The pseudoscalar mixing angle θ_{PS} from η and η' production in $\bar{p}p$ annihilation at rest

The Crystal Barrel Collaboration

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Abstract

We have determined ratios of branching ratios for $\bar{p}p$ annihilations at rest into $\pi^0\eta$ and $\pi^0\eta'$; into $\eta\eta$ and $\eta\eta'$; and into $\omega\eta$ and $\omega\eta'$. All reactions are observed in two different all-neutral final states. The ratios permit a determination of the pseudoscalar mixing angle. We find $\Theta_{\text{PS}} = -(17.3 \pm 1.8)^\circ$.

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We report on a derivation of the pseudoscalar mixing angle by comparison of the branching ratios for $\bar{p}p \rightarrow \eta X$ and $\eta' X$ with $X = \pi^0$, η , and ω making the assumption that the initial $\bar{p}p$ state couples only to the $u\bar{u} + d\bar{d}$ component of the η and η' wave functions. We refer to this assumption as the quark line rule. In support of the quark line rule, we notice that the branching ratios for $\bar{p}p$ annihilation into ϕX are small compared to the corresponding branching ratios into ωX [1]. The recent discussion of a possible $s\bar{s}$ component in the proton wave function at small momentum transfers is also relevant in this context (see, e.g. [2, 3] for recent reviews).

The two isoscalar mesons η and η' could possibly mix with their radial excitations [4] or with the lowest-mass pseudoscalar glueball [5]. The latter mixing scheme would require an additional glue-component in the η and η' wave functions. Data on η and η' production in $J/\psi(1S)$ decays into a vector (V) and a pseudoscalar (PS) meson indeed originally suggested a sizable glue component in the η' wave function [6]; but a later analysis showed that the inclusion of doubly OZI suppressed decays is needed to fit the various branching ratios and that a glueball component of the η or η' is not required [7, 8].

The η and η' wave functions are given by

$$|\eta\rangle = X_\eta \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + Y_\eta |s\bar{s}\rangle \quad (1)$$

$$|\eta'\rangle = X_{\eta'} \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + Y_{\eta'} |s\bar{s}\rangle \quad (2)$$

The ratio of the branching ratios for $\bar{p}p$ annihilation into $\eta' X$ to that for ηX is then, apart from phase space corrections, related to Θ_{PS} by [9]

$$r = \frac{X_{\eta'}^2}{X_\eta^2} = \tan^2 \{90^\circ + \Theta_{PS} - \Theta_{ideal}\} \quad (3)$$

For an ideal mixing angle $\Theta_{PS} = \Theta_{ideal} = 35.3^\circ$, $r = \infty$ since production of η mesons would then vanish due to the quark line rule. Note that the convention for the pseudoscalar mixing angle is to define the η meson as ‘mainly $s\bar{s}$ ’.

The branching ratios reported here have been measured with the Crystal Barrel detector at the Low-Energy Antiproton Ring (LEAR) at CERN. A detailed description of the detector is given elsewhere [10]. Here we recall only that the detector consists of central tracking chambers which are surrounded by 1380 CsI crystals covering nearly 4π . Tracking chambers and the crystal barrel are located in a magnetic field. The crystals are read out by photodiodes.

The branching ratios are determined from *all-neutral* annihilations. The restriction to *all-neutral* annihilations allowed the use of a selective trigger. The 8.2×10^6 events recorded with this trigger correspond to a total of 204 million antiproton annihilations. In these data the ω is observed through its $\pi^0\gamma$ decay mode, the η through its decay into either $\gamma\gamma$ or $\pi^0\pi^0\pi^0$, and the η' through the decay modes into $\gamma\gamma$ or $\pi^0\pi^0\eta$. In the offline analysis the data are replayed and any events containing tracks or track elements are rejected. Events with a photon having an impact point in a crystal close to beam entrance or exit hole of the barrel are also excluded. The data are split into data samples with different photon multiplicities; events with 4, 5, 8, and 9 photons are used for the present analysis. Energy and momentum conservation is imposed by a four-constraint (4C) kinematical fit.

Table 1: Branching ratios for η and η' production in $\bar{p}p$ annihilation at rest. The branching ratios are corrected for unseen decay modes. The errors denote the statistical (including errors from the variation of background assumptions) and the systematic uncertainties (from Monte Carlo statistics and reconstruction).

$\text{BR}(\bar{p}p \rightarrow \pi^0 \eta') / \text{BR}(\bar{p}p \rightarrow \pi^0 \eta)$	$=$	$(0.548 \pm 0.046 \pm 0.014)$
$\text{BR}(\bar{p}p \rightarrow \eta \eta') / \text{BR}(\bar{p}p \rightarrow \eta \eta)$	$=$	$(1.304 \pm 0.133 \pm 0.033)$
$\text{BR}(\bar{p}p \rightarrow \omega \eta') / \text{BR}(\bar{p}p \rightarrow \omega \eta)$	$=$	$(0.515 \pm 0.028 \pm 0.013)$

As an example we show in fig. 1 π^0 and η momentum distributions in the eight-photon final state. The invariant masses of all photon pairs are calculated, and a χ^2 method is used to combine pairs of photons to π^0 or η mesons. The data are then kinematically fit with eight constraints (8C) to the $4\pi^0$, $3\pi^0\eta$, or $2\pi^0 2\eta$ hypothesis imposing the four meson masses in addition to energy and momentum conservation. The peaks at fixed values in the momenta spectra give the signal strengths for the two-body reactions of interest.

Detection efficiencies are determined from Monte Carlo simulations. Monte Carlo data are treated exactly like real data, and have to pass the same analysis chain. Typical detection efficiencies are 50% for reactions leading to four photons, 20% for reactions with 8 photons.

Table 1 presents our final results on ratios of branching ratios for $\bar{p}p$ annihilations into ηX and $\eta' X$ for antiprotons stopping in liquid H_2 . The ratios are corrected for unseen meson decay modes using the decay branching ratios of the Particle Data Group [11]. Note that they are derived from two independent run periods and two different final states for η and η' decay. All results, in particular those with different meson decay modes, are fully consistent. Most systematic errors cancel in the determination of double ratios, therefore the errors are dominated by the statistical uncertainties.

In order to extract Θ_{PS} from these results, the measured branching ratios need to be corrected for the different phase space for η or η' production. Also dynamical corrections may not be negligible. Therefore r is not directly given by the ratio of branching ratios into $\eta' X$ and ηX . We write

$$r = \frac{\text{BR}(\eta' X)}{\text{BR}(\eta X)} \cdot \frac{f_{\text{dyn}}(\eta X)}{f_{\text{dyn}}(\eta' X)} \quad (4)$$

where f_{dyn} are dynamical factors which we shall now discuss. We try different dynamical factors and look for consistency between mixing angles calculated from $X = \pi^0$, η , or ω .

Results are collected in table 2. The first column lists the dynamical assumptions (5) to (11) which are explained in (a - f) below, and the second column gives a reference for f_{dyn} . The next columns give the corrected double ratio r defined in (4) for $X = \pi^0$, η , and ω . The branching ratio $\bar{p}p \rightarrow \eta \eta$ is multiplied by 2 to account for the two identical bosons. The weighted mean value of the three values and the corresponding mixing angle are given. The statistical error in Θ_{PS} is $\pm 0.7^\circ$. Finally we give the χ^2 per degree of freedom ($N_F = 2$); this estimates whether the three measurements are consistent. This consistency is a test of the dynamical assumption.

a) The dynamical factor f_{dyn} should contain two components, the phase space factor q and the momentum dependence of the amplitude T for the annihilation process. For small momenta, $T \sim q^l$, and

$$f_{\text{dyn}} = q^{(2l+1)}. \quad (5)$$

Figure 1: π^0 and η momentum distributions from events with 8 photons in the final state. (a) π^0 momentum distribution of events with four π^0 and (b) with three π^0 and one η . (c) η momentum distribution of events with three π^0 and one η and for events with two π^0 and two η mesons (d). The insets show expanded views of the signals and fits to the data. The numbers of signal events and their errors are listed.

Table 2: The table lists the dynamical assumptions (with references) which were used to correct the measured ratios of branching ratios for annihilation into ηX and $\eta' X$ with $X = \pi^0$, η , and ω . For correct dynamical assumptions we expect three compatible values of the corrected ratios r .

Equation	reference	$r(X = \pi^0)$	$r(X = \eta)$	$r(X = \omega)$	mean	Θ_{PS}	χ^2/N_F
(5)	[9]	1.177 ± 0.103	1.683 ± 0.178	3.381 ± 0.204	1.638 ± 0.082	-2.7°	46.7
(6,7)	[12]	0.719 ± 0.063	1.023 ± 0.108	1.107 ± 0.067	0.920 ± 0.042	-10.9°	9.5
(8)	[13, 14]	0.681 ± 0.060	0.909 ± 0.096	0.964 ± 0.058	0.839 ± 0.038	-12.2°	6.1
(9)	[15]	0.724 ± 0.063	0.937 ± 0.099	0.964 ± 0.058	0.868 ± 0.039	-11.8°	4.2
(10)	[16]	0.548 ± 0.048	0.652 ± 0.069	0.515 ± 0.031	0.541 ± 0.024	-18.4°	1.7
(11)	[17]	0.522 ± 0.046	0.560 ± 0.059	0.459 ± 0.028	0.487 ± 0.022	-19.8°	1.6

Annihilation into $\eta\omega$ and $\eta'\omega$ can occur from the $^3S_1$ and $^1P_1$ states, but here we neglect a possible contribution of P-wave annihilation [18]. The angular momentum between the two outgoing mesons is then $l = 1$ and the dynamics are sensitive to centrifugal barrier factors. Annihilations into two pseudoscalar mesons are allowed from the $^3P_0$ and $^3P_2$ states of $\bar{p}p$ atoms requiring $l = 0$ and $l = 2$, respectively. We assume that the two states contribute to the process according to their statistical weights. The $\chi^2/N_F = 46.7$ for this case is unacceptably high. The assumption that only the $^3P_0$ or $^3P_2$ state contributes to annihilation into two pseudoscalar mesons gives χ^2/N_F values of 81 and 30, respectively.

b) Next we consider the possibility that f_{dyn} is given by a Blatt–Weisskopf penetration factor [12]. For $l = 1$ it implies

$$f_{\text{dyn}} = \frac{(qR)^2}{1 + (qR)^2} \cdot q \quad (6)$$

and for $l = 2$

$$f_{\text{dyn}} = \frac{(qR)^4}{((qR)^2 - 3)^2 + 9(qR)^2} \cdot q \quad (7)$$

For small $q \ll \frac{1}{R}$ equations 6 and 7 are equivalent to equation 5. With the choice of $R = 1.2 \text{ fm}$ and a statistical mixture of the $^3P_0$ and $^3P_2$ states we find $\chi^2/N_F = 9.5$. For the $^3P_0$ state only, $\chi^2/N_F = 11.4$, for the $^3P_2$ state $\chi^2/N_F = 9.1$. A change of the range parameter to $R = 0.8 \text{ fm}$ yields very similar results.

c) Hartmann, Klempt, and Körner [13] and Carbonell, Ihle, and Richard [14] use only the phase space factor q and no angular momentum barrier factor:

$$f_{\text{dyn}} = q \quad (8)$$

The model still does not work satisfactorily even though the χ^2 is already considerably smaller. The smaller χ^2 suggests that angular momentum factors are inappropriate due to strong long-range attractive potentials which overwhelm the centrifugal barrier.

d) Mundigl, Vicente Vacas, and Weise [15] use in their calculations of branching ratios

$$f_{\text{dyn}} = q \cdot E_a \cdot E_b \quad (9)$$

where E_a and E_b are the (total) energies of the two outgoing mesons. Although an improvement on parameterization 5 to 8, the three double ratios are still inconsistent.

e) Obviously low-energy $\bar{p}p$ annihilations do not favour high momenta. Therefore Klempt [16] used a very simple dynamical model to extract the isospin decomposition of the $\bar{p}p$ atomic wave function;

$$f_{\text{dyn}} = 1 . \quad (10)$$

This assumption yields a consistent result for Θ_{PS} .

f) We also try a model suggested by Vandermeulen [17]. He assumes that the annihilation process prefers small energy transfers to the outgoing mesons. In his model, for two mesons with masses m_a and m_b ,

$$f_{\text{dyn}} = q \cdot \exp\{A(s - (m_a + m_b)^2)^{\frac{1}{2}}\} \quad (11)$$

where $s = 4m_p^2$ and $A = -1.2 \text{ GeV}^{-1}$ is obtained from a fit to data. With this assumption, he obtains a good description of $\bar{p}p$ annihilation in flight into multi-particle final states. We find that this model gives a consistent result for Θ_{PS} .

g) By varying the parameter A in (11) we find a best fit to the data with $A = -0.83 \text{ GeV}^{-1}$, $\chi^2 = 1.1$ ($N_F = 1$) and

$$\Theta_{\text{PS}} = -(17.3 \pm 1.8)^\circ . \quad (12)$$

We adopt this value as final result for Θ_{PS} . We point out that there is some model dependence in this result. For example, phase shifts for final state interactions could affect the result and so increase the error.

The test of various dynamical models has implications for $\bar{p}p$ annihilation dynamics. First we notice that the three double ratios give rather consistent results when no dynamical correction is applied. Also, the value for $\Theta_{\text{PS}} = -(18.4 \pm 0.7)^\circ$ from (10) agrees with our final result (12). The inclusion of centrifugal barrier factors of the form q^{2l+1} destroys completely this consistency; Blatt–Weisskopf corrections weaken the discrepancy but do not lead to consistency. Even the inclusion of the simple two-body phase space factor results in inconsistencies. The model (11) applies only very small corrections to (10) and gives the best result, if the parameter A is determined in a fit to our data.

Our result on Θ_{PS} is in good agreement with a determination by Gilman and Kauffman [19] of the pseudoscalar mixing angle. They find a consistent value of $\Theta_{\text{PS}} \simeq -20^\circ$ analyzing the Gell–Mann–Okubo quadratic mass formula (including one-loop chiral corrections [20]), two-photon decays of the η and η' mesons (again including chiral corrections [20]), $J/\psi(1S)$ radiative decays to η and η' mesons, $J/\psi(1S)$ decays to vector plus pseudoscalar particle, radiative decays of light mesons, π^-p scattering to $\eta/\eta'n$, and tensor meson decays. $SU(3)$ -symmetry was assumed for the $J/\psi(1S)$ radiative decays to η and η' . It was shown later [21, 22] that the inclusion of symmetry breaking effects does not change the conclusion of the mixing angle being close to -20° . A recent re-analysis [23] of (partly new) radiative and two-photon decay widths yields, however, a substantially smaller value for Θ_{PS} . Including $SU(3)$ -breaking effects in terms of the constituent quark mass ratio $m_s/\hat{m} = 1.45$, Bramon and Scadron [23] obtain $\Theta_{\text{PS}} = -14^\circ \pm 2^\circ$. Our value of the mixing angle is in fair agreement with -20° and -14° . The consistency of our result for Θ_{PS} with those obtained in other reactions indicates that (accepting the above assumptions) there is no need for mixing of η , η' with radial excitations, or with possible low mass pseudoscalar glueballs.

Summarizing, we have determined the pseudoscalar mixing angle to be $\Theta_{\text{PS}} = -(17.3 \pm 1.8)^\circ$. This value is in good agreement with other determinations. This fact supports the assumptions which have been made to arrive at our result: that the quark

line rule is valid, that there is no $s\bar{s}$ component in the proton wave function at low momentum transfers, and that there is no large dynamical corrections for $\bar{p}p$ annihilation branching ratios at rest. The centrifugal barrier does not appear to play any significant role in two-body branching ratios.

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