p are gauge invariant to all orders in α . Either of them may be used as a fundamental input parameter of the $SU_{2L} \times U_1$ theory.

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