Jet Broadening Measures in e^+e^- Annihilation*

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Abstract

Predictions are presented for the distributions of two new e^+e^- event shape measures which involve the summed scalar transverse momentum relative to the thrust axis. The calculations include resummation of leading and next-to-leading logarithms to all orders in QCD perturbation theory, together with non-logarithmic terms up to second order. In the two-jet region the new event measures are complementary to the jet-mass-related quantities that were resummed earlier, and so they could be helpful in further refining the determination of the strong coupling constant from e^+e^- event shapes.

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The perturbative calculations of certain e^+e^- hadronic event shape variables have recently been extended from second order to include the resummation of large logarithmic terms to all orders [1-4]. The resummation in these cases has been performed with sufficient precision to permit improved measurements of the strong coupling constant α_S . Such measurements have already been presented by several of the collaborations working at LEP [5]. Their results have confirmed the theoretical expectations [1] that resummation can be helpful in reducing theoretical uncertainties associated with renormalization scale dependence and hadronization corrections. In particular, after resummation there is no longer a preference for very small scales, and the region of event shapes in which the predictions are reliable is increased.

So far, most of the work on resummation has concentrated on the distributions of thrust [1] and heavy jet mass [2]. Both of these quantities are sensitive mainly to the longitudinal structure of jets and the resummed expressions for them turn out to be very similar. To check the consistency of this method of α_S -determination it would be very desirable to have resummed predictions for a selection of event shapes that were less correlated. For some of the familiar event shape variables the leading logarithms can be resummed, but in most cases the full set of next-to-leading terms required for a meaningful resummed determination of α_S is not yet known.

We present here the predicted distributions of two event shape variables which are not very familiar but for which resummation can be performed with the necessary precision. They are particularly sensitive to transverse jet development and so they should usefully complement the others that have been resummed previously. A variable of this type was proposed some time ago for e^+e^- physics [6] but has so far been used mainly for the study of jet broadening in hadron collisions [7,8].

For an e^+e^- annihilation event, we define the (*total*) *jet broadening* B and the *wide jet broadening* B_W as follows. First the thrust axis is found and the event is divided into two hemispheres S_+ and S_- by the plane perpendicular to this axis. For each hemisphere we define

$$B_{\pm} = \sum_{i \in S_{\pm}} |\mathbf{p}_i \wedge \mathbf{n}_T| / 2 \sum_i |\mathbf{p}_i| \quad (1)$$

where $\mathbf{n}_T$ is the thrust axis.[§] Then

$$B = B_+ + B_- , \quad B_W = \text{Max}\{B_+, B_-\} . \quad (2)$$

Thus B is similar to the sphericity [9], except that it is defined with respect to the thrust axis instead of being minimized with respect to the choice of axis. The thrust and sphericity axes coincide to order α_S but the former has better properties from several points of view. As far as we know, there is no convenient procedure for finding the sphericity axis and it does not permit the simple resummation of logarithmic contributions. In particular, the plane perpendicular to the sphericity axis can contain final-state momentum vectors (which is impossible for the thrust axis) and therefore it does not divide all events unambiguously into hemispheres.

To order α_S , $B = B_W = \frac{1}{2}O$ where O is the oblateness [10], and the mean and maximum values of B are $0.65\alpha_S$ [6] and $1/2\sqrt{3} = 0.29$ respectively. The complete analytic one-loop

[§]The factor of 2 in the denominator will be seen to simplify the resummed expressions.

result for the distribution of B is very cumbersome. Therefore we report here only its small- B expansion

$$\frac{B}{\sigma} \frac{d\sigma}{dB} = -\frac{\alpha_S}{\pi} C_F \left[4 \ln B + 3 + 4B - 4B^2 \ln B - 2B^2 + \mathcal{O}(B^3 \ln B) \right], \quad (3)$$

where $C_F = 4/3$ in QCD. It turns out that this expression describes the first-order distribution within an accuracy of a few per cent out to $B = 0.26$. In higher orders, the jet broadening and oblateness are not related. For a spherical event, for example, $B = 2B_W = \pi/8$, while $O = 0$. At present, no way is known to resum large logarithms in the oblateness distribution.

The jet broadening distribution, on the other hand, can be easily resummed, because it *exponentiates*. By this we mean that the fraction of events with broadening less than B , denoted by $R(B, \alpha_S)$, takes the form

$$R(B, \alpha_S(Q^2)) = C(\alpha_S(Q^2)) \Sigma(B, \alpha_S(Q^2)) + F(B, \alpha_S(Q^2)) \quad (4)$$

where Q is the centre-of-mass energy and

$$\begin{aligned} C(\alpha_S) &= 1 + \sum_{n=1}^{\infty} \alpha_S^n C_n, \\ \ln \Sigma(B, \alpha_S) &= \sum_{n=1}^{\infty} \alpha_S^n \sum_{m=1}^{n+1} G_{nm} \ln^m B, \\ F(B, \alpha_S) &= \sum_{n=1}^{\infty} \alpha_S^n F_n(B). \end{aligned} \quad (5)$$

Here C_n and G_{nm} are constants, while $F_n(B)$ are perturbatively computable functions that vanish at small B . Notice that although the perturbative expansion of $R(B, \alpha_S(Q^2))$ itself contains terms of the form $\alpha_S^n \ln^m B$ for all values of m up to $2n$, its logarithm involves only $m \leq n + 1$. We call this property exponentiation. A similar property holds for the fraction $R_W(B_W, \alpha_S)$ with wide jet broadening less than B_W . In our calculations, the leading and next-to-leading terms in $\ln \Sigma$, i.e. those with $m \geq n$, are summed to all orders, and sub-dominant terms with $m < n$, as well as C and F , are included to second order only.

If we now introduce a renormalization scale $\mu^2 \neq Q^2$, taking into account that R is a renormalization group invariant quantity, then in place of $\alpha_S(Q^2)$ in Eq. (4) we find $\alpha_S(\mu^2)$ together with an explicit Q^2/μ^2 dependence of the next-to-leading logarithms and fixed-order terms. Thus we can study the scale dependence of the resummed expression and compare it with that of the fixed-order result.

Let us now compute the jet broadening fractions R and R_W at small B . Determining the jet broadening distribution is a problem similar to that of finding the distribution of transverse energy emitted in Drell-Yan processes in hadron collisions [11,12]. The main difference is that here the transverse direction is defined relative to the jet itself rather than an external beam momentum. For this reason it is convenient to introduce a function $T_a(Q, \mathbf{k}_t; p_t)$, which describes the distribution of the summed scalar transverse momentum p_t in a jet of type a ($a = q, g$ for a quark or gluon jet), produced with vector transverse

momentum $\mathbf{k}_t$ at scale Q . Following the general procedure described in Ref. [4], we find that at small B each hemisphere contains a single quark jet and so

$$R(B, \alpha_S(Q^2)) \stackrel{B \ll 1}{=} \int_0^{2BQ} dp_{t1} \int_0^{2BQ-p_{t1}} dp_{t2} T_q(Q, \mathbf{0}; p_{t1}) T_q(Q, \mathbf{0}; p_{t2}) , \quad (6)$$

$$R_W(B_W, \alpha_S(Q^2)) \stackrel{B_W \ll 1}{=} \left[\int_0^{2B_W Q} dp_t T_q(Q, \mathbf{0}; p_t) \right]^2 . \quad (7)$$

The jet p_t distribution functions T_a may be calculated to next-to-leading logarithmic accuracy using the coherent branching formalism [13,14], from which one obtains the following integral equation

$$\begin{aligned} T_q(Q, \mathbf{k}_t; p_t) = & \delta(p_t - |\mathbf{k}_t|) + \int_0^Q \frac{d^2 \tilde{\mathbf{q}}}{\pi \tilde{\mathbf{q}}^2} \int_0^1 dz P_{qq}[\alpha_S(z^2(1-z)^2 \tilde{\mathbf{q}}^2), z] \\ & \cdot \left[\int_0^\infty dp_{t1} T_q(z|\tilde{\mathbf{q}}|, z\mathbf{k}_t + z(1-z)\tilde{\mathbf{q}}; p_{t1}) T_g((1-z)|\tilde{\mathbf{q}}|, (1-z)\mathbf{k}_t - z(1-z)\tilde{\mathbf{q}}; p_t - p_{t1}) \right. \\ & \left. \cdot \Theta(p_{t1} - |z\mathbf{k}_t + z(1-z)\tilde{\mathbf{q}}|) \Theta(p_t - p_{t1} - |(1-z)\mathbf{k}_t - z(1-z)\tilde{\mathbf{q}}|) - T_q(|\tilde{\mathbf{q}}|, \mathbf{k}_t; p_t) \right] . \end{aligned} \quad (8)$$

Here $P_{qq}[\alpha_S, z]$ represents the splitting function which gives the distribution of the longitudinal momentum fraction z of the final quark in the parton branching $q \rightarrow qg$. Its explicit form to next-to-leading accuracy in the $\overline{\text{MS}}$ scheme is

$$P_{qq}[\alpha_S, z] = \frac{\alpha_S}{2\pi} C_F \frac{1+z^2}{1-z} + \left(\frac{\alpha_S}{\pi} \right)^2 \frac{1}{2} C_F K \frac{1}{1-z} , \quad (9)$$

$$K = \frac{67}{6} - \frac{\pi^2}{2} - \frac{5}{9} N_f , \quad (10)$$

where N_f is the number of flavours. The gluon jet distribution function T_g in Eq. (8) satisfies a similar integral equation in terms of the gluon splitting functions P_{gg} and P_{gq} .

Introducing the Laplace transform

$$\tilde{T}_a(Q, \mathbf{k}_t; \nu) = \int_{|\mathbf{k}_t|}^\infty dp_t e^{\nu(|\mathbf{k}_t| - p_t)} T_a(Q, \mathbf{k}_t; p_t) , \quad (11)$$

we find that

$$\begin{aligned} \tilde{T}_q(Q, \mathbf{0}; \nu) = & 1 + \int_0^Q \frac{d^2 \tilde{\mathbf{q}}}{\pi \tilde{\mathbf{q}}^2} \int_0^1 dz P_{qq}[\alpha_S(z^2(1-z)^2 \tilde{\mathbf{q}}^2), z] \cdot \\ & \left[e^{-2z(1-z)\nu|\tilde{\mathbf{q}}|} \tilde{T}_q(z|\tilde{\mathbf{q}}|, z(1-z)\tilde{\mathbf{q}}; \nu) \tilde{T}_g((1-z)|\tilde{\mathbf{q}}|, -z(1-z)\tilde{\mathbf{q}}; \nu) - \tilde{T}_q(|\tilde{\mathbf{q}}|, \mathbf{0}; \nu) \right] . \end{aligned} \quad (12)$$

We are interested in the small- p_t region which corresponds to the large ν limit, where the exponential factor selects the region of small $z(1-z)\tilde{\mathbf{q}}$. Since the functions $\tilde{T}_q$ and $\tilde{T}_g$ in the integrand are regular in this region, and the splitting function P_{qq} is singular only at $z = 1$, to next-to-leading accuracy we can make the replacements

$$\tilde{T}_g((1-z)|\tilde{\mathbf{q}}|, -z(1-z)\tilde{\mathbf{q}}; \nu) \rightarrow \tilde{T}_g(0, \mathbf{0}; \nu) = 1 , \quad (13)$$

$$\tilde{T}_q(z|\tilde{\mathbf{q}}|, z(1-z)\tilde{\mathbf{q}}; \nu) \rightarrow \tilde{T}_q(|\tilde{\mathbf{q}}|, \mathbf{0}; \nu) . \quad (14)$$

The solution is then straightforward, viz.

$$\tilde{T}_q(Q, \mathbf{0}; \nu) = \exp \left\{ \int_0^{Q^2} \frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \int_0^1 dz P_{qq}[\alpha_S(z^2(1-z)^2\tilde{\mathbf{q}}^2), z] [e^{-2z(1-z)\nu|\tilde{\mathbf{q}}|} - 1] \right\}. \quad (15)$$

The exponentiated result in Eq. (15) follows from the next-to-leading approximations (13) and (14). In order to clarify their physical meaning, let us first notice that the p_t -distribution $\tilde{T}_g$ in Eq. (12) describes the soft gluon cascades produced in the jet evolution. Due to the coherence properties of soft radiation in QCD, these cascades develop in an angular region $\theta < (1-z)|\tilde{\mathbf{q}}|/Q$, smaller than the jet angular size $1/2\nu$. Therefore multiple (non-abelian) gluon correlations are completely integrated over the relevant two-jet region and cancel by unitarity as in Eq. (13). On the other side, the approximation (14) amounts to neglecting the small angular recoil $z(1-z)\tilde{\mathbf{q}}/Q$ of the quark which thus radiates soft gluons independently. The summed scalar transverse momentum p_t is therefore approximately conserved in the jet evolution, leading to the exponentiated result (15) in the space of the Laplace index ν .

To predict the jet broadening at small B we write Eq. (6) in the form

$$R(B, \alpha_S(Q^2)) \underset{B \ll 1}{=} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\nu}{\nu} \frac{e^{2\nu BQ}}{2\pi i} [\tilde{T}_q(Q, \mathbf{0}; \nu)]^2 \quad (16)$$

where the integration contour is to the right of all singularities of $\tilde{T}_q$. This gives the exponentiated factor Σ in Eq. (4). Similarly for the wide jet broadening

$$R_W(B_W, \alpha_S(Q^2)) \underset{B_W \ll 1}{=} \left[\int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\nu}{\nu} \frac{e^{2\nu B_W Q}}{2\pi i} \tilde{T}_q(Q, \mathbf{0}; \nu) \right]^2. \quad (17)$$

Both the integrals in Eq. (15) and Eqs. (16,17) can be performed analytically to next-to-leading logarithmic accuracy as explained in detail in Ref. [4]. Including the renormalization scale dependence in the way outlined earlier, we find

$$\Sigma(B, \alpha_S(\mu^2), Q^2/\mu^2) = \frac{\exp[\mathcal{F}(\ln B, \alpha_S(\mu^2), Q^2/\mu^2)]}{\Gamma[1 - 2\mathcal{S}(\ln B, \alpha_S(\mu^2))]}, \quad (18)$$

$$\Sigma_W(B_W, \alpha_S(\mu^2), Q^2/\mu^2) = \frac{\exp[\mathcal{F}(\ln B_W, \alpha_S(\mu^2), Q^2/\mu^2)]}{(\Gamma[1 - \mathcal{S}(\ln B_W, \alpha_S(\mu^2))])^2} \quad (19)$$

where Γ represents the Euler gamma function and (defining $\omega = 2\beta_0\alpha_S \ln B$ for brevity)

$$\begin{aligned} \mathcal{F}(\ln B, \alpha_S, Q^2/\mu^2) &= \frac{A^{(1)}}{\pi\beta_0^2\alpha_S} [\ln(1+\omega) - \omega] \\ &+ \frac{1}{\pi^2\beta_0^2} [A^{(1)}\pi\beta_0 \ln(Q^2/\mu^2) - A^{(2)}] \cdot \left[\ln(1+\omega) - \frac{\omega}{1+\omega} \right] \\ &+ \frac{B^{(1)}}{\pi\beta_0} \ln(1+\omega) + \frac{2A^{(1)}\gamma_E}{\pi\beta_0} \left[\frac{\omega}{1+\omega} \right] \\ &+ \frac{A^{(1)}\beta_1}{\pi\beta_0^3} \left[\frac{1}{2} \ln^2(1+\omega) + \frac{\ln(1+\omega)}{1+\omega} - \frac{\omega}{1+\omega} \right] \\ &+ \mathcal{O}(\alpha_S^{m+1} \ln^m B), \end{aligned} \quad (20)$$

	$R(B, \alpha_S)$		$R_W(B_W, \alpha_S)$	
	Fitted	Theory	Fitted	Theory
K_2	-1.4 ± 0.6	-1.50	-0.4 ± 0.3	-0.31
K_3	1.6 ± 0.6	1.77	1.6 ± 0.4	1.77
K_4	0.4 ± 0.5	0.36	0.4 ± 0.3	0.36

Table 1: Fitted vs calculated $\mathcal{O}(\alpha_S^2)$ log coefficients for $N_f = 5$.

$$\mathcal{S}(\ln B, \alpha_S) = \frac{A^{(1)}}{\pi\beta_0} \left[\frac{\omega}{1+\omega} \right], \quad (21)$$

$$\begin{aligned} A^{(1)} &= C_F, & B^{(1)} &= -\frac{3}{2}C_F, \\ A^{(2)} &= \frac{C_F}{2} \left(\frac{67}{6} - \frac{\pi^2}{2} - \frac{5}{9}N_f \right), \\ \beta_0 &= \frac{33 - 2N_f}{12\pi}, & \beta_1 &= \frac{153 - 19N_f}{24\pi^2}. \end{aligned} \quad (22)$$

Here $\gamma_E = 0.5772\dots$ is the Euler constant and N_f , the number of flavours, is 5 at LEP energies.

Having evaluated Σ to next-to-leading accuracy, we can obtain the non-logarithmic coefficient C_1 in Eq. (5) by comparison with the one-loop analytical result for $R(B, \alpha_S)$. We find

$$C_1 = \frac{C_F}{2\pi} \left(\pi^2 - \frac{17}{2} \right). \quad (23)$$

The solid curves in Figs. 1 and 2 show the resummed expressions (4), using Eqs. (18,19) and including C_1 , but setting other sub-dominant terms (i.e. those with fewer powers of $\ln B$ than of α_S) and the remainder functions F to zero. For definiteness we have assumed the (five-flavour) value $\Lambda_{\overline{\text{MS}}} = 200$ MeV, corresponding to $\alpha_S(Q^2) = 0.116$, with scale $\mu^2 = Q^2$.

Expanding the resummed results (18-20) to second order in α_S gives

$$R(B, \alpha_S) = 1 - \alpha_S \frac{C_F}{\pi} (2 \ln^2 B + 3 \ln B) + \alpha_S^2 \sum_{m=2}^4 K_m \ln^m B + \dots, \quad (24)$$

where the dots represent sub-dominant terms, and the leading and next-to-leading second-order coefficients K_m are as listed in Table 1. To this order, the full fixed-order prediction for all values of B can be calculated numerically using the program EVENT [15], based on the matrix elements of Ref. [16]. The corresponding fixed-order predictions are shown by the dashed curves in Figs. 1 and 2. Extrapolating to very small B and fitting the logarithmic terms, we obtain the fitted coefficients given in Table 1, which are seen to be consistent with those obtained by resummation. Next, by fixing these coefficients and

fitting the single-logarithmic term we can estimate the first sub-dominant coefficient G_{21} in Eq. (5):

$$G_{21} = -4.3 \pm 0.4 \quad \text{for } R(B, \alpha_S), \quad -4.7 \pm 0.2 \quad \text{for } R_W(B_W, \alpha_S). \quad (25)$$

Finally from the non-logarithmic term we can estimate C_2 :

$$C_2 = -2.1 \pm 0.4 \quad \text{for } R(B, \alpha_S), \quad -3.5 \pm 0.2 \quad \text{for } R_W(B_W, \alpha_S). \quad (26)$$

To obtain a prediction for the full second-order range of B (i.e. up to the four-parton phase space boundary) a number of procedures can be adopted [1-5] for matching the resummed and fixed-order expressions. The differences between the results obtained with various procedures may provide an estimate of the theoretical uncertainty due to missing higher order contributions. For illustration, we show in Fig. 3 the total jet broadening distribution using the simplest type of matching procedure, in which one writes

$$\frac{d\sigma}{dB} = \left[1 - f\left(\frac{B}{B_0}\right) \right] (\text{Resummed}) + f\left(\frac{B}{B_0}\right) (\text{Second-order}) \quad (27)$$

where $f(x)$ increases rapidly from zero to one as x passes through the point $x = 1$; for example

$$f(x) = [1 + x^{-5}]^{-1}. \quad (28)$$

The matching point B_0 should be chosen such that $\alpha_S \ln^2 B_0 \lesssim 1$. In Fig. 3, the value $B_0 = B_L = 0.09$ has been used, where B_L is the value of B at which the resummed and fixed order results coincide, corresponding to $\alpha_S \ln^2 B_0 = 0.67$. When comparing with data, an estimate of the matching ambiguity for this procedure could be obtained by varying B_0 within a reasonable range. More sophisticated matching procedures are discussed in Ref. [4].

An important aspect of resummation is its effect on the renormalization scale dependence of the predictions. For other event shape variables, it has been found that resummation reduces the scale dependence and eliminates the preference for small scales which is found in fixed order [1,5]. As illustrated in Fig. 4, this is also the case for the jet broadening distribution. The fixed-order and resummed expressions are shown at discrete values of B for a wide range of μ^2/Q^2 . For estimating the theoretical uncertainty associated with the scale dependence, a smaller range, say $\frac{1}{2} < \mu^2/Q^2 < 2$, should be sufficient. Note that the variation with μ^2/Q^2 is quite small (of the order of 2%) over this range. Having correctly treated the small transverse momentum region, we do not see any valid argument for using scales much different from Q^2 .

Another important contribution to the theoretical uncertainty in event shape predictions is the effect of hadronization. Here one can only use models to estimate the difference between the predictions at the partonic and hadronic levels. We have used the simulation program HERWIG [17] to construct a hadronization correction matrix for jet broadening, in the same way as was done earlier for the thrust distribution [1]. As shown by the dashed curve in Fig. 3, the resulting corrections are comparable to those that were found for the thrust distribution.

In conclusion, we find that jet broadening should be a good event shape measure for α_S determination in e^+e^- annihilation. Our predictions also provide a novel ‘blind test’ of

QCD: as far as we are aware, no measurement of the distribution of this quantity has ever been presented, at any energy. We await the data with interest and trepidation.

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Figure 1: Total jet broadening distribution in e^+e^- annihilation at 91 GeV. Solid: resummed expression (see text). Dashed: second-order prediction without resummation.

Figure 2: Wide jet broadening distribution in e^+e^- annihilation at 91 GeV. Solid: resummed expression (see text). Dashed: second-order prediction without resummation.

Figure 3: Predicted distribution of total jet broadening in e^+e^- annihilation at 91 GeV. Solid: perturbative prediction including resummation to all orders. Dashed: resummed prediction with HERWIG hadronization corrections.

Figure 4: Dependence on renormalization scale μ^2 of total jet broadening at various values of B . Solid: resummed expression (see text). Dashed: second-order prediction without resummation.