LOW INTENSITY RFQS

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ABSTRACT

The design of low intensity RFQs is different from the design of high intensity ones. In the former case, the requirements are less severe, resulting in shorter accelerating structures.

The establishment of “sound” design conditions for low intensity RFQs as well as the beam optics design of the RFQ for the lead ion project are presented.
INTRODUCTION

The design of RFQs follows some rules, which are particularly severe when dealing with high beam intensities. Essentially one tries to maintain constant beam dimensions in order to keep the same space charge forces, which are then, in turn, balanced by external RF forces. Constant beam dimensions, in particular along the longitudinal axis, have as consequence a very slow acceleration and hence lead to a relatively long RFQ.

When dealing with low intensities, one can depart from a constant bunch length (in real space) and try a quicker bunching, shortening in this way the RFQ, but without losing the beam quality. Of course, the change in the longitudinal beam dimension cannot be done too rapidly and, anyway, some design conditions have to be respected.

It is the establishment of such "sound" design conditions for low intensity RFQs, which makes the subject of this report. Following these conditions, one has designed the RFQ for the Lead Ion Project at CERN, shown here as example.

1. LOW INTENSITY RFQ VERSUS HIGH INTENSITY ONE

The design of high intensity RFQs at CERN followed essentially the procedure introduced by LANL. The RFQ is conveniently divided into four sections, the RMS (radial matching section), Shaper, Gentle Buncher and Accelerating section. The beam, continuous at the input, starts to get bunched in the Shaper, where the longitudinal RF field is slowly raised from zero to a certain value. The bunching is performed over a significant part of the longitudinal oscillation period (synchrotron oscillations). During this time, the bucket area is formed and progressively raised (adiabatically), together with the synchronous phase. The capture efficiency of the RFQ depends strongly on the delicacy of the parameter change in the Shaper.

The bunching is completed in the next section, the Gentle Buncher. The laws governing the beam dynamics are those established by Kapchinski [1]. Here the constancy of all the forces and beam dimensions is rigorously achieved. The phase compression is done slowly in a bucket of constant area. The following Accelerating section is intended to accelerate the beam to its final energy. Here one usually keeps the vane modulation constant, thus allowing for a slight increase in the bunch length. The consequence is a slight reduction in the transverse beam dimensions (the increased bunch diminishes the space charge forces), which is again useful to compensate for steering errors.

The design of low intensity RFQs does not follow the requirement of constancy of beam dimensions, but uses a quicker bunching, resulting thus in a smaller accelerator length. For low intensity RFQs one has to impose other conditions and in doing so we have essentially based our approach on Yamada’s work [2], introducing modifications, where we considered them useful.

Our approach for the design of low intensity RFQs is explained in more detail in the next sections. For the sake of completeness, we have presented the derivation of some formulae, which could also be found elsewhere.

It is assumed that the reader is familiar with the significance of various RFQ parameters.
2. LOW INTENSITY RFQ

An RFQ is usually divided in sections, each of which is designed so as to produce a well defined effect on the beam dynamics. Following Yamada we have divided our RFQ into six sections:

1. RMS (radial matching section)
2. Shaper
3. Prebuncher
4. Adiabatic buncher
5. Booster
6. Accelerating section

The fourth section, the adiabatic buncher, follows some new prescription proposed by Kapchinskij.

2.1. RMS

The radial matching section is where the beam is being matched to the time-varying transverse acceptance of the RFQ channel. This is achieved by slowly increasing the transverse focusing from zero to the final value, over a given number of cells, usually four or six. In this way it is possible to define constant transverse beam parameters at the input of the RFQ. This section is designed following the method developed at Los Alamos and already successfully applied in the RFQ2 project [3].

2.2. SHAPER

The shaper is dedicated to the creation of the bucket. It is composed of about ten or twenty cells, where a linear change of the synchronous phase and of the accelerating factor take places according to the following laws:

\[
\varphi_s(z) = \begin{cases} 
-\frac{\pi}{2} & z \leq z_0 \\
-\frac{\pi}{2} + \Delta \varphi_s \frac{z-z_0}{L_{sh}-z_0} & z > z_0 
\end{cases} \tag{2.2.1}
\]

with \( \Delta \varphi_s = \varphi_{sh} + \pi/2 \), where \( \varphi_{sh} \) is the synchronous phase at the end of the shaper, and

\[
A(z) = A_{sh} \frac{z}{L_{sh}} \tag{2.2.2}
\]
where $A_{sh}$ is the accelerating factor at the end of the section. In both expressions $L_{sh}$ is the length of the shaper and $z$ is the coordinate along the axis of the RFQ with origin at the beginning of the shaper.

The length $L_{sh}$ can be calculated by integrating the equation of motion:

$$
\frac{d\beta}{dz} = \frac{\pi}{2} \frac{QeV}{Mm_0c^2\lambda} \frac{A(z)\cos\varphi(z)}{\beta^2}
$$

(2.2.3)

where $Q$ and $Mm_0c^2$ are the charge state and rest energy of the ion, $\lambda$ is the wavelength of the rf field and $\beta$ is the relativistic factor. Substituting the expressions (2.2.1) and (2.2.2) in (2.2.3), the length can be expressed in the following way [4]:

$$
L_{sh} = \frac{2Mm_0c^2\lambda F}{3\pi QV} \frac{\beta^3_{sh} - \beta^3_i}{A_{sh}}
$$

(2.2.4)

where $\beta_{sh}$ and $\beta_i$ are the values of the relativistic parameter at the end of the shaper and at the input of the RFQ, respectively, and

$$
F = \frac{\Delta \varphi^2}{(1 - \Lambda)} \left[ -\Delta \varphi \cos \Delta \varphi + \sin \Delta \varphi + \frac{\Delta \varphi \Lambda (1 - \cos \Delta \varphi)}{1 - \Lambda} \right]
$$

(2.2.5)

with $\Lambda = z/L_{sh}$.

For reasons that we will see in the next sections, it is more convenient, in our design, to express $L_{sh}$ as a function of the area of the bucket $S_{sh}$ at the end of the shaper:

$$
L_{sh} = \frac{\lambda F \pi |\sin \varphi_{sh}|}{3S_{sh}^2} G_{sh}^2 \left( \beta^5_{sh} - \beta^3_i \beta^2_{sh} \right)
$$

(2.2.6)

where

$$
S_{sh} = \sqrt{\sigma^2_{Ish} \beta^2_{sh} \Phi_{sh}} \sqrt{1 - \frac{\varphi_{sh}}{\tan(\varphi_{sh})}}
$$

(2.2.7)

and

$$
G_{sh} = \Phi_{sh} \sqrt{1 - \frac{\varphi_{sh}}{\tan(\varphi_{sh})}}
$$

(2.2.8)

$\Phi_{sh}$ is the length of the bucket at the end of the shaper and $\sigma_{Ish}$ is the corresponding phase advance per period of the synchrotron oscillations.

## 2.3. PREBUNCHER

The prebuncher [2] is a section where the beam is rapidly compressed in phase, in roughly half of a synchrotron oscillation. The area of the bucket is kept constant and the rapid bunching is obtained by raising the phase advance per period
of the small amplitude synchrotron oscillations $\sigma_1$ as function of the relativistic factor $\beta$. The following expression has been proved to be successful:

$$\sigma_1^2(\beta) = \sigma_{lpb}^2 - \left(\sigma_{lpb}^2 - \sigma_{ish}^2\right) \left[\frac{\beta - \beta_{pb}}{\beta_{sh} - \beta_{pb}}\right]^n$$  

(2.3.1)

where $\sigma_{ish}$ and $\sigma_{lpb}$ are the synchrotron phase advances at the end of the shaper and of the prebuncher, respectively, and $\beta_{sh}$ and $\beta_{pb}$ are the corresponding relativistic factors. To calculate the length of the section $L_{pb}$, it is better to put the equation of motion in the following form:

$$\frac{d\beta}{dz} = \frac{1}{2\pi\lambda} \sigma_1^2(\beta) \frac{1}{\tan \varphi_s}$$  

(2.3.2)

and by substituting (2.3.1) in (2.3.2), one obtains, after a simple algebra:

$$L_{pb} = \frac{2\beta_{sh} \pi \lambda}{\sigma_{lpb}^2} \int_{1}^{\beta_{sh} - \beta_{pb}} \frac{\beta_s}{f(\xi)} \frac{d\xi}{\tan \varphi_s(\xi)}$$  

(2.3.3)

where

$$f(\xi) = 1 - \left(1 - \left(\frac{\xi - \beta_{pb}}{\beta_{sh}}\right)^n\right)$$  

(2.3.4)

with $\xi = \frac{\beta}{\beta_{sh}}$.

The integration of eq. (2.3.3) has to be carried out numerically and the value of $\varphi_s(\xi)$ involved in the integrand function can be calculated from the numerical inversion of the following system of two equations, with $\varphi_s$ and $\Phi$ as unknowns:

$$S = \sqrt{\frac{\sigma_1^2(\beta)}{2}} \beta^2 \Phi \sqrt{1 - \frac{\varphi_s}{\tan \varphi_s}} = \text{const}$$  

(2.3.5)

$$\tan \varphi_s = \frac{\sin \Phi - \Phi}{1 - \cos \Phi}$$  

(2.3.6)
The number of the small amplitude longitudinal oscillations is given by:

\[ \text{Nosc} = \frac{1}{2\pi} \int \sigma_1 \, d\eta \]  

(2.3.7)

where \( \eta \) is the normalized longitudinal variable \( \frac{z}{\beta \lambda} \). By means of the eq. (2.3.2) and (2.3.4), (2.3.7) becomes:

\[ \text{Nosc} = \frac{1}{\sigma_{lpb}} \int_1^{\beta/\beta^*} \frac{\tan \varphi_s(\xi)}{f(\xi)^{1/2}} \, d\xi \]  

(2.3.8)

2.4. ADIABATIC BUNCHER

This section, proposed by Kapchinskij [4], replaces the former Gentle Buncher, when dealing with low beam intensities.

The distribution of the electric field in an RFQ cell is, for a given vane geometry, unambiguously defined by the modulation factor \( m \), the cell length and the average aperture radius \( R_0 \). The peak surface electric field can be expressed as:

\[ E_s = \chi \frac{V}{R_0} \]  

(2.4.1)

where \( \chi \) is the field factor having the form:

\[ \chi \equiv \chi(m, kR_0) \]  

(2.4.2)

with \( k = 2\pi / \beta \lambda \).

![Figure 1](image.png)

Fig. 1 vs. \( kR_0 \) for different values of \( m \)
The fig. 1 shows plots of $\chi$ as a function of $kR_0$, for several values of $m$; the plots are valid for vane geometries with a constant radius of curvature $\rho$ (fig. 1 valid for $\rho = R_0$).

The design of the adiabatic buncher is carried out by changing the modulation factor $m$ in such a way as to keep $\chi$ constant at its maximum value it had at the end of the prebuncher. The maximum acceleration rate is thus achieved, without exceeding the maximum allowable surface field.

It is not possible to express (2.4.2) in an analytical form, so the relationship between $m$, $kR_0$ and $\chi$ is obtained by numerical computations and presented in tables [5]. However, a relation suggested by Kapchinskij:

$$m = b + \frac{c}{(kR_0)^2}$$

(2.4.3)

is a good approximation to $\chi(m, kR_0) = \text{const}$ for a wide range of $kR_0$, provided $b$ and $c$ are determined by a least square fit.

Concerning the phase law used in this section we did not apply the one suggested by Kapchinskij in [4], as we found that a relatively fast bunching and a good capture can be achieved by applying the following condition:

$$\beta^a \Phi = \text{const}$$

(2.4.4)

where $\Phi$ is the length of the bucket.

With $\alpha \leq 1$ and $\chi = \text{const}$, the area of the separatrix slightly increases in the adiabatic buncher.

The length of this section has to be computed cell by cell. The energy gain in a cell defines, with (2.4.3) and (2.4.4), the parameters in the successive one.

### 2.5. Booster

Once the bunching is completed, the accelerating field is rapidly raised in the booster. The modulation factor is increased and the bore radius is reduced until the design value for the transverse acceptance is reached. A good acceleration rate is achieved by keeping the frequency of the synchrotron oscillations and the synchronous phase constant.

The constancy of the synchrotron frequency yields:

$$\frac{A_{ab}}{\beta_{ab}^2} = \frac{A_b}{\beta_b^2}$$

(2.5.1)

where $A_{ab}, \beta_{ab}$ and $A_b, \beta_b$ are the accelerating and the relativistic factors at the end of the adiabatic buncher and of the booster, respectively.
Substituting eq. (2.5.1.) in the equation of motion, the length of the booster can be expressed as:

\[ L_b = \frac{2}{\pi} \frac{M m_0 c^2 \lambda}{A_{ab} Q V \cos \varphi_{ab}} \beta_{ab}^2 (\beta_b - \beta_{ab}) \]  (2.5.2)

where \( \varphi_{ab} \) is the synchronous phase at the end of the adiabatic buncheder.

In (2.5.2), \( \beta_b \) is the only unknown quantity. It can be calculated using the extra condition of a constant focusing factor \( B \), that holds in all the sections of the RFQ. After some calculations the following expression for \( \beta_b \) is found:

\[ \beta_b = \frac{\beta_{ab}^2}{A_{ab}} \left[ 1 - \left( \frac{M m_0 c^2 B}{Q V \lambda^2} + \frac{A_{ab} \pi^2}{\beta_{ab}^2 \lambda^2} \right) a_b^2 \right] \]  (2.5.4)

In this way, the energy at the end of the Booster is determined. The other parameter to determine is \( a_b \), given by the required acceptance at the end of the booster. In smooth approximation one has:

\[ a_b = \frac{\lambda A_n}{\sigma_f} \sqrt{\Psi} \]  (2.5.5)

where \( A_n \) is the normalized acceptance, \( \sigma_f \) the phase advance per period of the betatron oscillations and \( \Psi \) is the average wiggle factor, defined as the ratio between the maximum and the minimum amplitude of betatron oscillations:

\[ \Psi = \frac{1 + \frac{B}{4\pi^2}}{1 - \frac{B}{4\pi^2}} \]  (2.5.6)

### 2.6. ACCELERATING SECTION

In this section the beam is accelerated up to its final energy. The bore radius is unchanged and the modulation factor is raised to have a constant transverse acceptance. The synchronous phase is either left equal to the one at the end of the adiabatic buncheder or it is changed by a small amount. The changes of the accelerating factor and of the synchronous phase are linear with \( z \), as for the shaper. The length of the section is calculated by numerical integration of the equation of motion.

### 2.7. THE DESIGN CODE

The equations given in the previous sections have been included in the FORTRAN code RFQADB.
An important aspect of the implementation of RFQADB has been the choice of parameters to be used as inputs and where, along the RFQ, such parameters are to be defined.

A description of the whole set of input data would be long and tedious. It is, however, important to remark that a key point of the whole RFQ is the end of the prebuncher. In fact, numerical computations have revealed that the maximum surface electric field is situated there. Furthermore, the dynamics of both the prebuncher and adiabatic buncher is defined, once the modulation factor, the average radius, the energy and phase of the synchronous particle are fixed at that point.

The choice of the values to give to the whole set of input parameters (excluding, of course, specifications such as the range of energy and the type of particle) is either a matter of experience or based on simple physical considerations.

Great attention has to be paid to the choice of $R_0$, because it defines both the radius of curvature of the vanes and, to some extent, the transverse focusing force. If a small $R_0$ is taken, the vanes would be thin and difficult, if not impossible, to realize; on the other hand, a big $R_0$ would lead to great electric fields or a weak focusing force.

For the lead ion RFQ, that will be presented in details in the next section, the range of values of $R_0$, determined following the previous considerations, is found to be between 3.5 and 4.5 mm. Provided vanes with a radius of curvature equal to the average radius are adopted.

### 3. THE LEAD ION RFQ

The code RFQADB has been used to design an RFQ satisfying the specifications put forward by the CERN Lead Ion Injector Project.

The RFQ has to accelerate a beam of lead ions, with charge state $25^+$, mass 208 and a normalized emittance of $0.5\pi$ mm-mrad, from 2.5 keV/u up to 250 keV/u. The minimum transverse acceptance of the accelerator is $0.8\pi$ mm-mrad and the peak surface field is two times the Kilpatrick limit (22.8 MV/m at 101.28 Mhz); the length of the structure is $\leq 250$ cm.

![Diagram](image-url)  
*Fig. 2: Modulation factor vs. length [cm]*
Fig. 2 shows the total length of the RFQ as a function of the modulation factor at the end of the prebuncher, for $R_0$ equal to 3.5, 4, and 4.5 mm. All the cases plotted in fig. 2 have been designed so as to have a transverse acceptance of $0.8\pi$ mm-mrad. The energy in the last cell of the prebuncher has been always chosen to maximize the number of particles captured in the bucket. The synchronous phases at the end of shaper, prebuncher, adiabatic buncher and accelerating section are: -88, -60, -30 and -20, respectively.

The factor $\alpha$ of equation (2.5.4) is equal to 0.9, this value being a good compromise between a good longitudinal acceptance in the adiabatic buncher and a reasonable length of the section.

The above set of parameters, with $R_0=4.5$ mm and $m_{ab}=1.08$, gives, as shown in fig. 2, an RFQ 245 cm long, with a transmission efficiency > 90% and no emittance growth. However, the analysis of the transverse motion has revealed a growth of the beam envelope in the last cells of the adiabatic buncher and it has been found that the oscillations of the beam were due to the build-up of parametric resonances. Parametric resonances occur because of the coupling between the synchrotron and the betatron oscillations. In particular, it can be shown [6] that, in the smooth approximation the regions of instabilities are centered around:

$$\sigma_i^2 = \frac{n^2}{4} \sigma_i^2$$

where $n$ is an integer. The first region, defined by $n=1$, is the most dangerous because of the very rapid build-up of oscillations.

In the lead ion RFQ, it has been found that the beam passed through the first region of resonances in ten consecutive cells at the end of the adiabatic buncher. This was enough to trigger the resonance oscillations.

To get out of the resonance region, the electric field in the adiabatic buncher has been diminished in each cell by an amount of the order of 1/10000 of the maximum value. The effect of resonances on the beam envelope is shown in fig. 3, where the transverse beam profile is presented. Fig. 4 shows the equivalent plot with the electric field decreased as described above.

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**fig 3 - x[cm] vs cell number**

**fig 4 - x[cm] vs cell number**
The total drop of the electric field in the adiabatic buncher is about the 1%. This caused the vane length to increase to a value slightly bigger than the optimum shown in fig. 2. An additional optimization was then required to reach the length of 250 cm given by the specifications. The most significant parameters of the final RFQ are presented in fig. 5 as functions of the longitudinal coordinate (z).

As it has already been mentioned in chapter 2, one of the key concepts in the "low intensity" design method is the fast phase compression. The effect of this quick bunching can be appreciated comparing the phase profile of the lead ion RFQ, shown in fig. 6 with the one of an equivalent RFQ, designed following the "high current" method, shown in fig. 7.

Fig. 8 presents, for both the above cases, the change of the accelerating factor and of the energy per unit charge along the accelerator. The plots show clearly that in the "low current" case the accelerating factor is higher and exhibits a faster rate of change than in the "high current" case.

![Diagram](image)

**fig. 5**
Plots of the parameters of the RFQ as function of the length. 
B = focusing factor, A = accelerating factor, m = modulation factor, W = energy per unit charge [MeV], a = aperture [cm].

![Diagram](image)

**fig. 6** - Phase width [deg.] vs. cell number
The design method, described in the previous sections, has been developed assuming that the beam current was zero, which is correct when dealing with a beam of lead ions. However, in order to understand the performances of the new design when space charge is present, beams with increasing current have been analyzed. Fig. 9 shows the transmission efficiency as function of beam current for both the lead ion RFQ and the equivalent “high current” design. As one could expect, the high current design exhibits a better behaviour, which is of course paid by having a longer accelerator.
4. CONCLUSIONS

This work has been undertaken in order to find a design method suitable for low intensity RFQs. The method is mainly based on previous studies made in other laboratories, but it shows some new and original features. A computer code, called RFQADB, has been implemented and successfully used to design an RFQ satisfying the requirements of the lead-ion injector project. During the design phase, parametric resonances have been encountered and a method to avoid them has been found. The method being developed under the assumption of zero current, the performances of the RFQs strongly depend on the beam intensity. If a small current (of about 10 mA) is to be accelerated, space charge effects have to be taken into account and the design modified accordingly. Anyway, the procedure remains quite different from the one applied for the CERN high intensity RFQs.

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