**EVIDENCE FOR THE BOX ANOMALY
IN η AND η' DECAYS**

**Talk presented at the XXVI. International Conference on High Energy Physics,
Dallas, August 1992**

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representing the authors of ref. [1]

Abstract

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INTRODUCTION AND OUTLINE

Lots of experiments having reconstructed η' decays into $\pi^+\pi^-\gamma$ have noticed that the invariant $\pi^+\pi^-$ mass spectrum is not well described by the expectation of a single ρ resonance. We therefore reviewed and refitted all available data [1]. In order to get a reliable ρ resonance parametrization we also refitted $e^+e^- \rightarrow \pi^+\pi^-$ cross sections. Surprisingly, these are not well fitted by a single ρ with standard PDG values, which leaves some freedom in the analysis; and we adopt two extreme models.

We find that η and η' decays are much better described when a continuum term is introduced. A possible explanation is that it might stem from the box anomaly. We evaluate the experimental findings in the framework of Chanowitz equations [2] in terms of the pseudoscalar nonet decay constants f_π , f_8 and f_1 as well as the octet-singlet mixing angle θ_{PS} . We find parameters in the expected range. Internal consistency is achieved in both our ρ -models, however one solution needs integrally charged quarks.

TRIANGLE AND BOX ANOMALIES

The well known triangle anomaly shown in fig. 1a is responsible for the $\gamma\gamma$ decays of the π^0 , η and η' . The matrix element reads

$$M(X \rightarrow \gamma\gamma) = B_X(k_1 \cdot k_2) \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma$$

In this expression k_i and ϵ_i^δ ($i = 1, 2$) are the 4-momenta and polarization vectors of the two photons. At low energy, *i.e.* at the unphysical point where all 4-momenta vanish, the following relations can be established by e.g. PCAC and the Wess Zumino Lagrangian [2]:

$$\begin{aligned} B_\pi(0) &= \frac{\alpha_{em}}{\pi\sqrt{3}} \frac{\sqrt{3}}{f_\pi} \\ B_\eta(0) &= -\frac{\alpha_{em}}{\pi\sqrt{3}} \left[\frac{\cos \theta_{PS}}{f_8} - 2\sqrt{2} \xi \frac{\sin \theta_{PS}}{f_1} \right] \\ B_{\eta'}(0) &= -\frac{\alpha_{em}}{\pi\sqrt{3}} \left[\frac{\sin \theta_{PS}}{f_8} + 2\sqrt{2} \xi \frac{\cos \theta_{PS}}{f_1} \right] \end{aligned}$$

Here ξ is a discrete quantity which is equal to 1 for standard fractionally charged quarks and some special "designer" integral charge quark models (ICQM), but equal to 2 for all common formulations of ICQM. The couplings $B(k_1 \cdot k_2)$ are believed to exhibit only very weak energy dependence, and in the following we consider them to be energy independent.

Figure 1. a) Triangle anomaly; b) box anomaly

Using the measured $\gamma\gamma$ widths of the π^0 , η and η' [3], the coupling constants can be deduced from

$$\Gamma(X \rightarrow \gamma\gamma) = \frac{m_X^3}{64\pi} |B_X(0)|^2, \quad X = \pi^0, \eta, \eta'$$

to be $B(\pi^0) = [24.5 \pm 0.3] TeV^{-1}$, $B(\eta) = [-23.7 \pm 3.8] TeV^{-1}$ and $B(\eta') = [-32.1 \pm 0.9] TeV^{-1}$. Note the good accuracy of the prediction in case of the π^0 : the prediction using $f_\pi = 93.15 MeV$ [4] from $\pi^\pm \rightarrow \mu^\pm \nu$ is $B(\pi^0) = 24.9 TeV^{-1}$.

Although much less known, the same theories predict another anomaly occurring in processes involving three pseudoscalar particles: the so-called box anomaly depicted in fig 1b. The corresponding matrix element can be written

$$M(X \rightarrow \pi\pi\gamma) = E_X(p_+, p_-) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma$$

where $p_\pm$ are the 4-momenta of the outgoing $\pi^\pm$, k and ϵ are respectively the photon 4-momentum and its polarization vector. At low energies, the functions E_X are given by [2]:

$$\begin{aligned} E_\eta(0) &= -\frac{e}{4\pi^2\sqrt{3}f_\pi^2} \left[\frac{\cos\theta_{PS}}{f_8} - \sqrt{2}\frac{\sin\theta_{PS}}{f_1} \right] \\ E_{\eta'}(0) &= -\frac{e}{4\pi^2\sqrt{3}f_\pi^2} \left[\frac{\sin\theta_{PS}}{f_8} + \sqrt{2}\frac{\cos\theta_{PS}}{f_1} \right] \end{aligned}$$

Whereas the connection of the triangle anomaly constants to measurable decay rates is obvious, the occurrence of the ρ -resonance in the $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays obscures a direct connection of E_X and decay rates.

OUR MODEL FOR $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$

Knowing that a pure ρ resonance does not describe well experimental data we are led to the following model (see fig. 2): We assume that the radiative decays $\eta/\eta' \rightarrow \rho^0\gamma$ and the non-resonant production due to the anomaly are actually two distinct processes, and the overall amplitude is the coherent sum of both contributions. Furthermore, the anomaly strengths B_X and E_X are assumed to be energy independent. Finally, for quantitative fits to data we need a model for the ρ -resonance. We use a standard relativistic Breit-Wigner amplitude whose width is parametrized as

$$\Gamma_\rho(m) = \Gamma_0 \left[\frac{q_\pi(m)}{q_\pi(m_\rho)} \right]^3 \left(\frac{m_\rho}{m} \right)^\lambda$$

where the q^3 dependence is due to the p -wave matrix element and phase space, and any deviation of λ from 1 signals deviations from elementary pointlike couplings or the influence of left-hand singularities. This choice of parametrization is in accord with analyticity, for details see [1].

FITTING $e^+e^- \rightarrow \pi^+\pi^-$ IN THE ρ REGION

To get reliable parameters m_ρ , $\Gamma_\rho(m_\rho)$ and λ for the η/η' fits, we refitted the most recent $e^+e^- \rightarrow \pi^+\pi^-$ data, using ρ and ω Breit-Wigner amplitudes and a free $\rho - \omega$ -mixing angle φ . We chose this process since it is a clean, pure QED initial state which is believed to be free of uncertainties due to fi-

Figure 2. Model amplitude for $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$

Figure 3. Model M1 (solid line) and PDG value prediction (dotted line) compared to measured cross sections $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$

nal state interactions, interferences, production mechanisms etc.. To our surprise, the standard PDG ρ parameters do not give a good description (see dotted curve in fig 3; $\chi^2/ndf = 498/80$). We achieve a better fit by leaving m_ρ and Γ_ρ free (model M2, $\chi^2/ndf = 90/78$), but here we need a ρ mass almost degenerate to the ω mass, more than 20σ away from the PDG value ($m_\rho = 780.8 \pm 0.5 \text{ MeV}$, $\Gamma = 153 \pm 2 \text{ MeV}$, $\lambda = 0.66 \pm 0.05$). Another possibility is to allow for a small continuum term in the amplitude, and to fix the mass to the PDG value (model M1). Here we achieve a perfect description of the data with $\chi^2/ndf = 61/77$, $\Gamma = 142 \pm 2 \text{ MeV}$, $\lambda = 1.75 \pm 0.08$ and $\phi = (111.4 \pm 3.2)^\circ$, see solid curve in fig. 3. Note that the continuum cross section needed is tiny ($0.006 \mu\text{b}$ at the ρ mass); it is the interference term which is changing the details in the peak region. It is not clear whether such a term is theoretically justified, and for our argumentation of the continuum in the η/η' decays stemming from the box anomaly the need to introduce a

Figure 4. Results of fits for $E_{\eta'}$, using ρ parametrization M1

non-explained continuum into $e^+e^- \rightarrow \pi^+\pi^-$ is not very satisfactory.

FITS TO η AND $\eta' \rightarrow \gamma\pi^+\pi^-$

Using the two models M1 and M2 outlined above, we fitted all available $\pi^+\pi^-$ mass spectra. Some of the spectra were available only without acceptance correction; we tried to reconstruct corrected spectra wherever this was possible from formulae and curves given in the publications, for details see [1]. For the η decay, we used the data of Layter et al. [5], and for the η' data from PLUTO, TASSO, TPC/2 γ , CELLO, ARGUS, Grigorian et al., Lepton F and WA 76 (for references see [1]). Although there is some scattering in these data, the mean values extracted for $E_{\eta'}$ using both models are clearly incompatible with zero in both ρ parametrization models. The results are:

<i>model M1</i>	<i>model M2</i>
$E_\eta = -5.0 \pm 1.5$	$E_\eta = -4.5 \pm 0.7$
$E_{\eta'} = -5.1 \pm 0.7$	$E_{\eta'} = -2.1 \pm 0.5$

We recently learned that there are additional large clean η' samples from DM2 and Mark III in the reaction $J/\psi \rightarrow \gamma\gamma\pi^+\pi^-$ whose $\pi^+\pi^-$ mass spectra should be analyzed. Also the

data now taken at the BES experiment can improve the experimental situation soon.

QUANTITATIVE ANALYSIS

We now insert the experimental values into the four equations for B_η , $B_{\eta'}$, E_η and $E_{\eta'}$. First we treat ξ as a free parameter, with the result $\xi = 1.2 \pm 0.2$ in model M1 and $\xi = 2.7 \pm 0.7$. Since ξ can only take the values 1 or 2, we see that we achieve internal consistency in our model if either model M1 (i.e. a small continuum exists in $e^+e^- \rightarrow \pi^+\pi^-$) is correct and $\xi = 1$, i.e. quarks are fractionally charged; or model M2 (i.e. the ρ mass is almost as high as the ω mass) and $\xi = 2$, i.e. quarks are integrally charged. In both these cases, the results for the pseudoscalar nonet parameters are:

quantity	model M1	model M2
f_π/f_1	$0.96^{+0.03}_{-0.05}$	0.48 ± 0.02
f_π/f_8	$0.46^{+0.43}_{-0.38}$	0.58 ± 0.16
$\theta_{PS} [^\circ]$	$-26.7^{+9.6}_{-8.4}$	-24.1 ± 4.2
ξ	1	2
χ^2/ndf	0.54/1	0.03/1
χ^2/ndf alternate ξ	12/1	42/1

Thus, the parameters determined in both models are consistent with the expectations from e.g. mass formulae, chiral perturbation theory etc. (see [1]). From our model, it is presently impossible to distinguish between these solutions. In our view the unexpectedly large freedom in the ρ parameter determination and the apparent discrepancy between the precise $e^+e^- \rightarrow \pi^+\pi^-$ data and the PDG parameters without some additional contribution is an interesting question itself and needs further investigation.

SUMMARY

We have shown that a small continuum in addition to the dominating ρ resonance considerably improves fits to $\pi^+\pi^-$ mass spectra

in $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$ decays. We interpret this continuum as stemming from the box anomaly and evaluate the pseudoscalar coupling constants in the framework of Chanowitz equations. Using two possible ρ parametrizations, we get two solutions; one with $\xi = 1$ (conventional QCD) and another one with $\xi = 2$ (integer charged quarks). Both these solutions are self-consistent (4 equations, 3 parameters), and the parameters f_1 , f_8 and θ_{PS} are in the range expected from other determinations.

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