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Archives

Temperature-Dependent Structure in the Mesonic Channels of QCD⁺

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ABSTRACT

We study the mesonic spectrum of finite temperature QCD in the quenched approximation. To obtain more information on the spectral functions at non-zero temperature we use anisotropic lattices and measure meson propagators also in the "temperature" (Euclidean time) direction. We extract in this way "pole-masses", which have been inaccessible to conventional analyses and which, when compared to the "screening-masses" show a different behaviour from below until above T_c . This behaviour indicates plasma effects starting to build up already below T_c and having non-trivial consequences at intermediate distances.

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Introduction

Numerical simulations of lattice QCD have been expected to provide us with rich information about hadronic properties in the non-perturbative regions, although almost all Monte Carlo calculations so far have mainly tried to reproduce established experimental data. Now, QCD predicts also a transition from the low temperature confinement phase to the high temperature deconfinement phase. Experimental effort to find the so-called quark-gluon plasma phase has been started, and many phenomenological studies have been performed [1]. Large scale lattice Monte Carlo calculations, to decide the order of this finite temperature transition, have been or will be made with or without dynamical fermions [2], [3]. Lattice QCD calculations may, however, provide not only the determination of the phase transition character, but also more explicit information about hadrons at finite temperature. The heavy quark potential, for instance, has already been studied [4] and there are a number of results on screening lengths and spatial correlators in various channels [5]. This represents, however, only indirect information which does not always have a simple model-independent interpretation. One can ask therefore whether one can obtain more direct information about, say, the meson properties (partially, this question is addressed in [6]).

One of the quantities of interest is the momentum space structure of the hadron propagators, which gives information about the hadron masses. At zero temperature, on the Euclidean lattice we have complete symmetry between the different directions and the Feynman two-point functions of the Minkowski space theory can be obtained by a "rigid" Wick rotation from the corresponding Schwinger functions of the Euclidean theory. Not so at finite temperature, where the Lorentz invariance is broken. While the theory, of course, can still be formulated relativistically, the mixture implied by the thermal statistical ensemble leads to the alteration of the dispersion laws, the spectral densities becoming functions of energy and spatial momentum and not only of their invariant combination. In the "imaginary time formalism", on the Euclidean lattice this manifests itself by loss of the hypercubic symmetry and the specialization of the "Euclidean time" axis as "temperature" axis. The complex t -plane decays in horizontal analyticity strips and the relation between Minkowski and Euclidean n -point functions implies an analytic continuation, which cannot be achieved by a simple Wick rotation. The

zero temperature limit projects out again the vacuum expectation values, collapsing the statistical ensemble and re-establishing the hypercubic symmetry on the Euclidean lattice together with the simple Wick rotation interpretation. For a general discussion of these and related matters see, e.g., [7], [8]. We shall give an account of some points of interest for our discussion in Chapter I. Specifically, in Section 1 we shall recall the introduction of the temperature Green functions. Due to the anisotropy coming from the breaking of the Lorentz invariance, non-trivial self-energy effects (which imply more than a constant shift of the mass) become temperature and direction dependent (see Section 2). This leads to the necessity of measuring propagators in the "space" as well as "Euclidean time" directions. To obtain enough information about the spectral function, we must therefore have enough lattice points in each direction. This prohibits the realization of finite temperatures on isotropic lattices through reduction of the number of points in the "Euclidean time" direction and makes anisotropic lattices practically unavoidable. The redefinition of the problems on anisotropic lattices is presented in Sections 3 and 4.

In Chapter II of this paper we present and discuss an analysis of the quenched QCD at finite temperature realized with the help of anisotropic lattices. After a short description of the problem (Section 5) and an illustration of the temperature effects for the quark propagator in perturbation theory (Section 6), we present and discuss results for the meson propagators obtained in a quenched Monte Carlo simulation at various temperatures below and above the deconfining transitions (Section 7, 8). Part of these results have been presented in short form at previous conferences [9], [10], [11]. We also expect soon to be able to present further results concerning these problems, for which the discussion of this paper should then serve as a general introduction.

1. Temperature Green functions on anisotropic Euclidean lattices

1. Real time Green function at finite temperature and temperature Green functions

The basis of this and related analyses for finite temperature QFT implies three conceptual steps. i) We start from the QFT in Minkowski space (real time) at finite temperature and consider the Fourier transforms of its Green functions which have simple analytic properties when continued in the whole complex energy plane. ii) Since we envisage doing numerical analyses and hence use a Euclidean lattice, we employ the "imaginary time" formalism and write down "temperature Green functions" for which the Fourier transforms are defined only on a discrete set of points (the "Matsubara frequencies") on the imaginary axis of the complex energy plane. The relation between these two steps is given by the observation that the finite temperature, real time Green functions and the "temperature Green Functions" are related by identifying their Fourier transforms on their common domain, that is, at the Matsubara frequencies. iii) Finally, by discretizing the Euclidean QFT of step two we can obtain its "temperature Green functions" from a partition function defined on a lattice with periodic boundary conditions on its fourth axis, which is taken of fixed physical extension $1/T$, and which therefore may be called the "temperature axis". In the following we shall shortly review some aspects of this procedure. For general discussions and further aspects see, e.g. [7], [8].

We shall consider here the simple case of a scalar bosonic field ϕ at the finite temperature $T = 1/\beta$. (Notice that β has dimension of length.) The most fundamental quantities of the field ϕ for real time are

$$\begin{aligned} \langle \langle \frac{1}{i} [\phi(t, \mathbf{x}), \phi(t', \mathbf{x}')] \rangle \rangle &\equiv \frac{1}{Z} \text{Tr} \left(\frac{1}{i} [\phi(t, \mathbf{x}), \phi(t', \mathbf{x}')] e^{-\beta H} \right) \\ &= F_{\mathbf{x}-\mathbf{x}', \mathbf{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \Lambda_{\beta}(\omega, \mathbf{p}) \end{aligned} \quad (1.1)$$

and the retarded and advanced Green functions which have simple analytic properties in the frequency (energy) plane:

$$G_{\beta, M}^{ret/adv}(t, \mathbf{x}, t', \mathbf{x}') = \pm \theta(t-t') \langle \langle \frac{1}{i} [\phi(t, \mathbf{x}), \phi(t', \mathbf{x}')] \rangle \rangle$$

$$= F_{\mathbf{x}-\mathbf{x}', \mathbf{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} K_{\beta}^{ret/adv}(\omega, \mathbf{p}) \quad (1.2)$$

(the index M stands for "Minkowski", where the same definition for the expectation values at finite temperature as in Eq.(1.1) has been used. Here

$$\phi(t, \mathbf{x}) = e^{iH} \phi(0, \mathbf{x}) e^{-iH}$$

$$Z = \text{Tr}(e^{-\beta H}) \quad (1.3)$$

with H the Hamiltonian. F symbolizes the Fourier transform with, hopefully, transparent indexing. We have

$$K_{\beta}^{ret/adv}(\omega, \mathbf{p}) = i \lim_{\epsilon \rightarrow 0+} \int_{-\infty}^{\infty} \frac{d\varpi}{2\pi} \frac{\Lambda_{\beta}(\varpi, \mathbf{p})}{\omega - \varpi \pm i\epsilon} \quad (1.4)$$

correspondingly:

$$\Lambda_{\beta}(\omega, \mathbf{p}) = K_{\beta}^{ret}(\omega, \mathbf{p}) - K_{\beta}^{adv}(\omega, \mathbf{p}) \quad (1.5)$$

In the following we shall drop the $\mathbf{p}$ -dependence occasionally. Continuing to complex variable z we see from Eqs. (1.4) and (1.5) that we can define

$$K_{\beta}(z) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{\Lambda_{\beta}(\omega)}{\omega - z} \quad (1.6)$$

and

$$K_{\beta}^{ret/adv}(\omega) = \lim_{\epsilon \rightarrow 0+} K_{\beta}(\omega \pm i\epsilon)$$

$$\Lambda_{\beta}(\omega) = \lim_{\epsilon \rightarrow 0+} (K_{\beta}(\omega - i\epsilon) - K_{\beta}(\omega + i\epsilon)) \quad (1.7)$$

Hence, if we can obtain information on the function $K_{\beta}(z, \mathbf{p})$ we can reconstruct all the interesting quantities; in particular, its discontinuity Λ_{β} gives information on the spectrum of the theory.

Monte-Carlo simulations are performed on Euclidean lattices. It is therefore natural to use imaginary time formalism and introduce "temperature Green functions":

$$G_{\beta}(\tau, \mathbf{x}; \tau', \mathbf{x}') = \langle \langle T_{\tau} \phi(\tau, \mathbf{x}) \phi(\tau', \mathbf{x}') \rangle \rangle \quad (1.8)$$

with the same definition for the double brackets as in Eq.(1.1) - but notice the Euclidean time-ordering (see, e.g., [8]). We have:

$$\phi(\tau, \mathbf{x}) = e^{\tau H} \phi(0, \mathbf{x}) e^{-\tau H} \quad (1.9)$$

and the periodicity relation (KMS):

$$G_\beta(\tau, \mathbf{x}; 0, 0) = G_\beta(\tau + \beta, \mathbf{x}; 0, 0) \quad (1.10)$$

Fourier transforming again we get

$$K_\beta(\zeta_n, \mathbf{p}) = F_{\mathbf{x}-\mathbf{x}'; \mathbf{p}}^{-1} \int_0^\beta d\tau e^{-i\zeta_n(\tau-\tau')} G_\beta(\tau, \mathbf{x}, \tau', \mathbf{x}') \equiv K_{\beta, n}(\mathbf{p}) \quad (1.11)$$

with the discrete Matsubara spectrum:

$$\zeta_n = \frac{2\pi}{\beta} n, \quad n = 0, \pm 1, \pm 2, \dots \quad (1.12)$$

Then we have (again dropping the $\mathbf{p}$ -dependence when not relevant)

$$K_\beta(\zeta_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda_\beta(\omega)}{\omega - i\zeta_n} = iK_\beta(i\zeta_n) = K_{\beta, n} \quad (1.13)$$

Due to the analyticity properties of K_β and assuming "good" behaviour at infinity, we can then reconstruct $K_\beta(z)$ by continuation from a unique interpolation $K_\beta(\zeta)$ through the values $K_{\beta, n}$ at the Matsubara frequencies ζ_n (see Fig. 1). This result is the so-called Abrikosov-Gorkov-Dzyalozinski-Fradkin Theorem, and for a detailed discussion we refer to the original papers [12] and to reviews; see e.g. [7]. In the following we shall assume that the conditions ensuring uniqueness of these procedures are fulfilled.

We notice that one does not usually measure G_β , but the related functions G_β^μ obtained by partial Fourier transformation:

$$G_\beta^\mu(x_\mu, \{p_\nu, \nu \neq \mu\}) = \Pi_{\nu \neq \mu} F_{x_\nu, p_\nu}^{-1} G_\beta(\tau, \mathbf{x}; 0, 0) = F_{x_\mu, p_\mu} K_\beta(\zeta, \mathbf{p}) \quad (1.14)$$

where $p_4 = \zeta$ and for $\mu = 4$ the Fourier transform is taken discrete (with $\zeta = \zeta_n$). (See Appendix A. Notice that there is no analytic continuation involved at this stage.) Since we still have rotational symmetry, we shall only differentiate between the "Euclidean time" ("temperature") propagators $\mu = 4$ (G_β^T) and the "space" ones, $\mu \neq 4$ (G_β^s). Of course, for $\beta \rightarrow \infty$ they coincide, due to the restored Euclidean symmetry.

Going now from the Hamiltonian formulation to the Euclidean path integral, the temperature Green functions Eq.(1.8) are simply obtained as correlations taken

with an ensemble defined with periodic boundary conditions in τ (antiperiodic for fermionic variables), to ensure the corresponding KMS condition (see Eq.(1.10)):

$$G_\beta(\tau, \mathbf{x}) = \langle \phi(\tau, \mathbf{x}) \phi(0, 0) \rangle_\beta \equiv \frac{1}{Z} \int d[\phi] \phi(\tau, \mathbf{x}) \phi(0, 0) e^{-\int_0^\beta d\tau \int d\mathbf{x} \mathcal{L}_E(\phi)} \quad (1.15)$$

with Z the partition function, $\mathcal{L}_E$ the Euclidean Lagrangian density corresponding to H , and classical variables ϕ . Eq.(1.15) is well defined on the lattice.

A point to be stressed in this discussion is that the (periodic) "Euclidean time" axis is in fact a temperature axis and does not designate the measurable time intervals. As alluded to in the introduction, real time developments are not directly represented by the "Euclidean-time" dependence of the temperature Green function. One way to obtain the wanted information is to first extract the spectral function from the temperature Green function and then continue it analytically. Fig. 1 does not imply that we cannot have information about energies less than T , but that we have to obtain this information by interpolation from the directly available data at (discrete) Matsubara frequencies. Under the condition of uniqueness for the interpolation and continuation procedures involved, the discreteness of the original spectrum (the compactness of the temperature axis) does not limit, theoretically, the analysis. In practice, however, in lattice simulations where:

- 1) - the data for K_β are subjected to errors, and
- 2) - the number of points itself is limited (finite cut-off)

these procedures may quickly go out of control ("ill-posed problems") and we need some external bias in order to stabilize them (including a renormalization analysis). One possibility, and which we chose for our analyses [9], [10], [11], [13] is to use reasonable fit-ansätze for the spectral functions. But of course, we must try to improve on both points 1) and 2) by realizing better statistics and, first of all, by ensuring a large number of lattice points in the "Euclidean time"-direction.

Since the temperature is the inverse length of the lattice in this direction, it is clear that to have high temperature and many points we need a very fine discretization of the Euclidean time axis - while we still can accept a coarser space lattice, if the corresponding cut-off is already high enough to permit scaling. This situation can be well realized with anisotropic lattices, where, say, an $8^3 \times 16$ lattice with anisotropy factor $\xi \equiv a_\sigma/a_\tau = 4$ has the same physical extension as an isotropic $8^3 \times 4$ lattice (of course we could realize the same T also by going

nearer to the critical point - but then we would need a $32^3 \times 16$ lattice in order to have the same physical lengths in space direction as before - people involved in numerical simulations will appreciate the difference).

In the rest of this chapter we shall consider some details of this program.

2. More on the relation between the temperature Green functions and the spectral function at finite temperature

We shall first make the simplest assumption, namely that the spectral function at finite temperature can still be well described by a particle pole with usual dispersion law

$$\omega_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2} \quad (2.1)$$

to wit

$$\Lambda_{\beta}(\omega, \mathbf{p}) = \frac{2\pi}{2\omega} (\delta(\omega - \omega_{\mathbf{p}}) + \delta(\omega + \omega_{\mathbf{p}})) \quad (2.2)$$

where the mass m may depend on β . It then follows

$$K_{\beta}(\zeta_n, \mathbf{p}) = iK_{\beta}(i\zeta_n, \mathbf{p}) = \left(\left(\frac{2\pi n}{\beta} \right)^2 + \omega_{\mathbf{p}}^2 \right)^{-1} \quad (2.3)$$

Then, using Eq.(1.14) we have for $\mu = 4$, $\tau > 0$ (see, e.g. [7], [9])

$$G_{\beta}^{\tau}(\tau, 0) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} e^{-i\zeta_n \tau} iK_{\beta}(\zeta_n, 0) = \frac{\cosh(\omega_{\mathbf{p}}(\beta/2 - \tau))}{2\omega_{\mathbf{p}} \sinh(\omega_{\mathbf{p}}\beta/2)} \Big|_{\mathbf{p}=0, \omega_{\mathbf{p}}=m} \quad (2.4)$$

Notice that due to the Euclidean symmetry of the dispersion rule Eq.(2.1) we also have ($x > 0$):

$$G_{\beta}^{\tau}(x, \{p_2 = p_3 = \omega = 0\}) \approx \exp(-mx) \quad (2.5)$$

The mass defined by the correlation length appearing in the "space" propagator G_{β}^{τ} is usually called the "screening mass" (see, e.g. [5], [14]). We call the mass obtained from the correlation length of the "Euclidean time" propagator G_{β}^{τ} , the "pole mass", since this directly gives the position of the energy pole of the propagator at small momenta ($\mathbf{p} \sim 0$). In the above example they do, of course, coincide. In general, however, we expect a more complicated situation to occur, where cuts may overlap with poles in a temperature dependent way. To the extent that we can still recognize particle-like excitations, we expect them to show a dispersion

law reflecting the breaking of the Lorentz invariance at finite temperature, that is, a more complicated surface than the mass shell hyperboloid. A section obtained by fixing some of the momentum components will then not be enough to define this structure. Let us write, quite generally,

$$\omega_{\mathbf{p}} = f_m(\mathbf{p}), \quad f_m(0) = m \quad (2.6)$$

then G_{β}^{τ} is still given by Eq.(2.4), hence the pole mass is m . The expression for G_{β}^{τ} , however, depends on f_m and the screening mass therefore is not expected to be equal to m any longer.

Assuming $\omega_{\mathbf{p}}^2$ analytic in $\mathbf{p}^2$, the dispersion law may be reconstructed by measuring the "Euclidean time-propagator" at various momenta $\mathbf{p}$:

$$f_m(\mathbf{p}) \sim \frac{1}{\tau_1 - \tau_2} \ln \frac{G_{\beta}^{\tau}(\tau_2, \mathbf{p})}{G_{\beta}^{\tau}(\tau_1, \mathbf{p})} \quad (2.7)$$

In particular, let us assume that, at least at small T , it is reasonable to write [14]:

$$\omega_{\mathbf{p}}^2 = \mathbf{p}^2 + m^2 + \Delta(\mathbf{p}^2, \beta) \quad (2.8)$$

where for small momenta

$$\Delta(\mathbf{p}^2, \beta) = a(\beta) \mathbf{p}^2 + O(\mathbf{p}^4) \quad (2.9)$$

Then $G_{\beta}^{\tau}(\tau, 0)$ still behaves like Eq.(2.4)

$$G_{\beta}^{\tau}(\tau, 0) \approx \exp(-m\tau), \quad \tau < \beta/2 \quad (2.10)$$

while

$$G_{\beta}^{\tau}(x, \{p_2 = p_3 = \omega = 0\}) \approx \exp\left(-\frac{m}{\sqrt{1+a}}x\right) \quad (2.11)$$

Hence the pole mass is still given by m while the screening mass is

$$m_{\sigma} = m/\sqrt{1+a(\beta)}$$

(Notice that m may also depend on β .) This is understandable, since the two "masses" (correlation lengths, respectively) control different phenomena [11], [15].

3. The method of anisotropic lattices for finite temperature calculations

Anisotropic lattices have been already applied for some time [16], [17], [18], [19], [20]. We shall discuss here some aspects which are of direct interest for our calculations.

As a result of the (Euclidean) space-time discretization, the momentum spectrum of a lattice theory is bounded by π/a_μ in each direction, where a_μ is the lattice cut-off in the direction $\mu = 1, 2, 3, 4$. The continuum theory should obtain in the limit $a_\mu \rightarrow 0$. Since a_μ is not a parameter of the (dimensionless!) statistical mechanics model realized by the lattice theory, this limit is indicated by diverging dimensionless correlation lengths (measured in number of lattice points) - which can happen only at some "critical" values of the parameters of the lattice theory (bare couplings, etc.). Let N_μ denote the number of lattice points in the μ direction. The physical length achieved by the lattice, $N_\mu a_\mu$ will collapse at the critical point, unless N_μ (which is a parameter of the lattice model) is also taken to infinity. The kind of physics we describe in continuum depends on the interplay of these limits. We shall use the index σ for $\mu = 1, 2, 3$ and τ to indicate the direction $\mu = 4$.

In the general case, we start with a lattice model defined in terms of some couplings κ and coupling anisotropies γ , representing the direction dependence of the "gradient" terms in the action (links, plaquettes), e.g. we write for a nearest neighbour coupling $\kappa_\sigma(\phi_n \phi_{n+\sigma} + \gamma \phi_n \phi_{n+\tau})$ (with the standard notation for the neighbours in directions σ, τ). Then we do a scaling analysis in terms of a cut-off a and a cut-off anisotropy ξ ,

$$a \equiv a_\sigma = R(\kappa)/\Lambda(\kappa, \gamma), \quad \xi = a_\sigma/a_\tau = X(\kappa, \gamma) \quad (3.1)$$

and a number of physical quantities (e.g., masses). Here $R(\kappa)$ represents the renormalization group scaling, Λ has the dimension of a mass and κ, γ denote globally the various couplings and their anisotropies. The calibration then means tuning γ (and κ) to reproduce a desired a, τ at zero temperature after fixing the physical quantities, in accordance with the demand of physical isotropy. This implies determining the functions $X(\kappa, \gamma)$ and $\Lambda(\kappa, \gamma)$, which depend on the dynamics.

Consider that we measure a set of observables O_i . Then, assuming scaling, the isotropy condition at zero temperature provides a set of relations, say, of the form

$$\bar{O}_{m\sigma}(n; \gamma_1, \gamma_2, \dots) = \bar{O}_{m\tau}(\xi \gamma_1, \gamma_2, \dots); \quad m = 1, 2, \dots \quad (3.2)$$

where the averages $\bar{O}_{m\sigma}, \bar{O}_{m\tau}$ are some correlation function over n sites in space (time) direction, respectively. The solution of the calibration problem can then be written (formally) as

$$\gamma_i = \xi \gamma_i(\xi, a), \quad i = 1, 2, \dots, \gamma_i(1, a) = 1 \quad (3.3)$$

The function $\gamma_i(\xi)$ of Eq.(3.3) can be calculated perturbatively to lower orders [16], [18]. However, non-perturbative calculations [19] have shown that the non-perturbative effects may be very large, which makes the perturbative estimates of only limited use. Non-perturbative analyses have been performed for ϕ^4 [20], [10], for pure Yang-Mills theories [19], for the SU_2 Higgs model [11] and for full QCD [13]. For the former two cases the procedure is straightforward: one measures averages $\bar{O}$ at given γ and looks at what ξ fulfils the isotropy condition Eq.(3.3).

The calibration of models with more couplings is less simple, since the scaling Eqs.(3.3) establish relations between the various coupling anisotropies, γ_i , which therefore cannot be given independently. Assume, however, that we work at values of the anisotropies, γ'_i , near the "physical" ones, γ_i , (which fulfils Eqs.(3.2), (3.3) with a given ξ); then we can write for the observables entering Eqs.(3.2) [10]:

$$\bar{O}_m(\gamma_i) = \frac{\int O_m \exp(-S(\gamma_i))}{\int \exp(-S(\gamma_i))} = \frac{\int O_m \exp(-S(\gamma'_i) + \Delta S + C)}{\int \exp(-S(\gamma'_i) + \Delta S + C)} \quad (3.4)$$

with

$$\Delta S = S' - S = S(\gamma'_i) - S(\gamma_i) = - \sum_i Q_i \Delta \gamma_i \quad (3.5)$$

Expanding the exponentials we get for the isotropic averages

$$\bar{O}_m = \langle O_m \rangle = \langle O_m \rangle' + \sum_i \Delta \gamma_i (\langle O_m Q_i \rangle' - \langle O_m \rangle' \langle Q_i \rangle') \quad (3.6)$$

A fit to Eqs.(3.2) and (3.6) will then produce both γ_i and ξ without further tuning. C is of course redundant, but choosing it to be $- \langle \Delta S \rangle$ we see that it indicates that not ΔS (which is a global quantity) controls the quality of the approximation, but its fluctuation $\Delta S - \langle \Delta S \rangle$ which mostly only grows as the square root of the volume. This program has been applied to the SU_2 Higgs model and to full QCD with good results. One must notice, however, that in the first case the functions γ_i are near to 1 - the non-perturbative quantum corrections turn out to be, like

the perturbative ones, very small [10]. The success of this program for full QCD, where the quantum corrections η_i may be very large (up to 1.5 for the higher γ we work with), was to a large extent due to the lucky choice of the starting point γ' , allowing us to achieve the calibration within only one simulation [13]. In general we may expect to need some tuning, however.

Of particular interest may be the derivatives at $\xi = 1$,

$$\lambda_i \equiv \frac{\partial \gamma_i}{\partial \xi} \Big|_{\xi=1} = 1 - \frac{\partial \eta_i}{\partial \xi} \Big|_{\xi=1} \quad (3.7)$$

which enter the definition of thermodynamic quantities (e.g., the physical energy, the pressure, etc.). Fortunately, these parameters may be estimated non-perturbatively from simulations on isotropic lattices, hence avoiding the calibration procedure. This has been already discussed in [16]; we reproduce it here for completeness. Starting from the isotropy condition Eq.(3.2) and differentiating with respect to ξ at fixed a , κ_i (we assume for the moment Λ independent on γ_i) we obtain:

$$\sum_i \frac{\partial \gamma_i}{\partial \xi_i} \cdot \frac{\partial}{\partial \gamma_i} (\bar{O}_{m\sigma} - \bar{O}_{m\tau}) = n \frac{\partial}{\partial n_r} \bar{O}_{m\tau} \quad (3.8)$$

with

$$n_r = n\xi \quad (3.9)$$

Setting $\xi = 1$ we obtain a system of linear equations

$$\sum_i \lambda_i A_{im} = B_m, \quad m = 1, 2, \dots \quad (3.10)$$

where

$$\begin{aligned} A_{im} &= \frac{\partial}{\partial \gamma_i} (\bar{O}_{m\sigma}(n, \gamma_i) - \bar{O}_{m\tau}(n, \gamma_i)) \Big|_{\gamma_i=1} \\ &= \left\langle (O_{m\tau}(n) - O_{m\sigma}(n)) \frac{\partial S}{\partial \gamma_i} \right\rangle_{\gamma_i=1} > \gamma_i=1 \end{aligned} \quad (3.11)$$

$$B_m = n \frac{\partial}{\partial n} \bar{O}_{m\tau}(n, 1) = n \frac{\partial}{\partial n} \langle O_{m\tau}(n) \rangle_{\gamma_i=1} \quad (3.12)$$

Obviously both A and B can be obtained from averages on isotropic lattices ($\gamma_i = 1$) and the system of equation (3.10) can be solved or used for a best fit if we have enough averages O_m (notice that the index m may just represent various n -values).

Consider for illustration the pure Yang-Mills theory

$$S(\gamma) = \frac{1}{g^2} \sum_{\square} \left(\frac{1}{\gamma} \square_{\sigma\sigma} + \gamma \square_{\tau\tau} \right) \quad (3.13)$$

We obtain

$$\begin{aligned} \frac{\partial \Delta O}{\partial \gamma} \Big|_{\gamma=1} &= \left\langle \frac{\partial S}{\partial \gamma} \Delta O \right\rangle_{\gamma=1} - \left\langle \frac{\partial S}{\partial \gamma} \right\rangle_{\gamma=1} \langle \Delta O \rangle_{\gamma=1} \\ &= \beta \langle (O_{\sigma} - O_{\tau}) \sum_{\square} (\square_{\sigma\tau} - \square_{\sigma\sigma}) \rangle_{\gamma=1} \end{aligned} \quad (3.14)$$

(with, for instance, O_{σ} , O_{τ} Wilson loops in $\sigma\sigma$ and $\sigma\tau$ planes, respectively; notice that $\langle \Delta O \rangle_{\gamma=1} = 0$; $\square_{\mu\nu}$ is a shorthand notation for the plaquette in the $\mu\nu$ plane.) If we relax the assumption that Λ only depends on κ and also allow for a renormalization of the observables, leading to modified isotropy conditions at fixed a :

$$O_{m\sigma}(n, \gamma_i(\xi), \kappa_i(\xi)) = C_m(\xi) O_{m\tau}(\xi, \gamma_i(\xi), \kappa_i(\xi)) \quad (3.15)$$

to replace Eq.(3.2), the analysis becomes more involved but remains similar in principle. Of course one can use this procedure also in order to move from an isotropy point γ_i, ξ , to a new isotropy point γ'_i, ξ' , employing the averages obtained at the old point.

4. Thermal Green functions on anisotropic lattices

A non-zero physical temperature is achieved by fixing the physical "time" length t_r :

$$T = \frac{1}{t_r} = \frac{1}{N_r a_r} = \frac{\xi}{N_r a} = \frac{\xi}{N_r} \Lambda(\kappa, \gamma) / R(\kappa) \quad (4.1)$$

(see Eq.(3.1)). The usual scaling limit is then understood as

$$N_{\sigma} \rightarrow \infty$$

$$a \rightarrow 0 \quad (4.2)$$

$$N_r \rightarrow \infty, \quad a_r \rightarrow 0, \quad \text{fixed } T$$

such as to achieve an infinite volume but a finite temperature in the thermodynamic limit. With ξ at our disposal we can obtain a high enough temperature even with $N_r \gg 1$, such as not to violate this limit.

Numerical calculations require finite N_μ and non-zero a_μ . They are therefore affected by both finite cut-off and finite size corrections. Since the momentum spectrum is bounded, the complex energy plane decays in periodic horizontal strips, corresponding to the Brillouin zones of p_4 . The infinite discrete spectrum of Fig. 1 is replaced by a finite number of points, N_r , on the imaginary axis of each strip. Recalling the remarks at the end of Sect. 1, we notice that for the interpolation and analytic continuation procedure it is important to have many points (large N_r) and, of course, precise data (in fact we should envisage a scaling analysis, letting $N_r \rightarrow \infty$ at fixed T according to Eq.(4.2)). The analysis of the spectral function may be improved by taking into account lattice corrections to the fit ansatz.

With different discretization steps in space and time directions, a, a_r , we obtain for the free bosonic Euclidean action,

$$S_B^0 = a^3 a_r \sum_n \left(\frac{1}{2} \sum_{i=1}^3 \left| \frac{\phi_n - \phi_{n+i}}{a} \right|^2 + \frac{1}{2} \left| \frac{\phi_n - \phi_{n+i}}{a_r} \right|^2 + \frac{1}{2} m^2 |\phi_n|^2 \right) \quad (4.3)$$

and for the fermionic one (with Wilson's r),

$$S_F^0 = a^3 a_r \sum_n \left(-\bar{\psi}_n \sum_{i=1}^3 \gamma_i \frac{\psi_{n+i} - \psi_{n-i}}{2a} - \bar{\psi}_n \gamma_0 \frac{\psi_{n+i} - \psi_{n-i}}{2a_r} + m \bar{\psi}_n \psi_n \right) + a^3 a_r \sum_n \left(-\bar{\psi}_n \sum_{i=1}^3 \gamma_i \frac{\psi_{n+i} + \psi_{n-i} - 2\psi_n}{2a} - \bar{\psi}_n \gamma_0 \frac{\psi_{n+i} + \psi_{n-i} - 2\psi_n}{2a_r} \right) \quad (4.4)$$

Introducing dimensionless quantities

$$M = ma, \quad \Phi = \phi a, \quad \Psi = \psi a^{3/2} \quad (4.5)$$

we get for the free propagators in momentum space

$$G_0(p_\mu) = \left(\frac{M^2}{\xi} + \frac{4}{\xi} \sum_{i=1}^3 \sin^2 \frac{p_i}{2} + \xi \sin^2 \frac{p_4}{2} \right)^{-1}, \quad (4.6)$$

$$p_\mu = \frac{2\pi}{N_\mu} j_\mu, \quad j_\mu = 1, 2, \dots, N_\mu \quad (4.6)$$

and

$$S_0(p_\mu) = \left(\frac{M}{\xi} + \frac{r}{\xi} \sum_{i=1}^3 (1 - \cos p_i) + r(1 - \cos p_4) - \frac{1}{\xi} \sum_{i=1}^3 i \gamma_i \sin p_i - i \gamma_4 \sin p_4 \right)^{-1},$$

$$p_\mu = \frac{2\pi}{N_\mu} \left(j_\mu - \frac{1}{2} \right), \quad j_\mu = 1, 2, \dots, N_\mu \quad (4.7)$$

where N_μ is the lattice size in the μ -direction and periodic (antiperiodic) boundary conditions for the boson (fermion) were assumed. Eqs.(4.3), (4.4) imply

$$\xi \equiv a/a_r = \gamma \quad (4.8)$$

In formulating a pole ansatz for the 2-point function at positive temperature, we must then use Eqs.(4.6), (4.7). In this way, we obtain with Eq.(4.6) for the meson correlation function over the distance

$$\tau = l a_r \quad (4.9)$$

the result (here $N = N_r$ and we drop normalization factors):

$$-\langle T_r \Phi(\tau) \Phi(0) \rangle \simeq \frac{1}{N} \sum_{n=1-N/2}^{N/2} \frac{\exp(-i \frac{2\pi}{N} n l)}{M^2/\xi + 2\xi - 2\xi \cos(\frac{2\pi}{N} n)} = \frac{1}{2\xi N} \sum_{n=1-N/2}^{N/2} \frac{\cos(\frac{2\pi}{N} n l) + \cos(\frac{2\pi}{N} n (N-l))}{M^2/\xi^2 + 2 - 2 \cos(\frac{2\pi}{N} n)} \quad (4.10)$$

where we have made explicit the symmetry Eq.(1.10). Evaluating the sums we get:

$$-\langle T_r \Phi(\tau) \Phi(0) \rangle \approx z^{-l} + z^{l-N} = 2 \exp\left(\frac{N}{2} \ln z\right) \cosh\left[\left(\frac{N}{2} - l\right) \ln z\right] \simeq \cosh[(\beta/2 - \tau) \tilde{m}] = \cosh\left[\frac{1}{a}(\beta/2 - \tau) \tilde{M}\right] \quad (4.11)$$

(compare Eq.(2.4), see App.A) where

$$z = 1 + \frac{M^2}{2\xi^2} + \frac{M}{\xi} \sqrt{1 + \frac{M^2}{4\xi^2}} \quad (4.12)$$

and

$$\tilde{M} = \tilde{m}a = \xi \ln z \quad (4.13)$$

is the "physical" dimensionless slope. We notice, however, that M coincides with $\tilde{M}$ only at the critical point ($M \rightarrow \tilde{M}$ for $M/\xi \ll 1$) and therefore in general M does not have the right scaling properties, e.g. for $M/\xi \gg 1$

$$\tilde{M} \sim 2\xi \ln \frac{M}{\xi} \quad (4.14)$$

(Notice that this is true also for the standard isotropic lattice $\gamma = 1$!) Therefore, when employing the pole ansatz we should not use the slope $\tilde{M}$ for physical considerations, but extract the mass from the Fourier transforms, or invert Eqs.(4.12), (4.13), to wit

$$M = \xi \sqrt{2 \left(\cosh \frac{\tilde{M}}{\xi} - 1 \right)} \quad (4.15)$$

For small lattices, far from the thermodynamic limit, we should use directly Eq.(4.10) for the fit. For fermionic correlation functions we get, e.g. using Eq.(4.7) with $r=1$, a similar result, to wit

$$-(T_r \bar{\Psi}(\tau) \Psi(0)) \simeq \cosh(\sigma \sinh) \left[\frac{1}{a} (\beta/2 - \tau) \tilde{M} \right] \quad (4.16)$$

(where the precise expressions depend on the spin components - see App. A), with

$$\tilde{M} = \tilde{m}a = \xi \ln(1 + M/\xi), \quad M = \xi \left(e^{\tilde{M}/\xi} - 1 \right) \quad (4.17)$$

For a realistic, interesting case, where Eq.(4.8) does not hold any longer, one must first perform the calibration at $T \simeq 0$, i.e., determine the function $\eta_i(\xi)$ (Eq.(3.3)) on a physically isotropic lattice,

$$N_r = \xi N_\sigma \quad (4.18)$$

By lowering then N_r from the value Eq.(4.18) one increases the lattice temperature. Thereby one assumes the $\eta_i(\xi)$ not to change, since short-distance properties

should not depend on the temperature, at least in a limited temperature region. One should notice, however, that the scaling properties on which the analysis presented in section 3 rests - in particular Eqs.(3.2) and (3.15) - are only approximately valid when we depart from the continuum limit, as illustrated by the above discussion for the free field case.

Finally, the non-perturbative calculation of thermodynamic quantities involves essentially derivatives with respect to ξ . Considering for instance the physical energy,

$$\epsilon = -\frac{\partial}{\partial \beta} \ln Z \quad (4.19)$$

and observing that ($\beta = 1/T$):

$$\frac{\partial}{\partial \beta} = -T^2 \frac{\partial}{\partial T} = -\frac{\xi^2}{N_r a} \frac{\partial}{\partial \xi} \quad (4.20)$$

we obtain on an isotropic $N_\sigma^3 N_r$ lattice, but at finite temperature $T = 1/N_r a$ ($N_r < N_\sigma$)

$$\epsilon = \frac{1}{N_r a} \frac{1}{Z} \sum_i \lambda_i < \frac{\partial S}{\partial \gamma_i} > \quad (4.21)$$

Here λ_i (see Eq.(3.7)) can be obtained by a non-perturbative analysis on an isotropic, "low" temperature lattice, $\gamma_i = \xi = 1, N_r = N_\sigma$, as described in Sect.3.

II. QCD at finite temperature on anisotropic lattices

5. General formulation

The action for QCD on an anisotropic lattice is

$$S_{YM} = -\frac{1}{3}\beta \left[\frac{1}{\gamma_G} \text{ReTr}(\Box_{\sigma\sigma}) + \gamma_G \text{ReTr}(\Box_{\sigma\tau}) \right], \quad S_F = -\bar{\Psi} W \Psi \quad (5.1)$$

$$W = 1 - \kappa_\sigma \left(\sum_i (\Gamma_{+i} U_i T_i + \Gamma_{-i} T_i^\dagger U_i^\dagger) + \gamma_F (\Gamma_{+4} U_4 T_4 + \Gamma_{-4} T_4^\dagger U_4^\dagger) \right)$$

$$\Gamma_{\pm\mu} = \tau \pm \gamma_\mu, \quad \{\gamma_\mu, \gamma_\mu\} = 2\delta_{\mu\nu}$$

Here U_μ are the link variables ($SU(3)$ fundamental matrices) and $\Psi, \bar{\Psi}$ the site variables representing n_F flavours of Wilson fermions. T_μ are lattice translation operators. Periodic (antiperiodic) boundary conditions are assumed for the boson (fermion) field in all directions. The hopping parameter κ_σ is related to the bare quark mass by

$$\kappa_\sigma = \frac{1}{2(M + (3 + \gamma_F)r)} \quad (5.2)$$

and γ_G, γ_F are the two coupling anisotropies

$$\gamma_G = \sqrt{\beta_\tau/\beta_\sigma}; \quad \beta = \sqrt{\beta_\sigma\beta_\tau} = \frac{1}{g^2} \quad (5.3a)$$

$$\gamma_F = \frac{\kappa_\tau}{\kappa_\sigma}; \quad \kappa = \sqrt{\kappa_\sigma\kappa_\tau} \quad (5.3b)$$

They have to be tuned such that the renormalization at zero temperature can be done in terms of one cut-off a and one cut-off anisotropy ξ , and a number of physical quantities denoted globally by $\{m\}$, to wit:

$$\xi = a_\sigma/a_\tau, \quad a_\sigma = a \quad (5.4)$$

$$\gamma_G = \xi\eta_G(\xi), \quad \eta_G(1) = 1 \quad (5.5a)$$

$$\gamma_F = \xi\eta_F(\xi), \quad \eta_F(1) = 1 \quad (5.5b)$$

$$\beta = R_G(a, \xi, \{m\}) \quad (5.6a)$$

$$\kappa_\sigma = R_F(a, \xi, \{m\}) \quad (5.6b)$$

Here the functions R_G, R_F represent lines of constant physics given by the renormalization group equations and η_G, η_F represent the non-trivial effect of the interaction on the cut-off anisotropy. In principle η_G and η_F may also depend on β and κ_σ but this correction seems to be very small. Finally, finite lattice size effects (N_τ, N_σ dependences) are not taken into account in Eqs.(5.4-6) - but see further remarks below. Since we always work at $\xi > 1$, i.e. the "time" cut-off $1/a_\tau$ is always larger than the "space" cut-off $1/a$, the scaling limit is defined as $a \rightarrow 0$ with fixed m . Due to the asymptotic freedom this means

$$\beta \rightarrow \infty, \quad \kappa_\sigma \rightarrow \frac{1}{2(3 + \gamma_F)r} \quad (5.7)$$

In the scaling region we have, using the perturbative β -function to lowest order

$$a = \frac{1}{\Lambda(\xi)} e^{-b_0\beta}, \quad b_0 = 33/48\pi^2 \quad (5.8)$$

At each finite β we can take

$$\kappa_\sigma \rightarrow \kappa_\sigma^c(\beta)$$

defined such that the pion dimensionless mass vanishes (the corresponding lattice correlation length diverges); however this does not produce a reasonable continuum limit, since then all other physical masses would diverge (they stay finite in lattice units). In fact, the lines of constant physics run at a distance from the $\kappa_\sigma^c(\beta)$ line, concurring with this at $\beta \rightarrow \infty$. For each finite lattice, however, they cross over before to a regime dominated by "deconfining", perturbative effects and characterized by a lattice extension smaller than the correlation length, a region which we should not enter if we are interested in observing the hadronic regime. The usual approach, adopted from pure QCD, is to extrapolate from the confining region when scaling is reached but before entering the perturbative regime.

After this rather general remarks we shall proceed in the next section to an illustration of the aspects described in Sect.2 using results from a perturbative calculation of the quark propagator in QCD. Then in Sections 7 and 8 we shall present and discuss the results from a numerical simulation of quenched QCD.

6. The quark propagator at finite temperature

In this section we shall extend the results of [21] to discuss the quark propagator at finite temperature. Perturbative calculations for finite temperature are the

subject of extensive studies (see e.g., [22], [23]). The present considerations should not be considered as belonging to this ambitious program but are only meant as an illustration for the effects presented in Section 2 in the context of anisotropic lattices.

Starting from the formulae of Sections 4 and 5 we have

$$Z_2 S^{-1}(p) = S_0(p) + \Sigma(p) \quad (6.1)$$

where, to lowest order in the momenta and in g

$$\Sigma(p) = \tilde{g}^2 (M \Sigma_2 + r \Sigma_0 + \sum_{i=1}^3 \gamma_i p_i a_\sigma \Sigma_{1,\sigma} + \gamma_4 p_4 a_r \gamma_F \Sigma_{1,r}) \quad (6.2)$$

Here

$$\tilde{g}^2 \equiv \frac{N^2 - 1}{2\pi} g^2, \quad \Sigma \equiv \Sigma(p, M, r), \quad r \Sigma_0 = \Sigma(0, 0, r), \quad M = ma \quad (6.3)$$

and Σ_2 and $\Sigma_{1,\mu}$ are taken at $p = 0$ (for the explicit expressions see [21]). $\gamma_F = \xi(1 + O(g^2))$, we can set $\gamma_F = \xi$ in Eq.(6.2). Thus we have in general (to order p):

$$Z_2 a_\sigma^{-1} S^{-1} = m(1 + \tilde{g}^2 \Sigma_2) + \tilde{g}^2 r a^{-1} \Sigma_0 + (p_i \gamma_i)(1 + \tilde{g}^2 \Sigma_{1,\sigma}) + \frac{\gamma_F}{\xi} p_4 \gamma_4 (1 + \tilde{g}^2 \Sigma_{1,r}) \quad (6.4)$$

with

$$\Sigma_n = \Sigma_n(N_r, \xi, M, r) = \frac{1}{N_r} \sum_{j=1}^{N_r} \int_0^{2\pi} \frac{d^3 k}{(2\pi)^3} f_n(\mathbf{k}, k_4 = \frac{2\pi}{N_r}(j - \frac{1}{2})) \quad (6.5)$$

With $N_r = \infty$ we obtain the zero temperature case ($N_\sigma = \infty$ is already assumed)

$$\Sigma_n^0 \equiv \Sigma_n(\infty, \xi, M, r) = \int_0^{2\pi} \frac{d^4 k}{(2\pi)^4} f_n(\mathbf{k}, k_4) \quad (6.6)$$

At $T = 0$ we enforce the isotropy condition and perform the renormalization. The first step gives (see Eq.(6.4.))

$$\gamma = \xi(1 + \tilde{g}^2 \Sigma_{1,\sigma}^0) \quad (6.7)$$

with

$$\Sigma_{1,\sigma}^0 \equiv \Sigma_{1,\sigma}^0 - \Sigma_{1,r}^0 \quad (6.8)$$

We denote

$$m_0 \equiv m(1 + \tilde{g}^2(\Sigma_r^0 - \Sigma_{1,\sigma}^0)) + \tilde{g}^2 r a^{-1} \Sigma_0^0 \quad (6.9)$$

then the second step implies

$$m_0 : \text{fixed}, \quad a \rightarrow 0 \quad (6.10)$$

(this is not the exact scaling relation, since the quark mass itself is not an exact renormalization group invariant; for the following discussion, however, these corrections are not relevant). With:

$$M = ma \quad (6.11)$$

we have from Eq.(6.9)

$$M = am_0(1 - \tilde{g}^2(\Sigma_2^0 - \Sigma_{1,\sigma}^0)) - \tilde{g}^2 r \Sigma_0^0 \quad (6.12)$$

Hence we approach the continuum limit, $a \rightarrow 0$, by taking

$$\Delta M \equiv (M - M_c)/(1 - \tilde{g}^2(\Sigma_2^0 - \Sigma_{1,\sigma}^0)) = m_0 a \rightarrow 0 \quad (6.13)$$

At non-zero temperature we then have:

$$T = \frac{1}{N_r a_r} = \frac{\xi}{N_r a} = \frac{\xi m_0}{N_r \Delta M} \quad (6.14)$$

i.e.,

$$\frac{T}{m_0} \equiv \Theta = \frac{\xi}{N_r \Delta M} : \text{fixed} \quad (6.15)$$

and

$$N_r = \frac{\xi}{\Theta \Delta M} \rightarrow \infty \text{ for } \Delta M \rightarrow 0, \text{ fixed } \Theta \quad (6.16)$$

Introducing this in Eq.(6.4) (with Eqs.(6.7), (6.8)) we obtain for the propagator at positive temperature $\Theta > 0$, up to a constant factor:

$$S^{-1}(\Theta, p) \simeq \gamma_4 p_4 + (1 + \tilde{g}^2 \Delta_{1,r\sigma}(\Theta)) \sum_{i=1}^3 \gamma_i p_i + m_0(1 + \tilde{g}^2(\Delta_2(\Theta) - \Delta_{1,r}(\Theta)) + \tilde{g}^2 r \frac{1}{\Delta M} \Delta_0(\Theta)) \quad (6.17)$$

where we have introduced the subtracted quantities:

$$\Delta_n(\Theta, \xi, M, r) \equiv \Sigma_n(N_r) = \frac{\xi}{\Theta \Delta M} \xi, M, r) - \Sigma_n^0(\xi, M, r) \quad (6.18)$$

(see Eqs.(6.6), (6.16)). It can be shown, using the explicit formula for Σ (see Appendix B), that

$$\lim_{\Delta M \rightarrow 0} \Delta_0(\Theta)/\Delta M = 0 \quad (6.19)$$

$$\lim_{\Delta M \rightarrow 0} \Delta_{n \neq 0}(\Theta) = \text{finite, non-zero} \quad (6.20)$$

Hence in approaching M_c

$$S^{-1}(\Theta, p) \simeq \gamma_4 p_4 + (1 + \tilde{g}^2 \Delta_{1r\sigma}(\Theta)) \sum_{i=1}^3 \gamma_i p_i + m_0(1 + \tilde{g}^2(\Delta_2(\Theta) - \Delta_{1r}(\Theta))) \quad (6.21)$$

While this formula has been derived only for small p , it is clear that it indicates different, temperature-dependent slopes for the decay with the distance of propagators measured in the "time" or "space" directions:

$$m_r \simeq m_0(1 + \tilde{g}^2(\Delta_2(\Theta) - \Delta_{1r}(\Theta))) \quad m_\sigma \simeq m_0(1 + \tilde{g}^2(\Delta_2(\Theta) - \Delta_{1\sigma}(\Theta))) \quad (6.22)$$

where for $\Theta < 1$

$$\Delta_{1\sigma r}(\Theta) = \Delta_{1\sigma}(\Theta) - \Delta_{1r}(\Theta) \simeq \text{const } \Theta^2$$

(see Appendix B and Fig. 2).

7. Quenched QCD mesonic spectrum

a) Calibration

Since we have two nearest neighbour couplings — in the Yang-Mills and fermion actions — we also have two coupling anisotropies, γ_G and γ_F , which have to be tuned simultaneously to achieve a well-defined cut-off anisotropy ξ (see Sect.4). Thereby we expect that the quantum corrections will all be different, such that $\gamma_G \neq \gamma_F \neq \xi$ (see Sect.6). However, in the quenched approximation the fermions do not contribute to the dynamics, therefore we make the assumption

that the fermions only feel the physical anisotropy due to the Yang-Mills fields, that is we take $\eta_F(\xi) = 1$, and thus

$$\gamma_F = \xi, \quad (7.1)$$

while

$$\gamma_G \equiv \gamma = \xi \eta_G(\xi) \quad (7.2)$$

where $\eta_G \neq 1$ is to be fixed by the pure $SU(3)$ dynamics.

The calibration is done for each ξ at "zero" (in fact, just low) temperature using a physically symmetric lattice $N_r a_r \simeq N_\sigma a_\sigma$, that is,

$$N_r \simeq \xi N_\sigma. \quad (7.3)$$

In practice we give γ_G and then fix ξ by comparing Wilson loops in $\sigma \times \sigma$ and $\sigma \times \tau$ planes, extending thus the analysis of [19]. We work at four values of γ (different from 1.), namely $\gamma = 2., 2.5, 3.$ and 3.5 and use lattices of $8^3 \times N_r$ with $N_r = 8, 16, 24$ and 32 , trying to fulfil

$$\gamma_G N_\sigma \leq N_r \leq \xi N_\sigma \quad (7.4)$$

Since η_G may be up to 1.5 we may thereby deviate from the symmetry condition Eq.(7.3). However, the highest temperature generated in this way is of around $1/6a$, while we do not observe significant temperature effects below $1/3a$ at the value of β we stick with, namely $\beta = 5.68 - 5.7$. Hence the error introduced by using Eq.(7.4) instead of Eq.(7.3) in the calibration should remain small. This justifies our calling these lattices "zero" temperature. The results of the Yang-Mills calibration are presented in Fig. 3 (compare [19]) and Tables 1 and 2).

For the fermions we assume Eq.(7.1), and, moreover the naive relation

$$\frac{1}{\kappa_\sigma(\xi)} - \frac{1}{\kappa_\sigma(1)} = 2(\xi - 1), \quad (7.5)$$

to define the lines of constant physics (constant mass) in the (ξ, κ_σ) plane (see [9]). As can be seen from Fig. 4 these two assumptions lead to a reasonable approximation, in fact the optimal agreement seems to be achieved with

$$\frac{1}{\kappa_\sigma(\xi)} - \frac{1}{\kappa_\sigma(1)} = 1.9375(\xi - 1) \quad (7.6)$$

instead of Eq.(7.5). (This simple behaviour is lost in the unquenched case - see [13].) In particular, we obtain $\kappa_c(\xi = 4.2) \simeq 0.0833$ and $\kappa_c(\xi = 2.6) \simeq 0.112$, to be compared with $\kappa_c(\xi = 1) \simeq 0.171$. The results presented below include the correction corresponding to Eq.(7.6) (obtained by interpolation from the data taken using Eq.(7.5), a sensible procedure in view of the small deviations involved) as well as the finite cut-off correction Eq.(4.14).

b) Spectrum

We have measured the propagators in "time" τ and "space" σ directions:

$$G^{(\tau)}(t; T) \equiv G_\beta^{(4)}(x_4; p = 0), \quad (7.7)$$

$$G^{(\sigma)}(z; T) \equiv G_\beta^{(3)}(x_3; p_1 = p_2 = p_4 = 0), \quad (7.8)$$

(see Eq.(1.14)) in both the pseudoscalar and vector channels. The quark propagators have been calculated by the conjugate gradient method on each measured configuration, then mesonic propagators have been constructed and averaged. The data (with the statistical errors calculated in the usual way) have been fitted with a two pole ansatz in each channel:

$$G^{(\sigma)}(x_\alpha; T) = C_\alpha(T) \frac{\cosh(M_\alpha(T)(N_\alpha/2 - x_\alpha))}{2M_\alpha(T) \sinh(M_\alpha(T)N_\alpha/2)} + C'_\alpha(T) \frac{\cosh(M'_\alpha(T)(N_\alpha/2 - x_\alpha))}{2M'_\alpha(T) \sinh(M'_\alpha(T)N_\alpha/2)} \quad (7.9)$$

with $\alpha = \sigma, \tau$, $x_\alpha = z, t$ and $N_\alpha = N_3, N_r$, respectively. The fits have been done using *MINUIT*. The fit errors - and, correspondingly, the errors of the data in Tables 1 and 2 and in Figs. 5 and 6 - could not be obtained systematically. From the observation of the fit sensitivity to the variation of the parameters and to the range of points used in the fit, we estimate the overall error to be of the order of 5%. We used the two pole formulae Eq.(7.9) as a trial to isolate the low energy structure in the spectrum. We do not expect the higher mass M' to represent reasonably the next higher excitation, but it should allow us to collectively parametrize the high frequencies, such as to leave a cleaner low energy signal.

To estimate the temperature dependence of the parameters we have performed three kinds of measurements:

- i) In the first case we worked at fixed $\gamma = 2$ ($\xi = 2.6$) at $\beta = 5.7$, the calibration being done on a $8^3 \times 24$ lattice. We then increased the temperature by lowering N_r from 24 to 16, 10 and 8. On all lattices pole-masses M_r and screening masses M_σ were measured in the pseudoscalar and vector channels. We used ca. 20 well-decorrelated (2000 sweeps apart) configurations for each point. The results, as given in Fig. 5 and Table 1 correspond to temperatures below T_c and indicate steadily increasing pole masses M_r and a smaller screening mass M_σ , constant or slightly decreasing with T in the range $T_c/2 < T < T_c$ [10].
- ii) In the second case we worked at $\beta = 5.68$ on a fixed lattice of $N_z = N_y = 8, N_x = N_r = 16$ to avoid systematic effects from the different numbers of points, and varied $\gamma : 2, 2.5, 3, 3.5$ ($\xi : 2.6, 3.425, 4.2$ and 5.18 respectively). For calibration we used the $8^3 \times N_r$ lattices described at point a) above. We measured five, again well-decorrelated configurations, the higher ξ (i.e., T) value corresponding to the deconfined phase, as is indicated by the behaviour of the Polyakov loop. Screening mass data were obtained both in the "x" and in the "y" direction; the latter are as a rule slightly larger than the former, agreeing with these however within less than the expected 5% errors. The results are given in Table 2 and Fig. 6 (with the screening mass data from the "x" direction). We see a confirmation and extension of the conclusions from the first measurement (see (i) above): below the deconfining temperature, pole and screening masses show an opposite behaviour, above T_c the screening mass changes tendency and increases with T , like M_r does, but still staying below the latter. The effect appears in both the pseudoscalar and vector channels and is the more pronounced the lighter the masses are [11].
- iii) Finally, we checked that the behaviour of the measured masses as a function of the quark mass for the pseudoscalar is different in the confining and in the deconfining phase. This has been done at $\beta = 5.7$ on an 8^4 lattice, using $\gamma = \xi = 1$ to obtain the confined phase and $\gamma = 2$ ($\xi = 2.6$) to reach the deconfined phase. The results, given in Fig. 7, indicate that the typical m_π^2 behaviour of m_π in the confined phase seems to be lost above the deconfining transition - an effect similar to that observed for the screening mass [24]. A systematic analysis of this effect was not attempted [10].

8. Discussion

The data of Section 7 show that the behaviour of the "pole mass" in the mesonic propagators is different from that of the "screening mass". While such an anisotropy was expected on general grounds and found also in analytic (perturbative) analyses - see e.g. Section 6 - its amplitude is now shown by the Monte Carlo results, which indicate strong non-perturbative effects.

This raises further doubts about the possibility of interpreting the data for the "screening propagators" in order to describe the phenomena of high temperature QCD (see [15]). The mathematical problem is, of course, that these data only represent partial information, not allowing us to reconstruct the spectral function and perform the analytic continuation to the physical region. However, one may ask whether one could directly obtain physical information from these results. While this is undoubtedly possible, since these results are clearly physically relevant, we are warned against thinking of this as a short cut, taking over the effects seen in the "space-like" correlations and directly interpreting them in the usual terms of particle propagation, wave function, etc. for QCD at finite temperature. A certain intuition about what is going on here may be obtained by remembering what physical situation we observe when we proceed with one or the other "measurement". While doing this we should be clear that any interpretation requires in the background a relation to the real time theory at finite temperature (what we shall call in the following "the original problem"), even if one usually tends to leave this implicit and conduct the discussion in the Euclidean formalism. Further indications about the complexity of the phenomena in high temperature QCD are provided by perturbative calculations and related analyses, see, e.g., [22], [23], [25], [26], we should like here however to only make some general remarks, independent of an approximation scheme.

Let us first notice that if the length of the lattice in the "short" direction becomes smaller than the correlation length (that is, the temperature of the original problem becomes larger than the mass gap) it seems that this direction effectively compactifies and the finite size effects may be very large, leading to a kind of "dimensional reduction" [20], [10], [27], a phenomenon which has been also explored in perturbation theory, high temperature expansion and in effective models [25], [26]. This is a highly non-trivial limit, and it should also be clear that the

anisotropy concerning the physical interpretation of the various effects on the positive temperature Euclidean lattice also must tell how the "dimensionally-reduced" model is connected with the initial theory. In the following we shall try to review the basis of this anisotropy.

In terms of the transfer matrix, taking correlations between different (Euclidean) time slices means using the Hamilton operator of QCD (as defined independently of temperature and for infinite volume) to describe the time evolution of the states. The structure of the spectral functions entering the propagators in the various hadronic channels will account for the temperature effects (due to the interactions with the surrounding) by indicating the formation of "effective states" as superpositions of the eigenstates of the original, infinite volume Hamiltonian (see, e.g., [28] for a discussion about "particles" at finite temperature). As the temperature is increased above some T_c there should be a sharp change in behaviour to a high temperature regime which is believed to be dominated by the formation of the quark-gluon plasma. In fact, for pure gauge theory the existence of a finite temperature deconfining transition has been numerically established (for a recent review see [2]) and analytically proved [27], in the framework of the Euclidean formulation. For full QCD the transition was found to subsist for large quark masses, getting weaker with decreasing masses [29], while in the intermediate and small mass case the situation seems more complicated and also strongly dependent on the fermionic action, number of light and heavy flavours, etc. - see, e.g., [3]. The situation here is less transparent, since the signature of the transition is more subtle, and in fact it is possibly replaced by a crossover between two analytically connected phases. But independently of this, for QCD as also for pure gauge theory the Euclidean time correlations allow for a "simple" interpretation in terms of finite temperature effects referring to the original theory, since they give the structure in the energy plane (albeit modulo an ansatz to solve the interpolation and analytic continuation problem).

A quite different physics is implied when one is trying to interpret in the same way correlations taken between space-slices - say, y, z, t -planes. Here the corresponding transfer matrix (effecting translations in the direction x) describes the evolution of the system according to a Hamiltonian for y, z, t planes, which, unlike what happens at $T = 0$, because of the squeezing in t does not correspond to

the original (infinite volume) Hamiltonian. We can first try to obtain some insight into the various effects by taking the point of view of a zero temperature theory (since $aN_z = aN_\sigma \rightarrow \infty$) with spatial finite size effects (finite $a_r N_r = 1/T$).

Let us first look, for instance, at the $O(N)$ non-linear sigma model in two dimensions, since we have here on the Euclidean lattice a symmetric situation between finite temperature and finite volume. For this model on a cylinder of finite perimeter L one has for the mass gap of the finite "volume", zero temperature problem the convergent expansion

$$m = \Lambda_M S (ze^{b/2})^c \sum_{\nu=0}^{\infty} a_\nu z^\nu, \quad (8.1)$$

where

$$z = mL \quad (8.2)$$

and the parameters a, b, c depend on N [30]. Using the symmetry of the Euclidean formulation and interpreting L as inverse temperature:

$$z = m_\sigma/T = m_\sigma N_r a_r = M_\sigma N_r \xi \quad (8.3)$$

Eq.(8.1) gives the screening mass of the finite temperature, infinite volume problem, e.g., for $O(3)$:

$$m_\sigma \simeq a \frac{m_\sigma^2}{T^2} \Lambda_M S e^{bT/m_\sigma} \quad (8.4)$$

This shows a dip, above which m_σ increases as $T/\ln T$. In the infinite N -limit the model is solved in terms of a free field and, as we noticed in Section 2, "trivial" self-energy effects amounting to mere shifts of the mass cannot produce an anisotropy. Therefore in this case the pole and the screening mass coincide [31]. This, however, is due to the simplicity of the model and should not misguide us into expecting it to happen also in less trivial situations. But while the volume dependence of the mass gap and thus the temperature dependence of the screening mass for such models as $O(N)$ at finite N can still be estimated - see above -, no analytic, non-perturbative results about the temperature effects on the spectrum of these models are known to us. In the 4-dimensional case the situation is more involved, since only one of the space directions is compactified. We expect here also strong effects when the compactification radius becomes comparable to or less than the correlation length.

In the extreme case, in the limit of infinite temperature ($N_r a_r \rightarrow \infty$), we employ in fact the Hamiltonian of a 2+1-dimensional problem, with space directions y, z and time direction x , of length $N_\sigma a > 1$. Hence, in this limit we effectively study a zero temperature, 3-dimensional model; to the extent that this model resembles the 3-dimensional QCD, we expect it to confine at all couplings. It has been shown both analytically [32] and numerically [33] that for the pure gauge theory the string tension from Wilson loops in the space-space (x, y, z) planes varies very little with T (N_r) even when T increases beyond T_c , connecting smoothly to the string tension of the 3-dimensional theory.

To summarize: pole-masses, as well as other quantities obtained from correlations between "operators" in different time-slices (wave functions, etc.) are effective parameters describing the restructuring of the states of the original Hamiltonian, due to the interactions with the surrounding medium. There is a direct relation between the spectral densities of the Schwinger functions of the 4-dimensional, Euclidean problem defined in this way and those of the Green functions of the 3+1-dimensional (Minkowski), finite temperature QCD (see Section 2).

Screening-masses and other quantities obtained from correlations between space-slices are directly related to the parameters of a model at zero temperature but with a compactified space direction, and which we expect to show an effective dimensional reduction when the inverse compactification radius (the temperature of the original problem) becomes larger than the mass gap. There is a simple relation (by a Wick rotation) between the Schwinger functions of this problem (the "screening" Schwinger functions of the original, finite temperature QCD in Euclidean formulation) and those of a Minkowski, zero temperature QCD with one of its 3 space dimensions compactified. On the other hand, the relation to the original, finite temperature QCD problem in 3+1 Minkowski space is rather indirect and not well understood at present. It seems to us therefore that to obtain insight into the physics of finite temperature QCD it is unavoidable to study time-slice correlations and measure directly pole-masses, wave functions of quark-antiquark pairs propagating in the Euclidean time direction and related quantities (to determine the full dispersion rule it would be desirable, of course, to do calculations at non-zero spatial momentum.)

There are a number of questions raised now:

the region of critical temperature. This may change, however, at larger momenta (similar effects can be observed in perturbation theory [23]). (We should notice here that the quarks are rather heavy in our analysis and that we are far from the continuum limit. We cannot therefore observe short distances - large momenta -, while at large distances the uncertainties of the interpolation show up. This is also the reason why we do not try to make contact with the results of chiral perturbation theory - see, e.g., [35].) Using the estimation for T_c from the Polyakov loops as indicated in Fig. 6 we have a " π " and a " ρ " of about $3 - 4T_c$ with a m_π/m_ρ ratio of about 0.8 at the lowest temperature and quark mass we used. Hence the range of distances addressed by this calculation is around $1/3 - 2/3 fm$. The steady increase of the pole-masses with the temperature, starting at about $T_c/2$, indicates that already at this temperature the fluctuations of the strings begin to weaken the quark-antiquark potential, which is itself an effect of the string formation. The results for the screening masses, on the other hand, are quite similar with other data [5], showing the same tendency to approach the results for free-quark systems. Although we do not have good enough data to judge upon the existence of a jump at some T_c , this whole picture suggests that at high enough temperature most hadrons would have decayed into bleached quarks and gluons, in accordance with the usual image about the formation of the quark-gluon plasma, but that this process is rather intricate and involves more steps with different physics going on. In particular, it suggests that at lower temperatures the dispersion law of the hadrons may be modified in a non-trivial way. Since most of the present discussion seems to concern less the (still rather restrained) data and proofs and more the understanding of deconfinement as a well-defined set of phenomena - for some aspects see, e.g., [27], [14], [15], [36] [37], [38] - it is important to check the various parts of the above picture. To do this, and to get more insight into the structure of the hadrons at high temperature, it should be very useful to measure form factors or wave functions at space-like separation between a quark-antiquark pair propagating in the time direction. These analyses would be rather involved, since the separation of the states will imply many parameters-fits; however they seem feasible to us and should provide us with information which is not obtainable in other ways.

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- how reliable are our results for pole masses ?
- how can we interpret these results ?
- what other quantities can be measured - e.g., wave functions at fixed t , form factors, etc. ?

As to the first question, we note that our attitude has been to try (by having many Matsubara frequencies and by using two-pole fits) to isolate the low-lying effective structure in spite of the small physical Euclidean time length $N_\tau a_\tau = 1/T$. However, at this stage of our analysis, we did not use non-local sources, smooth propagators or other methods to improve the low energy signal (partly because it was not clear to us how to define these procedures in agreement with our search for effective structures - remember that we do not look for a threshold but for a bump). Therefore we cannot exclude the possibility of an admixture of higher states, that is of a poor separation of different particle-like excitations. With increasing temperature, $N_\tau a_\tau$ decreases, and this admixture may increase, mimicking a genuine temperature dependence of the spectral function. Against this, however, speaks the appearance of the fits which have a low χ^2 , seem to be rather stable against the variation of the fit region and are characterized by a rather large distance between the two peaks at all temperatures. But the essential difficulty (and challenge!) of this analysis remains the fact that we look for a structure in the spectral functions at energies comparable to or smaller than the energies at which the actual calculations are done (at least $2\pi T$). We must therefore rely on an interpolation and we depend on a good fit and on the reasonableness of the ansatz for it to obtain a sensible result.

Assuming that the effects we observe are genuine, that is, they describe the properties of some effective states and are not due to a poor resolution of the spectral function: then our results imply that the original mesons are overlapped by plasma effects and become effective excitations, which move towards higher energies with increasing temperature. These effective excitations control also the intermediate distances and intermediate duration phenomena in plasma, hence they should lead to directly observable effects. Among others, the usual intuition that the plasma leads to a slow-down of the "particle" modes [34] seems contradicted by the above results: in the intermediate momentum range where our analysis has chances of being meaningful, the contrary effect seems to occur, especially in

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Appendix A

We provide here some formulae within the imaginary time formalism, recast for anisotropic lattices.

Calculations at finite temperature usually involve sums over the Matsubara frequencies

$$\zeta_n = \frac{2\pi}{N}(n + \frac{\eta}{2}) \quad (A.1)$$

with $\eta = 0(1)$ accounting for periodic (antiperiodic) boundary conditions and $N = N_\tau$ the lattice size in the fourth direction. With the notation

$$\frac{1}{N} \sum_{n=-N/2}^{N/2-1} = \sum_n^N \quad (A.2)$$

what we must evaluate is

$$F_N^\eta = \sum_n^N f(\zeta_n) \quad (A.3)$$

where the index η refers to the spectrum Eq.(A.1). Here f typically involves trigonometric functions of ζ and, independently of the lattice boundary conditions,

$$f(\zeta + 2\pi) = f(\zeta) \quad (A.4)$$

For special forms of f we can use the "lattice step function" formula (here we assume N to be a power of 2):

$$aI_N^\eta = \sum_n^N \frac{a}{a - e^{i\zeta_n}} = a^2 I_{N/2}^\eta(a^2) = \dots = \frac{a^N}{a^N - (-1)^\eta} \quad (A.5)$$

In general, however, the standard procedure is to use the Sommerfeld-Watson transformation on the contour C of Fig. 8, taking into account the momentum space periodicity Eq.(A.4):

$$\begin{aligned} F_N^\eta &= \int_C \frac{d\zeta}{2\pi} \frac{f(\zeta)}{(-1)^\eta e^{i\zeta N} - 1} \\ &= \int_{-\pi-i0}^{\pi-i0} \frac{d\zeta}{2\pi} \left(\frac{f(\zeta)}{(-1)^\eta e^{i\zeta N} - 1} - \frac{f(-\zeta)}{(-1)^\eta e^{-i\zeta N} - 1} \right) \\ &= \int_{-\pi}^{\pi} \frac{d\zeta}{2\pi} f(\zeta) + \int_{\Gamma} \frac{d\omega}{2\pi i} \frac{f(-i\omega) + f(i\omega)}{(-1)^\eta e^{\omega N} - 1} \end{aligned}$$

$$= \int_{\Gamma} \frac{d\omega}{2\pi i} \frac{f(-i\omega)(-1)^N e^{\omega N} + f(i\omega)}{(-1)^N e^{\omega N} - 1} \quad (A.6)$$

Here the contour Γ encloses all singularities of f in the half-plane $\text{Re } \omega > 0$, where

$$\zeta = -i\omega \quad (A.7)$$

and the last equality in Eq.(A.6) assumes

$$\omega f(\omega) \rightarrow 0 \text{ for } \text{Re } \omega \rightarrow \infty$$

Moreover f is assumed to be regular at $\text{Re } \omega = 0$. We shall exemplify here this formula for the $\mathbf{p} = 0$ propagator of a free fermionic field (for the bosonic case the formula is given in the text - see Eqs.(4.10)-(4.13)). For the sake of generality we also introduce a chemical potential α according to [39], that is, we shift in Eq.(4.8)

$$p_4 \rightarrow p_4 - i\alpha \quad (A.8)$$

Then we obtain for the time-slices correlations (see Eq.(1.14)) at distance

$$l = \tau/a_r \quad (A.9)$$

with $r = 0$:

$$S_N^r(l, \mathbf{p} = 0; \alpha) = \frac{\xi}{2\sqrt{M^2 + \xi^2}} \cdot \left(\frac{e^{(\alpha-b)l}}{e^{(\alpha-b)N} + 1} (\Gamma_{+4} + (-1)^l \Gamma_{-4}) - \frac{e^{(\alpha+b)l}}{e^{(\alpha+b)N} + 1} (\Gamma_{-4} + (-1)^l \Gamma_{+4}) \right) \quad (A.10)$$

with

$$b = a \sinh\left(\frac{M}{\xi}\right) \quad (A.11)$$

For $r = 1$ we obtain:

$$S_N^r(l, \mathbf{p} = 0; \alpha) = \frac{\xi}{2M} \left(\frac{e^{(\alpha-c)l}}{e^{(\alpha-c)N} + 1} \Gamma_{-4} - \frac{e^{(\alpha+c)l}}{e^{(\alpha+c)N} + 1} \Gamma_{+4} \right) \quad (A.12)$$

with

$$c = \ln\left(\frac{M}{\xi} + 1\right) \quad (A.13)$$

(the simple $\sinh$, $\cosh$ structure reappears at $\alpha = 0$). Here

$$\Gamma_{+4} = 1 + \gamma_4, \quad \Gamma_{-4} = 1 - \gamma_4$$

The naive scaling is obtained with:

$$M = ma, \quad \alpha = \mu a_r = \mu a / \xi, \quad (A.14)$$

$$N = \beta / a_r = \beta \xi / a, \quad l = \tau / a_r = \tau \xi / a$$

in the limit

$$a \rightarrow 0; \quad \beta, \xi, \mu : \text{fixed} \quad (A.15)$$

Appendix B

Notice first, that according to Eq.(6.16) the continuum limit at fixed temperature is obtained as (with somewhat simplified notations)

$$N = N_r = \frac{1}{\Theta \Delta M} \rightarrow \infty; \quad \Theta = T / m_0 \quad (B.1)$$

The expressions for the self-energy terms can be found in [21]. To prove Eqs.(6.19), (6.20) we only need the observation obtained by simple inspection, that after the $d^3 p$ integration, for the remaining p_4 integration (sum) the integrand of Σ_2^0 is regular together with its first derivative, and its second derivative can lead to a logarithmic divergence, while for $\Sigma_{1\sigma}^0, \Sigma_{1r}^0$, the integrand itself leads to a logarithmic divergence.

Hence in the first case the integral is of the form

$$\begin{aligned} \sigma_0 &= \int_0^1 dx f(x) = \sum_{n=1}^N \int_{\frac{n-1}{N}}^{\frac{n}{N}} dx f(x) \\ &= \sum_{n=1}^N \int_{\frac{n-1}{N}}^{\frac{n}{N}} dx \left(f_n + \left(x - \frac{n-1/2}{N} \right) f'_n + \frac{1}{2} \left(x - \frac{n-1/2}{N} \right)^2 f''_n + \dots \right) \end{aligned} \quad (B.2)$$

where

$$f_n = f\left(\frac{n-1/2}{N}\right)$$

Then

$$-\delta_0 = \sigma_0 - \frac{1}{N} \sum_{n=1}^N f_n = \frac{1}{24N^2} \sum_{n=1}^N f''_n + \dots = o(1/N) \quad (B.3)$$

hence at every Θ

$$\frac{1}{\Delta M} \delta_0 \rightarrow 0 \text{ for } \Delta M \rightarrow 0 \quad (B.4)$$

In the second case the problem is of the form

$$\begin{aligned}\delta_1 &= \frac{1}{N} \sum_{n=1}^N \frac{1}{\Delta M + n/N} - \int_0^1 \frac{dx}{\Delta M + x} = \sum_{n=1}^N \left(\frac{1}{\Theta + n} - \int_{\frac{n-1}{N}}^{\frac{n}{N}} \frac{dx}{\Delta M + x} \right) \\ &= \sum_{n=1}^N \left(\frac{1}{\Theta + n} - \ln \frac{n+1/\Theta}{n-1+1/\Theta} \right) = - \sum_{k=2n=1}^{\infty} \sum_{k=2n=1}^{\infty} \frac{1}{k(n+1/\Theta)^k} \quad (B.5)\end{aligned}$$

From the last two expressions one can see, using the dominated convergence theorem, that $-\infty < \delta_1 < 0$ for all finite Θ and in fact $\delta_1 \simeq \Theta$ in continuum limit.

A more precise analysis gives

$$\begin{aligned}\Delta_{1,\sigma r} &\simeq \frac{1}{N} \sum_{n=-N/2+1}^{N/2} \int_{-\pi}^{\pi} \frac{d^3 q}{(2\pi)^3} \frac{\sin^2 q_a - \sin^2 q_1}{(1/(\Theta N)^2 + \sum_{\nu} \sin^2 q_{\nu}) (\sum_{\nu} (1 - \cos q_{\nu}))^2} \\ &\simeq \frac{N\Theta^4}{8\pi^2} \sum_n \int_{-1}^1 dx (n^2 - x^2 N^2) \left(\ln \frac{n^2 + x^2 N^2}{(1/\pi\Theta)^2 + n^2 + x^2 N^2} + \frac{1}{\pi^2 \Theta^2} \frac{1}{n^2 + x^2 N^2} \right) \\ &\simeq \text{const } \Theta^2 \quad (B.6)\end{aligned}$$

where the first $\simeq$ sign means that we only considered the singular part in the integrand and the other $\simeq$ signs indicate farther approximations. The curve in Fig. 2 is obtained numerically starting from the upper expression in Eq.(B.6).

REFERENCES

- 1) For recent reviews see: H. Satz, "High Energy Nuclear Collisions: Probing the States of Matter in QCD", plenary talk at the Joint International Lepton Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, 1991, prep. CERN-TH 6216/91 and J. Schukraft, "Ultrarelativistic Heavy Ion Collisions: Searching for the Quark-Gluon Plasma", prep. CERN-PPE-92-92 and S. Nagamiya, Proceedings of the Conference "Physics of High Energy Ion Collisions", Eds. S. Date and S. Ohta, page 197, RIKEN, Saitama, 1992
- 2) For a recent review see: A. Ukawa, Nucl. Phys. B (Proc. Suppl.) 17 (1990) 118
- 3) For recent reviews see: S. Gottlieb, Nucl. Phys. B (Proc. Suppl.) 20 (1991) 247, N.H. Christ, "Finite Temperature QCD", talk at the International Conference Quark Matter '91, Gatlinburg, 1991, prep. CU-TP-544, D. Toussaint, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 3
- 4) See, e.g., J. Engels, F. Karsch and H. Satz, Nucl. Phys. B315 (1989), 419 and B. Petersson, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 332
- 5) C. DeTar and J. Kogut, Phys. Rev. Lett. 59 (1987) 399 and Phys. Rev. D36 (1987) 2828 and K.D. Born et al., Phys. Rev. Lett. 67 (1991) 302
- 6) S. Gupta, "The Physical Degrees of Freedom in Hot Quenched QCD", prep. CERN-TH.6509/92
- 7) N.P. Landsman and Ch.G. van Weert, Phys. Rep. 145 (1987) 141 and J.J. Kapusta, "Finite Temperature Field Theory", Cambridge University Press, Cambridge 1989
- 8) S.A. Fulling and S.N.M. Ruijsenaars, Phys. Rep. 152 (1987) 135
- 9) A. Nakamura et al., Nucl. Phys. B (Proc.Suppl.) 9 (1989) 373 and T. Hashimoto et al., "Vector and Pseudoscalar Meson Masses at Finite Temperature on Anisotropic Lattices", prep. HD-THEP-89-2
- 10) I. Bender et al., Nucl. Phys. B (Proc. Suppl.) 17 (1990) 387
- 11) I. Bender et al., Nucl. Phys. B (Proc. Suppl.) 20 (1991) 329 and T. Hashimoto, A. Nakamura and I.O. Stamatescu, contribution to the Kyoto Conference 1991
- 12) A.A. Abrikosov, L.P. Gorkov and I.E. Dzyakoshinski, Sov. Phys. JETP Volume 36(9) (1959) 636 and E.S. Fradkin, Sov. Phys. JETP Volume 36(9) (1959) 912
- 13) T. Hashimoto, A. Nakamura and I.O. Stamatescu "Finite Temperature Effects in the Mesonic Spectrum of QCD with Dynamical Wilson Quarks", in preparation
- 14) C. DeTar, Phys. Rev. D 32 (1985) 276 and Phys. Rev. D 37 (1988) 2328
- 15) C. Bernard et al., Phys. Rev. Lett. 68 (1992) 2125
- 16) F. Karsch, Nucl. Phys. B205 [FS5] (1982) 239
- 17) J. Engels, F. Karsch and H. Satz, Nucl. Phys. B205 [FS5] (1982) 285, S. Das and J. Kogut, Nucl. Phys. B265 [FS15] (1986) 303, J.C. Sexton and H.B. Thacker, Phys. Rev. Lett. 57 (1986) 2131, R.V. Gavai et al., Nucl. Phys. B (Proc. Suppl.) 26 (1992) 539
- 18) R. Trinchero, Nucl. Phys. B227 (1983) 61, U. Heller and F. Karsch, Nucl. Phys. B251 [FS13] (1985) 254 and Nucl. Phys. B258 (1985) 29

- 19) G. Burgers, F. Karsch, A. Nakamura and I.O. Stamatescu *Nucl. Phys.* B304 (1988) 587
- 20) L.v. Lönneysen, R. Schrock and I.O. Stamatescu, *Phys. Lett.* B205 (1988) 321
- 21) F. Karsch and I.O. Stamatescu, *Phys. Lett.* B227 (1989) 153
- 22) D.J. Gross, R.D. Pisarski and L.G. Yaffe, *Rev. Mod. Phys.* 53 (1981) 43, T. Toimela, *Int. J. of Theor. Physics* 24 (1985) 901, J. Frenkel and J.C. Taylor, *Nucl. Phys.* B374 (1992) 156, R. Kobes, G. Kunstatter and K. Mak, *Phys. Rev. D* 45 (1992) 4632
- 23) O.K. Kalashnikov, *Fortschr. Phys.* 32 (1984) 525
- 24) Y. Iwasaki, Y. Tsuboi and T. Yoshie, *Phys. Lett.* B220 (1989) 602
- 25) See, e.g., T. Appelquist and R.D. Pisarski, *Phys. Rev. D* 23 (1981) 2305, N.P. Landsman, *Nucl. Phys.* B322 (1989) 498 and *Comm. Math. Phys.* 125 (1989) 643, S. Nadkarni, *Phys. Rev. D* 38 (1988) 3287, A. Irbäck et al., *Nucl. Phys.* B363 (1991) 34
- 26) See, e.g., S. Nadkarni, *Nucl. Phys.* B334 (1991) 559, T. Reisz, *Z. Phys.* C53 (1992) 169, P. Lacroix, D.E. Miller and T. Reisz, *Nucl. Phys.* B369 (1992) 501
- 27) C. Borgs and E. Seiler, *Nucl. Phys.* B215 [FS7] (1983) 125 and *Comm. Math. Phys.* 91 (1983) 329
- 28) H. Narnhofer, M. Requardt and W. Thirring, *Comm. Math. Phys.* 92 (1983) 247
- 29) P. Hasenfratz, F. Karsch and I.O. Stamatescu *Nucl. Phys.* B334 (1991) 559, Ph. de Forcrand and I.O. Stamatescu, *Nucl. Phys.* B261 (1985) 613
- 30) M. Lüscher, *Phys. Lett.* 118B (1982) 391
- 31) M. Dine and W. Fischler, *Phys. Lett.* 105B (1981) 207
- 32) C. Borgs, *Nucl. Phys.* B261 (1987) 455
- 33) H. Satz, talk at the LATTICE86 Conference, Brookhaven 1986, unpublished, E. Manoutsakis and J. Polonyi, *Phys. Rev. Lett.* 58 (1987) 847
- 34) E.V. Shuryak, *Phys. Rev. D* 42 (1990) 1764
- 36) J. Polonyi, "The Confinement-Deconfinement Mechanism", in vol. "Quark Gluon Plasma", R.C. Hwa ed., World Scientific 1989 and M. Oleszczuk and J. Polonyi, "Dynamical Symmetry Breaking at High Temperature", prep. MIT-CTP-1984 (1991)
- 37) E.V. Shuryak, *Nucl. Phys.* A533 (1991) 761 and V. Koch, E.V. Shuryak, G.E. Brown and A.D. Jackson, "The Propagation of Quarks in the Spatial Direction in Hot QCD", prep. SUNY-NTG-92-4

- 38) J. Bros and D. Buchholz, "Particles and Propagators in Relativistic Thermo Field Theory", prep. DESY 92-051
- 39) P. Hasenfratz and F. Karsch, *Phys. Lett.* B125 (1983) 308

Table Captions

Table 1: Lower masses from two-pole fits in the pseudoscalar and vector channels at $\beta = 5.7$ with $\gamma = 2(\xi = 2.6)$ and at three temperatures realized by taking $N_\tau = 16, 10$ and 8. Here $\kappa_\sigma = 0.11$ has been used, which corresponds, by the calibration relation Eq.(7.6), to $\kappa = 0.1669$ at $\gamma = 1$.

Table 2: Lower masses from two-pole fits in the pseudoscalar and vector channels (columns 4 to 7) at $\beta = 5.68$ with $N_\tau = 16$ and at different temperatures $T = \xi/N_\tau$, realized by changing $\gamma : 2., 2.5, 3.$ and 3.5. The two ranges of values for κ_σ (the first four and the last four lines of column 3) correspond to $\kappa = 0.1449$ and $\kappa = 0.1669$, respectively, for $\gamma = 1$ and are calculated using the calibration relation Eq.(7.6). Also given is the position of the second pole, which however does not have a clean interpretation.

Figure Captions

- 1 Energy plane with Matsubara frequencies for a scalar field.
- 2 Deviation from 1 of the pole- over screening-quark mass ratio to $O(g^2)$ vs. the temperature divided by the quark mass at zero temperature, $\Theta = T/m_0$.
- 3 Anisotropy correction factor η vs. γ at $\beta = 5.68$ for the pure SU(3) theory. These results are obtained on "zero-temperature" lattices (see Eq.(7.4)). The dashed curve is a rough representation of the earlier results [19].
- 4 Test of the calibration relation Eq.(7.5). The improved relation Eq.(7.6) leads to exactly parallel straight lines, on which the values of κ used in Table 2 correspond to horizontally aligned sequences of points.
- 5 Pole-masses in the pseudoscalar and vector channels obtained as the lower masses from two-pole fits at $\beta = 5.7$ with $\gamma = 2$ ($\xi = 2.6$) and different temperatures realized by changing N_τ . Here the value $\kappa_\sigma = 0.11$ used corresponds to $\kappa = 0.1669$ at $\gamma = 1$. See also Table 1.

6 Pole-masses in the pseudoscalar and vector channels obtained as the lower masses from two-pole fits at $\beta = 5.68$ with $N_\tau = 16$ and for different temperatures realized by changing γ . Here two quark-masses have been used corresponding to $\kappa = .1669$ and to $\kappa = .1449$, respectively, at $\gamma = 1$. See also Table 2.

7 Pole-mass squared vs. quark-mass in the pseudoscalar channel at $\beta = 5.7$ for two values of γ corresponding to temperatures below, respectively above the deconfining transition for the 8^4 lattice.

8 The integration contours for finite temperature sums in the complex plane ω . The crosses, respectively full circles on the imaginary axis represent the Matsubara frequencies for bosons (fermions). The large crosses illustrate some possible singularities of the integrand at $Re\omega > 0$. Notice that due to the periodicity in $\zeta = Im\omega$ the upper and lower ends of the various contours shown coincide (we can imagine welding together the two horizontal dashed lines to obtain a torus).

<i>lattice</i>	$1/N_r$	ps, t	ps, x	v, t	v, x
$8^3 \times 16$	0.0625	0.837	0.884	0.960	1.080
$8^3 \times 10$	0.1000	0.920	0.872	1.232	1.100
$8^3 \times 8$	1.2500	1.059	0.850	1.457	1.036

Table 1

γ	ξ	κ_σ	M_{ps}^τ	M_{ps}^σ	M_v^τ	M_v^σ	$M_{ps}^{\tau l}$	$M_{ps}^{\sigma l}$	$M_v^{\tau l}$	$M_v^{\sigma l}$
2.0	2.6	.1000	1.70	1.80	1.74	1.85	3.56	3.58	3.61	3.31
2.5	3.425	.0862	1.74	1.75	1.80	1.80	3.83	3.32	3.90	3.48
3.0	4.2	.0763	1.83	1.63	1.88	1.67	4.13	2.93	4.19	3.16
3.5	5.18	.0667	1.97	1.82	1.99	1.89	4.49	3.48	4.49	3.29
2.0	2.6	.1100	0.90	0.89	1.08	1.09	2.88	2.58	2.97	3.17
2.5	3.425	.0936	0.93	0.79	1.15	1.08	2.98	2.09	3.24	2.68
3.0	4.2	.0820	1.04	0.67	1.32	0.82	3.32	2.32	3.55	2.53
3.5	5.18	.0710	1.32	1.11	1.46	1.29	3.81	2.89	3.88	2.75

Table 2

Fig.1

Fig.3

Fig.2

Fig.5

Fig.4

Fig.7

Fig.6

Fig.8