SUPER SELF-DUALITY AS ANALYTICITY IN HARMONIC SUPERSPACE

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Abstract

A twistor correspondence for the self-duality equations for supersymmetric Yang-Mills
theories is developed. Their solutions are shown to be encoded in analytic harmonic super-
fields satisfying appropriate generalised Cauchy-Riemann conditions. An action principle
yielding these conditions is presented.

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1. There has recently been a revival of interest in self-duality equations, arising from numerous confirmations of a remarkable suggestion [1] that all integrable systems are obtainable by dimensional reduction from 4D self-dual theories. The purpose of this letter is to show that for supersymmetric Yang-Mills theories [2] the self-duality equations can be written in a form in which their integrability becomes manifest and their solutions can be constructed in terms of superfields which are in a definite sense holomorphic. In other words, we shall establish the so-called “twistor correspondence” for the supersymmetric self-duality equations analogous to that for the ordinary (N = 0) self-duality equations [3], which in the harmonic space language [4-6] involves a splitting of the coordinate $x^{\alpha \dot{\alpha}}$ into $x^{+\alpha} = u^\beta_{\beta} x^{\alpha \beta}$ and $x^{-\alpha} = u^-_{\dot{\beta}} x^{\alpha \dot{\beta}}$, where $u^\pm_{\beta}$ are harmonics on the two-sphere, which appears in the harmonisation of the rotation group [4-6], and $\alpha$ and $\dot{\beta}$ are two-spinor indices. The gist of ordinary self-duality is the Cauchy-Riemann-like equation

$$\nabla^+_\alpha \phi = 0,$$

where $\nabla^+_\alpha$ is the covariant derivative in $x^{-\alpha}$. We shall show that this construction can be extended naturally to supersymmetric gauge theories. In the N = 1 case we shall use the harmonic superspace with coordinates

$$x^{\pm \alpha} \equiv u^\pm_{\beta} x^{\alpha \beta}, \quad \vartheta^{\pm} \equiv u^\pm_{\alpha} \bar{\vartheta}^{\dot{\alpha}}, \quad u^\pm_{\alpha}.$$

Now super-self-duality is the condition for the integrability of the equation

$$\bar{D}^+ \phi = 0,$$

where $\bar{D}^+$ is the gauge-covariant spinorial derivative with respect to the variable $\bar{\vartheta}^-$. N = 1 supersymmetric gauge theories respect chirality, so we may, without loss of generality, take the field $\phi$ in (3) to be a chiral superfield $\phi(x^{+\alpha}, x^{-\alpha}, \bar{\vartheta}^+, \bar{\vartheta}^-)$ independent of $\vartheta^\alpha$, i.e.

$$D_\alpha \phi = 0,$$

where $D_\alpha$ is the covariant spinorial derivative with respect to the variables $\vartheta^\alpha$, which may be made flat by definition; and we shall set, throughout this letter, the corresponding connections to zero,

$$A_\alpha = 0.$$

Of importance is the fact that consistency of (3) and (4) implies (1) in virtue of the algebra of spinorial derivatives, so N = 1 self-duality implies the usual N = 0 self-duality. We shall demonstrate that all solutions of the supersymmetric self-duality conditions are encoded in a “holomorphic” chiral superfield which satisfies generalised Cauchy-Riemann conditions; in other words, we shall establish the famous twistor correspondence [3] for the supersymmetric self-duality equations. By holomorphicity we mean that there is a basis in which this superfield is independent of $x^{-\alpha}, \bar{\vartheta}^-$ and $\vartheta^\alpha$. The resulting formulation greatly simplifies the problem of constructing gauge superpotentials $A_{\alpha \dot{\beta}}, A_{\dot{\beta}}$ solving the
super-self-duality equations. It also helps in the search for an action principle for super-self-duality.

Although our considerations are rigorous only for 4D Euclidean space, we shall remain in the complexified picture with Lorentz group $SL(2, \mathbb{C})_L \times SL(2, \mathbb{C})_R$, with $\alpha$ and $\dot{\alpha}$ labelling fundamental representations of $SL(2, \mathbb{C})_L$ and $SL(2, \mathbb{C})_R$, respectively. In [6] it was shown that consideration of the complexified picture is required by conformal invariance of the self-duality equations. Reality conditions appropriate for the required signature of four dimensional real space may be imposed; e.g. by identifying undotted and dotted spinors as representations of the two different $SU(2)$’s in the 4D Lorentz group $SU(2)_L \times SU(2)_R$ corresponding to a Euclidean signature; or by identifying them as representations of two different $SL(2, \mathbb{R})$’s, with Lorentz group $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ corresponding to a $(2,2)$ signature. In the latter case we expect intriguing peculiarities due to the non-compactness of $SL(2, \mathbb{R})$ and an appropriate harmonic space needs to be considered (see e.g. [7]).

2. In complexified superspace $\mathcal{M}_{4|4}$ of complex dimension $(4|4)$ with coordinates $(x^{\alpha\dot{\beta}}, \vartheta^\alpha, \bar{\vartheta}^{\dot{\alpha}})$, the $N = 1$ super Yang-Mills theory is conventionally described in terms of two spinorial field strengths $w_\alpha, \bar{w}_{\dot{\alpha}}$ defined by

$$[\bar{D}_{\dot{\alpha}}, D_{\alpha\dot{\alpha}}] = \epsilon_{\dot{\beta} \dot{\alpha}} w_\alpha$$
$$[D_\beta, D_{\alpha\dot{\alpha}}] = \epsilon_{\beta \alpha} \bar{w}_{\dot{\alpha}},$$

where the gauge-covariant derivatives $D_A \equiv \partial_A + A_A = (D_{\alpha\dot{\beta}}, D_\alpha, \bar{D}_{\dot{\alpha}})$ satisfy the familiar constraints

$$\{D_\alpha, D_\beta\} = 0 \quad (7a)$$
$$\{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \quad (7b)$$
$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = 2 D_{\alpha\dot{\beta}} \quad (7c)$$

and the supertranslations

$$\partial_A = (\partial_{\alpha\dot{\beta}}, D_\alpha, \bar{D}_{\dot{\beta}}) \equiv (\frac{\partial}{\partial x^{\alpha\dot{\beta}}}, \frac{\partial}{\partial \vartheta^\alpha}, \frac{\partial}{\partial \bar{\vartheta}^{\dot{\beta}}} + 2 \vartheta^\alpha \frac{\partial}{\partial x^{\alpha\dot{\beta}}},)$$

realise the free superalgebra

$$[D_\beta, D_{\alpha\dot{\beta}}] = [\bar{D}_{\dot{\alpha}}, D_{\alpha\dot{\beta}}] = \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0$$
$$[D_\alpha, D_{\dot{\beta}}] = 2 \partial_{\alpha\dot{\beta}}.$$

The self-duality equations for the superconnection $A_A$ take the form of the following further (in comparison with (6)) constraints

$$[D_{\beta}, D_{\alpha\dot{\alpha}}] = 0,$$

which say that $\bar{w}_{\dot{\alpha}}$ vanishes. That this is the supersymmetrisation of the usual ($N = 0$) self-duality condition is evident from the dimension $(1|1)$ Jacobi identity which yields the following superfield equations

$$f_{\dot{\alpha}\dot{\beta}} = 0 \quad (a), \quad D_\alpha w_\beta = 2 f_{\alpha\beta} \quad (b)$$
for the self- and anti-dual vector field strengths \( f_{\dot{\alpha}\dot{\beta}} \), \( f_{\alpha\beta} \) appearing in the definition

\[
[D_{\alpha\dot{\alpha}}, D_{\beta\dot{\beta}}] \equiv \epsilon_{\alpha\beta} f_{\dot{\alpha}\dot{\beta}} + \epsilon_{\dot{\alpha}\dot{\beta}} f_{\alpha\beta}.
\]

The dimension \( (3,1|1) \) and \( (1,3|2) \) Jacobi identities are then satisfied, respectively, if \( w_\alpha \) and \( f_{\alpha\beta} \) satisfy the (anti-) chirality equations

\[
D_\gamma f_{\alpha\beta} = 0 \quad (c), \quad \bar{D}_\dot{\beta} w_\alpha = 0 \quad (d).
\]

All other Jacobi identities are then automatic, requiring the introduction of no further superfield strengths. Equations (8) are the superfield self-duality equations. They indeed imply the equations of motion \( \epsilon^{\gamma\alpha} D_\gamma f_{\alpha\beta} = \epsilon^{\gamma\alpha} D_\gamma w_\alpha = 0 \). We have therefore shown that the constraints (7), (8) for \( A_A \) imply the superfield equations (9) for the superfield-strength \( w_\alpha \) and superfield vector-potential \( A_{\alpha\dot{\beta}} \). These in turn imply the ordinary space supersymmetric self-duality equations for the component fields (which we denote by the same symbols as the superfields of which they are the leading components)

\[
f_{\dot{\alpha}\dot{\beta}} = 0, \quad \epsilon^{\gamma\alpha} D_\gamma w_\alpha = \bar{w}_{\dot{\alpha}} = 0,
\]

on eliminating all gauge degrees of freedom depending on the anticommuting superspace coordinates. The converse, that given a set of component fields satisfying these component equations, one can reconstruct superfields satisfying (9), and in turn the superconnection \( A_A \) satisfying (7), (8) is also true. The proof closely follows the methods of [8].

3. Our main purpose here, however, is to introduce yet another piece of data corresponding to the above three: a ‘holomorphic’ prepotential in harmonic superspace. We shall show that the constraints (7), (8) imply generalised Cauchy-Riemann (CR) conditions for a prepotential in harmonic superspace and that any superconnection satisfying (7), (8) may be expressed in terms of such holomorphic prepotentials. (We shall use ‘holomorphic’ in this generalised sense; to describe solutions of these generalised CR conditions). This construction is a realisation of the twistor construction [3] for supersymmetric self-dual systems in the harmonic superspace framework.

In the present complex setting, the harmonics \( u^\pm_\alpha \) remain, as usual, \( S^2 \) harmonics, the 2-sphere being a coset of the \( SL(2,\mathbb{C})_R \) part of the Lorentz group with its maximal parabolic subgroup. In this setting, \( u^+ \) and \( u^- \) are not complex conjugates of each other; and are defined up to parabolic subgroup transformations. They obey the usual constraints \( u^+\dot{\alpha} u^-_{\dot{\alpha}} = 1 \). Details of this construction, the role of conformal invariance, as well as appropriate reality conditions may be found in [6]. For our \( N = 1 \) harmonic superspace the derivatives \( \partial^\pm_\alpha \equiv u^\pm\dot{\alpha} \partial_{\alpha\dot{\alpha}} \), \( \bar{D}^+ = u^+\dot{\alpha} \bar{D}_{\dot{\alpha}} \), \( D_\alpha \), together with harmonic ones

\[
D^{\pm} = u^{\pm\dot{\alpha}} \frac{\partial}{\partial u^{\pm\dot{\alpha}}}, \quad D^0 = u^{+\dot{\alpha}} \frac{\partial}{\partial u^{+\dot{\alpha}}} - u^{-\dot{\alpha}} \frac{\partial}{\partial u^{-\dot{\alpha}}},
\]

realise the free superalgebra

\[
\{D_\alpha, \bar{D}^\pm\} = 2 \partial^\pm_\alpha, \quad [D^{++}, D^{--}] = D^0, \quad [D^0, D^{\pm\pm}] = \pm 2 D^{\pm\pm} \quad (10a)
\]

\[
[D^{++}, D^{--}] = D^0, \quad [D^0, D^{\pm\pm}] = \pm 2 D^{\pm\pm} \quad (10b)
\]
\[ [D^{\pm\pm}, \bar{D}^\mp] = \bar{D}^{\pm}, \quad [D^{\pm\pm}, \partial_\alpha^\mp] = \partial_\alpha^{\pm}, \quad (10c) \]

with all other commutators vanishing. Now, in terms of the gauge-covariant derivatives

\[ \nabla_\alpha^+ = \partial_\alpha^+ + A^+_\alpha = u^{+\hat{\alpha}} D_{\alpha \hat{\alpha}}, \quad \bar{D}^+ = \bar{D}^+ + \bar{A}^+ = u^{+\hat{\alpha}} \bar{D}_{\alpha}, \quad D_\alpha = D_{\alpha}, \]

(recall that \( A_\alpha = 0 \)) obeying the commutation relations

\[ [D^{++}, \bar{D}^+] = 0, \quad (11a) \]
\[ [D^{\pm\pm}, D_\alpha] = 0, \quad (11b) \]
\[ [D^{++}, \nabla_\alpha^+] = 0, \quad (11c) \]

the constraints (7), (8) are equivalent to the Cauchy-Riemann system

\[ [\bar{D}^+, \nabla_\alpha^+] = 0, \quad (12a) \]
\[ [D_\beta, \nabla_\alpha^+] = 0, \quad (12b) \]
\[ [\bar{D}^+, D_\alpha] = 2 \nabla_\alpha^+. \quad (12c). \]

Remarkably, these are precisely the integrability conditions for equations (1), (3) and (4). We therefore have the following pure-gauge-like expressions for \( A^+_\alpha \) and \( \bar{A}^+ \)

\[ A^+_\alpha = -\partial_\alpha^+ \phi \phi^{-1}, \quad \bar{A}^+ = -\bar{D}^+ \phi \phi^{-1}. \quad (13) \]

4. Equations (11), (12) are therefore equivalent to the constraints (7), (8). Let us now choose a coordinate basis, which we shall call the analytic frame, in which the derivatives take the forms

\[ \hat{D}_\alpha = \phi^{-1}[D_\alpha] \phi = \frac{\partial}{\partial y^\alpha}, \]
\[ \hat{\nabla}_\alpha^+ = \phi^{-1}[\nabla_\alpha^+] \phi = \frac{\partial}{\partial x^{-\alpha}}, \]
\[ \hat{D}^+ = \phi^{-1}[\bar{D}^+] \phi = \frac{\partial}{\partial y} + 2 \phi \frac{\partial}{\partial x^{-\alpha}}, \quad (14) \]
\[ \hat{D}^{++} = \phi^{-1}[D^{++}] \phi = D^{++} + V^{++}, \]
\[ \hat{D}^{--} = \phi^{-1}[D^{--}] \phi = D^{--} + V^{--}. \]

In this basis the covariant derivatives \( \hat{D}^+ \) and the \( \hat{\nabla}_\alpha^+ \) become flat, losing their connections (\( \hat{D}_\alpha \) remains flat), while the harmonic derivatives \( D^{\pm\pm} \) clearly acquire the connections

\[ V^{++} = \phi^{-1} D^{++} \phi, \quad (15a) \]
\[ V^{--} = \phi^{-1} D^{--} \phi. \quad (15b) \]
In order to preserve the operator $D^0$ as a charge counting operator, we have used (as usual, see e.g. [3]) the conventional gauge in which it does not acquire a connection:

$$[\hat{D}^{++}, \hat{D}^{--}] = D^0. \quad (16)$$

In this basis the dynamical content is contained entirely in (11), the rest of the equations being kinematical. Using the identity

$$A(Bff^{-1}) \equiv f(B(f^{-1}Af))f^{-1} + [A, B]ff^{-1},$$

for arbitrary differential operators $A$ and $B$, eqs. (11) take the form of generalised CR conditions

$$\frac{\partial}{\partial \bar{\vartheta}^{-}} V^{++} = 0,$$

$$\frac{\partial}{\partial \vartheta^{\alpha}} V^{\pm \pm} = 0,$$

$$\frac{\partial}{\partial x^{- \alpha}} V^{++} = 0. \quad (17)$$

$V^{++}$ is therefore holomorphic: it depends on $x^{+ \alpha}$, $\vartheta^{+}$ and $u^{\pm}$, being independent of $x^{- \alpha}$, $\bar{\vartheta}^{-}$ and $\vartheta^{\alpha}$; both $V^{++}$ and $V^{--}$ are chiral. Note that the third equation is a consequence of the other two. We have shown that to any solution of the super self-duality constraints (8), there corresponds a chiral holomorphic superfield $V^{++}$ taking values in the gauge algebra and having component expansion

$$V^{++}(x^{+ \alpha}, \vartheta^{+}, u^{\pm}_{\dot{\alpha}}) = v^{++}(x^{+ \alpha}, u^{\pm}_{\dot{\alpha}}) + \bar{\vartheta}^{+}\chi^{+}(x^{+ \alpha}, u^{\pm}_{\dot{\alpha}}). \quad (18)$$

The superfield $V^{++}$ is defined modulo gauge transformations

$$\delta V^{++} = [\hat{D}^{++}, \lambda],$$

where $\lambda$ is an arbitrary holomorphic superfield. Note that due to the presence of the fermion mode there is an important difference with the $N = 0$ case: whereas the $N = 0$ connection was encoded in one function on the 2-sphere, in the present case we have two functions, $v^{++}$ and $\chi^{+}$, instead.

5. The converse statement, that any chiral analytic superfield prepotential $V^{++}$ encodes a superconnection $A_A$ satisfying (7), (8) also holds. To reconstruct the superconnection $A_A$ there are two options.

a) We can start with the chiral superfield $\phi$. In this case we need to solve (15a) for $\phi$; $V^{++}$ being given. Equations on the 2-sphere of this kind are not too easy to solve and they appear in many applications of the harmonic-twistor approach, see, e.g. [4]. In order to determine the corresponding superconnection solving (7), (8), we need to insert the $\phi$ thus obtained into the following formulae

$$A_{a\dot{a}} = 2 \int d^2 u \ u_{\dot{a}}^{-} \phi \partial_{a}^{+} \phi^{-1}, \quad A_{\dot{a}} = 2 \int d^2 u \ u_{a}^{-} \phi D^{+} \phi^{-1}, \quad (19)$$
which follow immediately from (13); and $A_\alpha$ is of course zero.

b) Instead of $\phi$, we can start with the harmonic connection $V^{--}$. It follows from (16) that

$$Z \equiv D^{++}V^{--} - D^{--}V^{++} + [V^{++}, V^{--}] = 0. \quad (20)$$

For a given holomorphic $V^{++}$ taking values in the gauge algebra, it contains a set of coupled first-order linear equations for the gauge algebra components of $V^{--}$. These may be solved [9] somewhat more easily than (15a). Now, as further consequences of gauge-covariantising the harmonic derivatives (14), we have, from (10),

$$[\hat{D}^{--}, \hat{D}^{-+}] = \hat{D}^{--}, \quad [\hat{D}^{--}, \hat{D}^{-}] = 0, \quad [\hat{D}^{--}, \partial^{+}_\alpha] = \nabla^{-}_\alpha, \quad [\hat{D}^{--}, \nabla^{-}_\alpha] = 0, \quad (21)$$

from which we obtain superconnections

$$\hat{A}^{-} = -\hat{D}^{+}V^{--}, \quad A^{-}_\beta = -\partial^{+}_\beta V^{--}, \quad (22)$$

in terms of solutions of (20). Now, superconnections satisfying (7), (8) may be recovered by harmonic integration similar to (19). In fact, from $V^{--}$ we may also directly construct the superfield strength $w_\alpha$ satisfying the superfield equations (9); namely, from (6),

$$w_\alpha = -[\hat{D}^{+}, \nabla^{-}_\alpha] = \hat{D}^{+}\partial^{+}_\alpha V^{--}. \quad (23)$$

An alternative to equation (20) follows from the following commutators contained in the superalgebra (10) (in the analytic frame)

$$[\hat{D}^{-}, \nabla^{-}_\alpha] = 0 = [\nabla^{-}_\alpha, \nabla^{-}_\beta].$$

These yield the alternative equations for $V^{--}$:

$$L^{--} \equiv -\partial^{+}_\alpha \hat{D}^{-}V^{--} + \partial^{-}_\alpha \hat{D}^{+}V^{--} + [\hat{D}^{--}, \partial^{+}_\alpha V^{--}] = 0, \quad (24)$$

$$L^{-} \equiv \partial^{+\alpha} \partial^{-}_\alpha V^{--} + [\partial^{+\alpha}V^{--}, \partial^{+}_\alpha V^{--}] = 0. \quad (25)$$

The latter equation is in fact the one introduced for the $N = 0$ case in [10]. We note the following interesting interrelations amongst the left-hand-sides of equations (20), (24) and (25):

$$\hat{D}^{+}L^{-} = \partial^{+\alpha}L^{-}_\alpha, \quad \nabla^{-}_\alpha \partial^{+\alpha}Z = \hat{D}^{++}L^{-}, \quad \nabla^{-}_\alpha \hat{D}^{+}Z = \hat{D}^{++}L^{-}_\alpha.$$

6. We now present an action for super self-duality. Since all we need to have is the generalised CR conditions (17), with $V^{++}$ expressed in terms of the field $\phi$, as a variational equation, we plug this condition into an action functional with the help of a Lagrange multiplier-type auxiliary field. The latter does not propagate if it contains only gauge degrees of freedom. For the chiral superfield $\phi$, the action functional

$$S = \int d^4x \ d^2\bar{\theta} \ du \ \text{tr} \ (D^{+}\zeta^{-3}\phi^{-1}D^{++}\phi) \quad (26)$$
yields, on varying the auxiliary field $\zeta^{-3}$, the CR condition $\bar{D}^+ V^{++} = 0$, $V^{++}$ is chiral by definition; and the final condition $\frac{\partial}{\partial x^-} V^{++} = 0$ is a consequence. Now, on varying $\phi$, we obtain

$$-\phi^{-1} D^{++} [\phi \bar{D}^+ \zeta^{-3} \phi^{-1}] = 0. \quad (27a)$$

It follows (cf. [11]) that

$$\bar{D}^+ \zeta^{-3} = 0. \quad (27b)$$

All solutions of this equation have the form

$$\zeta^{-3} = \bar{D}^+ y^{-4}. \quad (28)$$

However $\zeta^{-3}$ enters the action via $\bar{D}^+ \zeta^{-3}$, so it is only defined modulo the addition of $\bar{D}^+ y^{-4}$. $\zeta^{-3}$ therefore does not represent any additional physical degree of freedom. For $N = 0$ an analogous action was discussed in [11,12].

Alternatively, we may choose $V^{--}$ (instead of $\phi$) as the dynamical field, express $V^{++}$ in terms $V^{--}$ with the help of (20), and construct an action having analyticity conditions for the functional $V^{++}[V^{--}]$ as variational equations.

There also exists the possibility (analogous to the $N = 0$ action considered in [10]) of writing an action for eq.(25) trilinear in $V^{--}$. It explicitly contains a constant harmonic factor of charge $+4$, say $(u_1^+ u_2^+)^2$, and is consequently not Lorentz invariant.

7. To conclude we generalise our construction to $N$-extended harmonic-superspace with coordinates $x^\pm$, $\varphi^{\alpha i}$, $\bar{\varphi}^j_\beta$, $u_\pm^\pm$, where $i, j = 1, \ldots, N$. Solutions of the generalised CR conditions

$$\frac{\partial}{\partial \bar{\varphi}^j_\beta} (\phi^{-1} D^{++} \phi) = 0,$$

$$\frac{\partial}{\partial \varphi^{\alpha i}} (\phi^{-1} D^{\pm\pm} \phi) = 0,$$

$$\frac{\partial}{\partial x^-} (\phi^{-1} D^{++} \phi) = 0,$$

encode $N$-extended self-dual superconnections. It may be verified that the superconnection components

$$A_{\alpha \dot{\alpha}} = \int d^2 u^- u_\alpha^- \phi \bar{\varphi}^+_\alpha \phi^{-1},$$

$$A^j_\dot{\alpha} = \int d^2 u^- u_\alpha^- \phi \bar{D}^+_j \phi^{-1},$$

$$A_{\dot{\alpha} i} = 0,$$

with $\phi$ satisfying (29), automatically satisfy the self-dual restrictions of the conventional extended superconnection constraints

$$F_{i\alpha j\beta} = 0 = F^{ij}_{\dot{\alpha} \beta} + F^{ij}_{\beta \dot{\alpha}}$$

$$F^i_{\alpha \beta j} = 0.$$
That integrability conditions for these equations yield (29) follows from reasoning parallel to that for the N=1 case above.

For the full (non-self-dual) N = 2 and 3 theories, for which harmonic superspace formulations (harmonising the internal automorphism group) exist [4,6], the self-duality conditions are also equivalent to “double” analyticity conditions arising from Lorentz as well as internal automorphism group harmonisation. We intend to return to these theories elsewhere.

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