Global Analysis of Precision Electroweak Data
within the
Minimal Supersymmetric Extension of the Standard Model

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Abstract

We present detailed results of a global analysis of precision electroweak data within the context of the Minimal Supersymmetric Extension of the Standard Model (MSSM). We present constraints on the parameters of the MSSM in 3 planes that are commonly used in phenomenological analyses, namely the $(m_A, \tan \beta)$ plane describing the supersymmetric Higgs sector, the $(\mu, m_{\tilde{g}})$ plane describing the chargino/neutralino sector, and the $(m_0, m_{\tilde{g}})$ plane describing sfermions. In each case, we compare the indirect limits that we derive with the results of direct searches and the prospects for future experiments. We find that the indirect constraints are not yet very significant in the supersymmetric Higgs sector, although they may have the potential to become so, particularly if the top quark mass is relatively light. The indirect constraints also give only modest extensions of direct bounds on charginos and neutralinos. However, the indirect constraints already give interesting improvements of existing direct searches for sfermions, and may even pre-empt some of the physics reach of LEP II and the FNAL Tevatron Collider.

1. Introduction

It was pointed out some years ago that electroweak radiative corrections were sensitive to the masses of unseen particles [1], such as the top quark and the Higgs boson of the Standard Model. Precision experiments at LEP and elsewhere have in recent years become sensitive to these radiative corrections, and enabled m_t and M_H to be constrained [2]. Within the Standard Model, indications are that m_t is within about 20% of 130 GeV, whilst M_H is probably within an order of magnitude of the present experimental lower limit from direct searches at LEP [3]. Radiative correction analyses also give important constraints on possible extensions of the Standard Model, such as composite Higgs models and supersymmetry. Present indications seem to go against at least the simplest realizations of the composite idea [4], [5], whilst it is well known that the Minimal Supersymmetric extension of the Standard Model (MSSM) predicts the existence of a Higgs boson with a mass below about 130 GeV [6], as apparently favoured by the updating of the precision electroweak data [3]. Therefore we have embarked on a programme to include supersymmetric radiative corrections in global fits to the electroweak data, checking consistency with the MSSM and eventually constraining its parameters [5].

Electroweak radiative corrections within the MSSM have been calculated in many places [7], and also extensively reviewed [8]. Various aspects of their implications for global fits to the electroweak data have also been discussed. For example, the effects of electroweak isospin breaking due to stop and sbottom quark loops on fit estimates of m_t have been explored [9], it has been pointed out that spin- $\frac{1}{2}$ particles with masses close to $M_Z/2$ could alter the values of certain LEP observables [10], the radiative corrections due to the Higgs sector in the MSSM have been incorporated in a global fit [11], and some constraints on other MSSM parameters have been reported [3]. In this paper we incorporate all vacuum

polarization (oblique) corrections in the MSSM into a new global analysis of all the precision electroweak data available from experiments conducted before the end of 1991, as reviewed in Section 3, and discuss the resulting constraints on the parameters of the MSSM.

We perform our analysis in the MSSM treating exactly the oblique self-energy functions, and treating vertex and box diagrams, 2-loop effects and the resummation of higher orders exactly as in the Standard Model. Thus we avoid the approximations and uncertainties inherent in the model-independent approach of refs. [12], where the oblique self-energies are treated in a linear approximation, and 2-loop and resummation effects are either ignored or not treated exactly [5]. It is possible to be more precise in a well-defined model such as the MSSM which we study here. We also include the tree-level Z decays into sparticle pairs whenever they are kinematically allowed.

The gauge and Yukawa interactions of sparticles in the MSSM are identical with those in the Standard Model, and its extra parameters are mostly related to supersymmetry breaking, the exception being a Higgs superfield mixing parameter μ [13]. The masses of all the sparticles are in principle arbitrary once supersymmetry is broken, but they are in fact interrelated in a large class of models motivated by GUTs, supergravity and superstring ideas. In many such models, the different spin-0 sparticle masses are universal at the supersymmetric grand unification scale, as are the different spin- $\frac{1}{2}$ gaugino masses, and the physical masses of all the sparticles are calculable in terms of two primordial supersymmetry-breaking parameters m_0 and $m_{\frac{1}{2}}$, as we review in Section 2. As also discussed there, one must take into account the presence of supersymmetry-breaking trilinear couplings between scalar fields, but these are of limited importance for the fitting that we undertake later. The supersymmetric Higgs sector is characterized at the tree level by two additional parameters, which can be taken as the pseudoscalar Higgs boson mass m_A and the ratio $\tan\beta = v_2/v_1$ of supersymmetric Higgs vacuum expectation values (v.e.v.'s). There are also important one-loop corrections to the supersymmetric Higgs sector [6], which are mainly associated with m_t and so do

not add to the list of parameters in the MSSM. This consists essentially of the 5 parameters μ , m_0 , $m_{\frac{1}{2}}$ (or equivalently the gluino mass $m_{\tilde{g}}$), m_A and $\tan\beta$.

Several two-dimensional subspaces of this 5-parameter space have come to be used conventionally in phenomenological analyses of the MSSM. They are the $(m_A, \tan\beta)$ plane for effects in the supersymmetric Higgs sector, the $(\mu, m_{\tilde{g}})$ plane for the charged and neutral spin- $\frac{1}{2}$ sparticles (the charginos and neutralinos), and the $(m_0, m_{\tilde{g}})$ plane to compare searches for sleptons, squarks, gluinos and charginos. We analyze the constraints imposed by our new global analysis in each of these planes in subsequent Sections of this paper, mentioning in each case the sensitivity to the other MSSM parameters.

In Section 4 we discuss the constraints imposed by the precision electroweak data in the $(m_A, \tan\beta)$ plane, comparing them with the present and prospective direct limits from search experiments. We find that there are at the moment non-trivial $\Delta\chi^2 > 1$ limits on the Higgs parameter space only if m_t is below about 120 GeV. Values of m_A around 35 GeV are preferred for all values of $\tan\beta$ and m_t , but this preference is not significant for larger values of m_t . The minimum value of χ^2 is similar to that in the Standard Model, and becomes independent of m_A , and equal to that of the Standard Model for the corresponding value of the lighter scalar Higgs mass m_A , when m_A becomes large.

In Section 5 we discuss the constraints in the $(\mu, m_{\tilde{g}})$ plane and compare them with direct limits from LEP and hadron-hadron colliders. We find only modest improvements on the direct limits in this plane, reflecting the previously-mentioned fact [10] that the chargino/neutralino effects on LEP observables are significant only if the chargino mass is very close to the direct experimental limit.

In Section 6 we discuss the constraints in the $(m_0, m_{\tilde{g}})$ plane and again compare them with direct limits from LEP and hadron-hadron colliders. Although we again find only modest improvements in the limits $m_{\tilde{g}} \gg m_0$ and $m_0 \gg m_{\tilde{g}}$, we find more significant improvements at intermediate ratios of $m_0/m_{\tilde{g}}$. These already have the effect of pre-empting part of the direct discovery potential of

LEP 2 for charged sleptons, and of the FNAL $p\bar{p}$ collider for squarks.

Finally, in Section 7 we review our results and the prospects for future indirect limits as the precision of the electroweak data improves, and some uncertainties such as the mass of the top quark are possibly removed. We also give values of $\sin^2\theta_w$ in terms of the MSSM model parameters, which can then be compared with the predictions of supersymmetric GUTs.

2. Parametrization of the MSSM

In this Section we review in more detail our standard parametrization of the MSSM [13], including supersymmetric GUT renormalization effects, and describe the one-loop vacuum polarization diagrams that we have included in our global analysis.

The supersymmetric interactions of the MSSM [13] are characterized by the standard $SU(3) \times SU(2) \times U(1)$ gauge couplings and the following Yukawa superpotential written in terms of superfields

$$\mathcal{W} = \sum_{L, L^c} \lambda_L LL^c H_1 + \sum_{Q, U^c} \lambda_u QU^c H_2 + \sum_{Q, D^c} \lambda_d QD^c H_1 + \mu H_1 H_2, \quad (1)$$

where L are lepton doublets, L^c are singlet charged leptons, Q are quark doublets, U^c are charge- $\frac{2}{3}$ quark singlets, D^c are charge- $\frac{1}{3}$ quark singlets, the first three terms provide the lepton, charge- $\frac{2}{3}$ and charge- $\frac{1}{3}$ masses respectively, and the last is the Higgs mixing term mentioned in the Introduction. In addition to the superpotential (1), we must include in the Lagrangian soft supersymmetry breaking effects which we parametrize as follows:

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & - \sum_i m_{0i}^2 |\phi^i|^2 - \sum_{ijk} (A_{ijk} \lambda_{ijk} \phi^i \phi^j \phi^k + \text{h.c.}) \\ & - \sum_{ij} (B_{ij} \mu_{ij} \phi^i \phi^j + \text{h.c.}) - \sum_{a=1}^3 M_a \tilde{V}_a \tilde{V}_a, \end{aligned} \quad (2)$$

where the ϕ^i are generic scalar fields, the m_{0i} are spin-0 masses, the A_{ijk} and B_{ij} are trilinear and bilinear soft supersymmetry-breaking parameters, and the M_a ($a = 3, 2, 1$) are the masses of the SU(3), SU(2) and gauginos $\tilde{V}_a$. The bilinear parameter B_μ corresponding to the μ term in (1) is irrelevant for our purposes, and we will not discuss it further. In many supersymmetric GUT models based on supergravity or inspired by string ideas, the m_{0i} , A_{ijk} and M_a are each universal at the grand unification scale. We denote the universal scalar and gaugino masses by m_0 , $m_{\frac{1}{2}}$ respectively. The physical values of the soft supersymmetry breaking parameters are then renormalized in calculable ways by the gauge interactions in particular.

For the physical gaugino masses, one has simply

$$M_a = \frac{\alpha_a}{\alpha_{\text{GUT}}} m_{\frac{1}{2}}, \quad (3)$$

which in particular for the gluino reads $m_{\tilde{g}} = (\alpha_s/\alpha_{\text{GUT}}) m_{\frac{1}{2}}$. The SU(2) and U(1) gauginos mix with higgsinos through well-known 4×4 (for the charginos) and 2×2 (for the neutralinos) mass matrices which we do not reproduce here [13]. They depend on the Higgs mixing parameter μ as well as $M_{2,1}$ and the ratio $\tan \beta = v_2/v_1$ of Higgs v.e.v.'s, which therefore determine not only the chargino and neutralino mass eigenstates, but also their compositions as mixtures of the gaugino and higgsino states, and the mixing angles that enter into their couplings. For the physical spin-0 masses, one must add to the soft supersymmetry breaking terms D-term contributions, so that

$$m_{\tilde{f}_{L,R}}^2 = m_0^2 \pm M_Z^2 \cos 2\beta (T_{\tilde{f}_{L,R}}^3 - Q_{\tilde{f}_{L,R}} s_w^2) + C_{\tilde{f}_{L,R}} m_{\tilde{g}}^2 \quad (\tilde{f} = \tilde{u}, \tilde{d}, \tilde{e}, \tilde{\nu}), \quad (4)$$

where the renormalization coefficients take the following approximate numerical values [14]:

$$\begin{aligned} C_{\tilde{\nu}} &\cong 0.06, & C_{\tilde{u}_L} &\cong 0.83, & C_{\tilde{u}_R} &\cong 0.78, \\ C_{\tilde{e}_L} &\cong 0.06, & C_{\tilde{d}_L} &\cong 0.83, & C_{\tilde{d}_R} &\cong 0.78, \\ C_{\tilde{e}_R} &\cong 0.01. \end{aligned} \quad (5)$$

The mass matrix for the stop squarks is more complicated, since it involves important non-trivial off-diagonal entries:

$$\mathcal{M}_t^2 = \begin{bmatrix} m_{\tilde{t}_{LL}}^2 & m_{\tilde{t}_{LR}}^2 \\ m_{\tilde{t}_{LR}}^2 & m_{\tilde{t}_{RR}}^2 \end{bmatrix}, \quad (6)$$

where

$$\begin{aligned} m_{\tilde{t}_{LL}}^2 &= m_{\tilde{u}_L}^2 + m_t^2, \\ m_{\tilde{t}_{RR}}^2 &= m_{\tilde{u}_R}^2 + m_t^2, \\ m_{\tilde{t}_{LR}}^2 &= m_t \left(A_t + \frac{\mu}{\tan \beta} \right). \end{aligned} \quad (7)$$

Both the mass eigenstates and the mixing angle between the left- and right-handed stops are fixed by the formulae (6) and (7).

The masses of the Higgs bosons and the mixing angles in their couplings are fixed by the following potential at the tree level:

$$\begin{aligned} \mathcal{V} = & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + \text{h.c.}) + \\ & \frac{1}{8} g^2 \left(H_2^\dagger \tilde{\sigma} H_2 + H_1^\dagger \tilde{\sigma} H_1 \right)^2 + \frac{1}{8} g'^2 (|H_2|^2 - |H_1|^2)^2, \end{aligned} \quad (8)$$

so that, for example, the CP-odd (pseudoscalar) and pair of CP-even (scalar) neutral Higgs masses are given at the tree level by

$$\begin{aligned} m_{A,A'}^2 &= \frac{1}{2} \left[m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \right], \\ m_{H^\pm}^2 &= m_A^2 + M_W^2. \end{aligned} \quad (9)$$

The effective potential (8) receives substantial one-loop radiative corrections, of which the most important are those quartically-dependent on the large top quark mass:

$$\Delta^2 = \frac{3g^2 m_t^4}{8\pi^2 M_W^2 \sin^2 \beta} \log \left(\frac{m_t^2}{m_t^2} \right). \quad (10)$$

We use the full one-loop potential (8), (10) to evaluate the Higgs boson mass matrix, which we then diagonalize to extract the physical values of m_A , m_h , $m_{A'}$ and the coupling mixing angles, which now also depend on m_t .

We evaluate the one-loop vacuum polarization diagrams using these formulae in the standard expressions found in refs. [6]. In this way, we treat completely the one-loop radiative corrections associated with the second- and third-generation squarks (except for $Z \rightarrow b\bar{b}$) and Higgses. However, we do not include vertex and box diagrams, which are potentially important for small values of the -ino, slepton and squark masses. We leave their inclusion to a future study, in the belief that they are no longer very important for sparticle masses respecting the constraints that we find below.

3. Electroweak Data Set

In this Section we briefly review the electroweak data set considered in the analysis. The most important amount of experimental data comes from the four

LEP experiments. Their different results are collected and unified into a well-specified set of information, regularly updated as the statistics improves. Here we consider the updating presented at La Thuile in 1992 [15], which collects all the experimental data taken until the end of '91. In particular, assuming lepton universality, we consider the data concerning the lineshape, with the measurements of M_Z , Γ_Z , σ_0^h and $R = \Gamma_h/\Gamma_t$, the leptonic and b forward-backward asymmetries A_{FB}^e , A_{FB}^b , and the τ polarization asymmetry A_{pol}^τ . All correlation effects are carefully taken into account. The high-energy subset of experimental data is completed by the measurements of the vector boson mass ratio M_W/M_Z performed at the $\bar{p}p$ colliders (CDF at Fermilab and UA2 at CERN), as presented at the Lepton-Photon & EPS High Energy Physics Conference held in Geneva at the end of July 1991 [16].

To the high-energy data we add the contribution (whose importance has been stressed in the conclusions drawn in ref. [3] about a comparison of the data with the SM predictions) coming from low-energy neutral current (NC) analyses. These include the four squared NC chiral couplings u_L^2 , d_L^2 , u_R^2 , d_R^2 derived from a quark-parton model approach to the $\nu\mathcal{N}$ deep-inelastic scattering on isoscalar and non-isoscalar target [17]. Important information also comes from the purely leptonic νe sector, which involves all the data of the experiments measuring the two NC couplings g_V and g_A from $\nu_\mu e$, $\bar{\nu}_\mu e$, $\nu_e e$, $\bar{\nu}_e e$ scattering (in particular we include the recent data of the CHARM II Collaboration [18]). Finally, we include the estimate of the three NC couplings C_{1s} , C_{1d} , $C_{2s} - \frac{1}{2}C_{2d}$ that characterize the $e-q$ interaction, in particular from the parity violation effects in atoms, dominated by the Boulder experiment on parity violation in Cesium [19].

4. Constraints on the Supersymmetric Higgs Sector

In this Section we explore the dependence of the quality of global fits on the parameters of the supersymmetric Higgs sector. At the tree level, these are just m_A and $\tan \beta$. However, the important radiative corrections to the supersymmetric

Higgs masses depend on m_t , $m_{\tilde{t}_{1,2}}$, and μ . The μ -dependence of the fits induced via the Higgs sector is not very strong, and we simply take $\mu = 800$ GeV in this Section. The m_t -dependence of the fits arises from their dependences on the unmixed squark masses and on the trilinear soft supersymmetry-breaking parameter A_t . The dependence on A_t is not significant, and we set $A_t = 0$. The dependence on $m_{\tilde{t}}$ is gradual and monotonic, and we choose for illustrative purposes $m_0 = m_{\tilde{t}} = 800$ GeV, corresponding to a squark mass $m_{\tilde{t}}$ of about 1 TeV. Thus we are left with the dependences of the global fit on m_A , $\tan\beta$ and m_t as the main topics of discussion in this Section.

As a general rule, the other Higgs bosons decouple in the limit $m_A \rightarrow \infty$, so that the χ^2 of the fit becomes equal to that of the Standard Model for the corresponding value of m_A , if μ , m_0 and $m_{\tilde{t}}$ are large enough. The effects of the radiative corrections tend to increase with $\tan\beta$, but are in general rather weak. The quality of global fits is known to vary rapidly with m_t , and this remains true in the MSSM. The value of m_t at the global minimum of the χ^2 function is not very sensitive to m_A and $\tan\beta$, as we can see in Fig. 1, although it may be very slightly reduced if the Higgs masses are small. We notice that the minimum of the χ^2 function is slightly dependent on m_A . We are aware that oft-denied rumour and a recent theoretical analysis [20] favour the discovery of $m_t = 130$ GeV at the Fermilab $p\bar{p}$ collider, but shall not insist on this value. We shall generally use this as just one of our illustrative values, along with one lower and one higher value.

We show in Fig. 2 the variation of χ^2 with m_A for various different choices of m_t and $\tan\beta$. We see that indeed the value of the χ^2 function tends towards a constant as m_A increases. The absolute minimum of χ^2 occurs when m_A is about 35 GeV, which is very close to the present experimental lower limit on m_A , almost independently of m_t and $\tan\beta$. However, the significance of this minimum is not very strong, and tends to disappear for large m_t and/or low $\tan\beta$. We show in Fig. 3 the variation of χ^2 with $\tan\beta$ for $m_t = 130$ GeV and $m_A = 200$ GeV, together with the associated spectrum of sparticle masses. We see that there is no significant indication yet on the preferred range of $\tan\beta$. The small visible

variation in the χ^2 function is tightly correlated with the Higgs mass shown in the bottom half of the figure.

It is apparent from this analysis that the constraints on the supersymmetric Higgs parameters are not yet very significant. Moreover, if m_t turns out to be in the upper part of the preferred range, i.e. above about 130 GeV, the present weak indications in the $(m_A, \tan\beta)$ plane shown in Fig. 4 will become even less important. However, they will surely improve as the precision of LEP data improves further. Moreover, as we can see from Fig. 4, they have the capability of being complementary to LEP searches if they continue to disfavour large values of m_A . Therefore the analysis of radiative corrections in the $(m_A, \tan\beta)$ plane is likely to have an interesting future.

5. Constraints on Charginos and Neutralinos

In this Section we explore the sensitivity of the precision electroweak data to the parameters of the chargino and neutralino (generically, $\tilde{\chi}$) sector of the MSSM. These parameters are 3 in number: μ , $m_{\tilde{g}}$ and $\tan\beta$. On general principles, we expect that the $\tilde{\chi}$ -inos will decouple and the radiative correction effects disappear as μ and/or $m_{\tilde{g}} \rightarrow \infty$. Moreover, it is a general feature of the radiative corrections that they are larger for larger values of $\tan\beta$. The radiative corrections become very large in the regions of parameter space already excluded by direct $\tilde{\chi}$ -ino searches in Z^0 decays, and might have interesting effects if the $\tilde{\chi}$ -ino masses are very close to the LEP 1 limits [10].

We show in Fig. 5 the dependence of the χ^2 function on μ for several values of $\tan\beta$ and the default choice $m_t = 130$ GeV. We see that indeed the effects of the radiative corrections disappear for large μ as expected. Comparison with the bottom half of each figure shows that the central rise in χ^2 is correlated with small chargino masses. Fig. 6 shows the dependence of the χ^2 function on $m_{\tilde{g}}$, also for several values of $\tan\beta$ and the same value of m_t . It is a common feature of both

these analyses that the $\Delta\chi^2 = 1$ line gives a modest improvement of up to 5 GeV on the direct LEP 1 limit on the chargino mass $m_{\chi^\pm} > \frac{1}{2}M_Z$. Conversely, if the chargino mass is just above the present direct LEP 1 search limit, there can be a significant reduction in the preferred value of m_t , as seen in Fig. 7.

We show in Fig. 8 the $\Delta\chi^2 = 1$ contour in the $(\mu, m_{\tilde{g}})$ plane for two representative choices $\tan\beta = 2, 8$, and the default choice $m_t = 130$ GeV. Also shown for comparison by dotted lines are the limits in this plane that come from direct searches for $Z \rightarrow$ chargino and neutralino pairs.

In view of the modest improvements afforded by the present radiative correction analysis, and the likelihood that this will not become much more sensitive in the future, we believe that in this plane at least direct searches will in the future be the more significant.

6. Constraints on Sfermions

In this Section we explore the sensitivity of the precision electroweak data to the parameters of the sfermion (slepton and squark) sector of the MSSM. The three most important of these parameters are the primordial scalar mass m_0 , the gluino mass $m_{\tilde{g}}$, and $\tan\beta$. However, sensitivities to m_t , μ and A_t appear via the stop sector. If the soft supersymmetry-breaking contributions to the sfermion masses were negligible, the stop contribution to effective one-loop isospin breaking in the Standard Model would be the same as that of the top quark, and upper limits on m_t would be correspondingly strengthened by $\frac{1}{\sqrt{2}}$ [21]. However, it seems that the squark masses are larger than $M_{W,Z}$ and m_t , and the squarks and sleptons decouple in the limit where $m_{0,\tilde{g}} \rightarrow \infty$, so their effects on fits to the electroweak data are likely to be much less dramatic. Nevertheless, we should point out that the direct experimental limits on $m_{\tilde{t}}$ and $m_{\tilde{g}}$ are somewhat ambiguous, since they depend on the assumed decay modes, which in turn depend on other MSSM parameters [22], whereas the constraints coming indirectly from analyses

of radiative corrections depend in principle only on the masses of the sparticles under consideration.

We show in Fig. 9 the dependence of the χ^2 function on m_0 for several values of $\tan\beta$. The rise at low values of m_0 gives us a lower bound, which does however depend on the assumed values of m_t and μ , as well as on $\tan\beta$, both directly and also indirectly if the $-\text{ino}$ sector is included simultaneously in the fit.

The resulting bounds in the $(m_0, m_{\tilde{g}})$ plane, both with and without the gaugino contributions to the radiative corrections, are shown in Figs. 10 and 11, for two different values of μ . We see that, although the bounds on $m_{\tilde{g}}$ at large m_0 and on m_0 at small $m_{\tilde{g}}$ are only slightly stronger than the limits coming from direct searches for sneutrinos and charginos, the bounds at intermediate values of $m_{\tilde{g}}/m_0$ can be significantly stronger than these direct searches. Indeed, the radiative correction analysis can be competitive with the direct searches for squarks at $p\bar{p}$ colliders, depending on the values of m_t , μ and $\tan\beta$.

We show in Fig. 12 the dependence of the χ^2 function on m_t for three different cases corresponding to relatively light sfermions. We see the feature, mentioned earlier, that the preferred range of m_t may be significantly reduced if the sfermion masses are small. This means that there would not be much scope for a light sparticle spectrum if m_t were in the upper part of the range indicated in the Standard Model. On the other hand, a relatively low value of m_t could give some legitimate hope that there might also be light sparticles and/or Higgses.

7. Conclusions and Prospects

We have presented in this paper the detailed results of a first global analysis of precision electroweak data within the context of the MSSM. Within the general 5-parameter space of essential parameters $(m_0, m_{\tilde{g}}, \mu, m_A, \tan\beta)$, we have explored 3 subspaces. The first of these was the $(m_A, \tan\beta)$ plane which characterizes the supersymmetric Higgs sector at the tree level and, together with m_t , largely

characterizes it at the one-loop level. The data are highly consistent with supersymmetric Higgses, and we find in the limit where the other sparticles are heavy a marginal preference for small values of m_A . This tendency is less pronounced for larger m_t , where it is without statistical significance so far.

In the $(\mu, m_{\tilde{g}})$ plane characterizing charginos and neutralinos, we find that the precision electroweak data improve marginally the best available direct limits on the chargino mass. However, a chargino in the mass range accessible to LEP 2 certainly cannot yet be excluded. The indirect limits from precision electroweak data can improve significantly constraints in the $(m_0, m_{\tilde{g}})$ plane for intermediate values of the ratio $m_{\tilde{g}}/m_0$, at least if $m_t = 130$ GeV.

Of intrinsic general interest, and specifically in comparison with the predictions of various GUTs, is the value of $\sin^2\theta_w(M_Z)|_{\overline{\text{MS}}}$. It is not feasible to quote here the variation of this quantity over the full multi-dimensional parameter space of the MSSM. However, we can make a simplified, and hopefully useful, presentation in the following way. We have seen that the data are completely consistent with, and even prefer, large values of μ , m_0 and $m_{\tilde{g}}$, whilst they are essentially independent of A_t . Therefore we quote the range of $\sin^2\theta_w(M_Z)|_{\overline{\text{MS}}}$ for the values $-\mu = m_0 = m_{\tilde{g}} = 1$ TeV and $A_t = 0$, leaving m_t free and considering $M_Z \leq m_A \leq 1$ TeV, $2 \leq \tan\beta \leq 8$:

$$\sin^2\theta_w(M_Z)|_{\overline{\text{MS}}} = (0.2323 \text{ to } 0.2324) \pm 0.0006. \quad (11)$$

The corresponding range of $\sin^2\theta_w$ defined in the on-shell prescription [23] is

$$\sin^2\theta_w = (0.2272 \text{ to } 0.2273) \begin{matrix} +0.0026 \\ -0.0024 \end{matrix}. \quad (12)$$

In each case the errors are those associated with the allowed variation in m_t , and the parenthesis gives the range associated with uncertainties in m_A and $\tan\beta$. We note that the values of m_A and $\tan\beta$ are almost irrelevant to the ranges of

eqs. (11), (12), whereas the uncertainty in m_t is more important, and particularly large, in (12).

It would clearly be a great help in this type of MSSM analysis if the top quark were discovered and its mass determined. We have mentioned several cases where the strength of bounds on MSSM parameters, even their existence, depends on the precise value of m_t .

We have not attempted in this paper a best fit in the multi-parameter space of the MSSM, as we do not consider that this would be very meaningful at the present level of precision in the data. However, we hope and expect that some more definite conclusions about the MSSM parameters could be drawn in the future, as the number of Z^0 events observed at LEP increases, polarized Z^0 decays become available from the SLC, M_W is measured more precisely by CDF/D0 and at LEP 2, better low-energy data on νN deep-inelastic and νe elastic scattering emerge, and better data on atomic physics parity violation become available.

We believe that precision electroweak data are on the verge of providing important indirect probes of virtual supersymmetry, which complement and extend the programme of direct searches at the FNAL $\bar{p}p$ collider, LEP 2 and future high-energy accelerators.

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Fig. 1 — The value of χ^2 as a function of m_t within the MSSM, for three choices of m_A (60 GeV, 100 GeV, 300 GeV) and the values (a) $\tan \beta = 2$, (b) $\tan \beta = 8$. The χ^2 function is insensitive to A_t , which we set to zero, and only weakly sensitive to μ , m_0 and $m_{\tilde{g}}$, which we set each to 800 GeV.

Fig. 2 — The value of χ^2 as a function of m_A , for three choices of $\tan \beta$ (2, 4, 8) and the values (a) $m_t = 100$ GeV, (b) $m_t = 130$ GeV, (c) $m_t = 150$ GeV. The statistical importance of the dependence on m_A is significant only for low m_t and large $\tan \beta$.

Fig. 3 — The $\tan \beta$ -dependence of the χ^2 function for $m_t = 130$ GeV and $m_A = 200$ GeV, and the corresponding variations in the masses of the $\tilde{q}$ (SQU), $\tilde{\ell}$ (SLE), lighter chargino (CHA), lightest Higgs h (HIG), lightest neutralino (NEU), and the $t_{1,2}$ mass splitting (STS). We see that the value of χ^2 is correlated essentially with m_A .

Fig. 4 — Contours of the χ^2 function in the $(m_A, \tan \beta)$ plane for $m_t = 130$ GeV. Its variation is not presently significant.

Fig. 5 — The dependence of the χ^2 function on μ for the three choices (a) $\tan \beta = 2$, (b) $\tan \beta = 4$, (c) $\tan \beta = 8$ and $m_t = 130$ GeV. The corresponding particle masses are shown in the bottom part of the figure, using the same notation as in Fig. 3. The behaviour of χ^2 is correlated essentially with the lighter chargino mass.

Fig. 6 — The dependence of the χ^2 function on m_2 for the three choices (a) $\tan \beta = 2$, (b) $\tan \beta = 4$, (c) $\tan \beta = 8$ and $m_t = 130$ GeV. We again see from the bottom part of the figure that χ^2 is correlated essentially with the lighter chargino mass.

Fig. 7 — The dependence of the χ^2 function on m_t for the choice $m_{\chi^\pm} = 50$ GeV, 60 GeV, 70 GeV. We see that the preferred value of m_t decreases somewhat as $m_{\chi^\pm}$ increases, whilst the minimum value of χ^2 increases by a small amount.

Fig. 8 — The $\Delta\chi^2 = 1$ contours in the $(\mu, m_{\tilde{g}})$ plane for (a) $\tan\beta = 2$, (b) $\tan\beta = 8$ (solid lines) compared with the limits from direct Z decays into chargino and identical neutralino pairs (dashed lines).

Fig. 9 — The dependence of χ^2 on m_0 for the three choices (a) $\tan\beta = 2$, (b) $\tan\beta = 4$, (c) $\tan\beta = 8$ and $m_t = 130$ GeV. Note the correlation of χ^2 with the slepton mass.

Fig. 10 — The $\Delta\chi^2 = 1, 2.7$ contours in the (m_0, m_3) plane for $m_t = 130$ GeV, $\mu = -300$ GeV, $m_A = 1$ TeV and (a) $\tan\beta = 2$, (b) $\tan\beta = 8$, shown as full heavy lines. The heavy dashed line is the limit due to the absence of the direct decay into the lightest chargino with mass $< M_Z/2$. Dash-dotted heavy lines indicate the limits when the chargino and neutralino effects are not included. Also shown is the excluded zone corresponding to $m_3^2 < 0$, together with dashed curves for constant $m_{\tilde{g}}$ and dotted curves for constant $m_{\tilde{u}_R}$.

Fig. 12 — The dependence of the χ^2 function on m_t for the three different choices $m_0 = 70$ GeV, 100 GeV, 300 GeV. We see that the preferred value of m_t decreases significantly for smaller m_0 .

Fig. 11 — As in Fig. 10, for $\mu = -150$ GeV.