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THE TOP DECAY $t \rightarrow b W$ IN THE TWO HIGGS DOUBLET MODEL[†]

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Abstract

Electroweak radiative one-loop corrections to the top decay width $\Gamma(t \rightarrow b W^+)$ are calculated in the Two Higgs Doublet extension of the Standard Model. Whereas in the Minimal Standard Model the corrections are always below 2% and positive if the lowest order width is parametrized by G_F and M_W they may become negative and much larger in the general Two Higgs Doublet Model. In particular for a light scalar Higgs boson and enhanced top quark Yukawa couplings (corresponding to a large top mass) the corrections owing to the additional Higgs bosons easily exceed -10%. For supersymmetric values of the parameters, however, the non standard corrections hardly reach 1%.

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1 Introduction

Apart from the nonabelian gauge couplings, which will soon be investigated at LEP200, the experimentally missing ingredients of the Minimal Standard Model (MSM) are the top quark and the Higgs sector. The analysis of radiative corrections to precision experiments indicates that the top quark mass range is between 100 and 200 GeV. Accordingly the next generation of e^+e^- - and pp -colliders will be ideally suited to investigate its production and decay in detail. If the top quark is heavier than the W boson the dominant MSM decay is the one into a W boson and a bottom quark. This decay has a sizable branching ratio even in extensions of the MSM allowing for decays into charged and neutral Higgs bosons.

The present experimental information on the Higgs sector is rather poor. The only essential restriction is that the ρ -parameter $\rho = M_W^2/(M_Z^2 \cos\theta_W)$ is close to one. This is naturally accomplished in models which contain only Higgs doublets because these yield $\rho = 1$ at tree level. The simplest extension of the MSM in this class is the one with two Higgs doublets. It exhibits many of the general features of the models with more Higgs doublets and reproduces, for appropriate values of its parameters, the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM) [1].

We consider the general Two Higgs Doublet Model (THDM) with no theoretical constraints on its parameters apart from a discrete symmetry to avoid flavour changing neutral currents [2]. This symmetry is arranged such that the masses of the up- and down-type quarks are generated by the vacuum expectation values v_2 and v_1 , respectively, of the two Higgs doublet fields. The THDM contains five physical spin 0 bosons, namely two neutral scalars H_1, H_2 , one neutral pseudoscalar H_3 and one pair of oppositely charged Higgs bosons $\Phi^\pm$. Besides the Higgs boson masses $M_{H_1}, M_{H_2}, M_{H_3}, M_\Phi$ the model involves two mixing angles α and β as additional free parameters. The angle β is directly related to the vacuum expectation values of the Higgs doublets

$$\tan\beta = \frac{v_2}{v_1}. \quad (1)$$

Even if experimental searches for the Higgs boson give only lower mass limits, these particles manifest themselves in virtual states, i.e. within radiative corrections, and thus influence physical observables. In the THDM particularly large radiative corrections can be obtained because the Yukawa couplings typically contain factors like

$$\frac{m_f}{M_W} \tan\beta, \quad \frac{m_f}{M_W} \cot\beta$$

and can thus be enhanced by tuning β . Since the Yukawa couplings involve the fermion mass these effects are more important for heavier fermions such as the top or the bottom quarks.

In this paper we discuss the radiative corrections to the decay $t \rightarrow bW$ arising from a second Higgs doublet with particular emphasis on the large effects due to enhanced Yukawa couplings. The Feynman rules are taken from [1]. The QCD and electroweak radiative corrections in the context of the MSM have been given in refs. [3, 4, 5, 6]. Recently a first analysis of the additional electroweak corrections within the THDM has been carried through [7]. Here we give the complete analytical formulae and an extended discussion of the numerical results.

This paper is organized as follows: The notation and the lowest order decay width are given in Section 2. Section 3 contains the necessary formulae for the first order virtual electroweak Higgs corrections. The results are evaluated and discussed in Section 4. Section 5 contains a short summary.

2 Notation and Lowest Order Decay Width

In lowest order the decay

$$t(p_1) \rightarrow b(p_2) W^+(k) \quad (2)$$

is described by the Feynman diagram in fig. 1. It yields the amplitude

$$\mathcal{M}_{Born} = \frac{ie}{\sqrt{2}s_W} \mathcal{M}_0 V_{tb}, \quad \mathcal{M}_0 = \bar{u}(p_2) \not{\epsilon}(k, \lambda) \omega_- u(p_1), \quad (3)$$

where V_{tb} is the corresponding Kobayashi-Maskawa matrix element, $\epsilon(k, \lambda)$ the polarization vector of the W boson, $\omega_{\mp} = \frac{1}{2}(1 \mp \gamma^5)$ the projection operator on left- and right-handed fermions, respectively, and $s_W^2 = 1 - c_W^2 = 1 - M_W^2/M_Z^2$. Summing over the boson polarizations and the fermion helicities the amplitude (3) leads to the lowest order width [8]

$$\Gamma_0 = \frac{\alpha_{em}}{4} \frac{m_t}{2s_W^2} |V_{tb}|^2 \left[\frac{m_t^2 + m_b^2}{2m_t^2} + \frac{(m_t^2 - m_b^2)^2}{2m_t^2 M_W^2} - \frac{M_W^2}{m_t^2} \right] \times \frac{1}{m_t^2} \sqrt{(m_t^2 - (M_W + m_b)^2)(m_t^2 - (M_W - m_b)^2)} \quad (4)$$

Figure 1: Lowest-order Feynman diagram for the decay $t \rightarrow bW$.

3 Electroweak One-Loop Corrections

The virtual corrections are calculated in the complete on-shell renormalization scheme as worked out in ref. [9] for the MSM. It is symmetric in the treatment of the fermions and may easily be adjusted to the extended Higgs sector within the THDM. Furthermore, for the process considered here there is no need for renormalization of the Higgs sector and we can directly take over the results¹. The renormalized parameters are the electrical charge e , the masses of the gauge bosons M_Z, M_W , the fermions m_f and the Higgs bosons $M_{H_1}, M_{H_2}, M_{H_3}, M_\Phi$ and the mixing angles α and β . For our purpose we only need the renormalization constants of the electrical charge e , δZ_e , and of the sine of the Weinberg angle s_W , δs_W , as well as the field renormalization constants for the W boson, δZ_W , and the left-handed fermion fields, δZ_{ii}^L , with $i = t, b$. Owing to the small bottom mass the Kobayashi-Maskawa matrix can be neglected in the $\mathcal{O}(\alpha_{em})$ corrections; it appears simply as a multiplicative factor in the corrected amplitude [10].

At first order the decay width $\Gamma(t \rightarrow bW)$ gets QCD and electroweak corrections, each consisting of virtual and real (bremsstrahlung) contributions. The QCD corrections and the real photonic corrections are identical to the ones in the MSM given in [3, 5, 6]. Therefore we need only calculate virtual electroweak corrections within the THDM, more precisely only those involving the five Higgs bosons, because the corrections containing only virtual gauge bosons and fermions are identical in both models.

We now specify explicitly the virtual electroweak corrections involving Higgs bosons. We omit those diagrams containing only the charged Higgs ghost because these are the same as in the MSM. With $\mathcal{M}_0$ defined in eq. (3) and the three further Dirac matrix elements

$$\begin{aligned}\mathcal{M}_1 &= \bar{u}(p_2)\not{\omega}_+u(p_1), \\ \mathcal{M}_2 &= \bar{u}(p_2)\not{\omega}_+u(p_1)\epsilon \cdot p_1, \\ \mathcal{M}_3 &= \bar{u}(p_2)\not{\omega}_-u(p_1)\epsilon \cdot p_1\end{aligned}\tag{5}$$

the electroweak one-loop corrections to the amplitude can be written as follows

$$\begin{aligned}\mathcal{M}_{virt}^{ew} &= \frac{ie}{\sqrt{2}s_W}V_{tb}\frac{1}{2}\left\{\mathcal{M}_0\left(\delta Z_{tt}^L + \delta Z_{bb}^L + \delta Z_W + 2\delta Z_e - 2\frac{\delta s_W}{s_W} + \delta_v^{(0)}\right)\right. \\ &\quad \left.+ [\mathcal{M}_1\delta_v^{(1)}m_tm_b + \mathcal{M}_2\delta_v^{(2)}m_t + \mathcal{M}_3\delta_v^{(3)}m_b]\right\}.\end{aligned}\tag{6}$$

The corrections due to loop diagrams are summarized in $\delta_v^{(i)}$, ($i = 0, \dots, 3$). The renormalization constants read in terms of the unrenormalized self energies Σ^{xy}

¹Clearly we have to take into account the additional Higgs contributions in the self energies.

$$\begin{aligned}
\delta Z_e &= \frac{1}{2} \frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2} \Big|_{k^2=0} - \frac{s_W}{c_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \\
\frac{\delta s_W}{s_W} &= -\frac{1}{2} \frac{c_W^2}{s_W^2} \text{Re} \left(\frac{\Sigma_T^{WW}(M_W^2)}{M_W^2} - \frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} \right), \\
\delta Z_W &= -\text{Re} \frac{\partial \Sigma_T^{WW}(k^2)}{\partial k^2} \Big|_{k^2=M_W^2}, \\
\delta Z_{ii}^L &= -\text{Re} \Sigma_{ii}^L(m_i^2) - m_i^2 \frac{\partial}{\partial p^2} \text{Re} [\Sigma_{ii}^L(p^2) + \Sigma_{ii}^R(p^2) + 2\Sigma_{ii}^S(p^2)] \Big|_{p^2=m_i^2}.
\end{aligned} \tag{7}$$

The self energies in the THDM were already calculated in ref. [11]. Complete explicit expressions for the unrenormalized self energies are listed in ref. [12]. With

$$\begin{aligned}
G_0 &= \sum_{\text{pol}} |\mathcal{M}_0|^2 = m_t^2 + m_b^2 - 2M_W^2 + \frac{(m_t^2 - m_b^2)^2}{M_W^2}, \\
G_1 &= m_t m_b \sum_{\text{pol}} \mathcal{M}_0 \mathcal{M}_1^\dagger = -6m_t^2 m_b^2, \\
G_2 &= m_t \sum_{\text{pol}} \mathcal{M}_0 \mathcal{M}_2^\dagger = -m_t^2 \left[m_t^2 + m_b^2 - \frac{1}{2} M_W^2 - \frac{(m_t^2 - m_b^2)^2}{2M_W^2} \right], \\
G_3 &= m_b \sum_{\text{pol}} \mathcal{M}_0 \mathcal{M}_3^\dagger = \frac{m_b^2}{m_t^2} G_2
\end{aligned} \tag{8}$$

the full virtual electroweak corrections to $\Gamma(t \rightarrow bW)$ are given by

$$\begin{aligned}
\delta \Gamma_{virt}^{ew} &= \Gamma_0 \delta_{virt}^{ew} \\
&= \Gamma_0 \left(\delta Z_{tt}^L + \delta Z_{bb}^L + \delta Z_W + 2\delta Z_e - 2 \frac{\delta s_W}{s_W} \right. \\
&\quad \left. + \delta_v^{(0)} + \frac{G_1}{G_0} \delta_v^{(1)} + \frac{G_2}{G_0} \delta_v^{(2)} + \frac{G_3}{G_0} \delta_v^{(3)} \right).
\end{aligned} \tag{9}$$

The IR singularities contained in δ_{virt}^{ew} are compensated by the corresponding bremsstrahlung corrections, which have to be added to obtain the full electroweak corrections δ^{ew} to the decay width.

Using the Feynman rules in ref. [1] and denoting the couplings of the Z^0 boson

Figure 2: Feynman diagrams for the Higgs part of the tbW -vertex corrections. The letters correspond to the indices of the functions $\mathcal{V}$, $\mathcal{W}$, $\mathcal{X}$.

to the right- (+) and left-handed (-) fermions by

$$\begin{aligned} g_t^+ &= -Q_t \frac{s_W}{c_W}, & g_t^- &= \frac{\frac{1}{2} - Q_t s_W^2}{s_W c_W}, \\ g_b^+ &= -Q_b \frac{s_W}{c_W}, & g_b^- &= \frac{-\frac{1}{2} - Q_b s_W^2}{s_W c_W}, \end{aligned} \quad (10)$$

where

$$Q_t = \frac{2}{3} \quad \text{and} \quad Q_b = -\frac{1}{3}, \quad (11)$$

and introducing the shorthand notation

$$\sigma = \sin(\alpha - \beta), \quad \kappa = \cos(\alpha - \beta) \quad (12)$$

we get from the Feynman diagrams of fig. 2 the following form for the Higgs contribution to $\delta_v^{(i)}$, ($i = 0, \dots, 3$):

$$\begin{aligned} \delta_{v,H}^{(0)} &= \left(\frac{\alpha_{em}}{4\pi} \right) 2 \times \\ &\left\{ \frac{1}{4s_W^2} \frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \left[\mathcal{V}_c(m_t^2, M_W^2, m_b^2, M_{H_1}, m_t, m_b) \right. \right. \\ &\quad \left. \left. - \mathcal{V}_c(m_t^2, M_W^2, m_b^2, M_{H_2}, m_t, m_b) \right] \right. \\ &\quad \left. + \frac{1}{2s_W^2} \left[\frac{1}{\sin\beta} \left(\kappa \sin\alpha \mathcal{V}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right) \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\sigma \cos\alpha \mathcal{V}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \Big) \\
& + \frac{1}{\cos\beta} \Big(\kappa \cos\alpha \mathcal{V}_d(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \\
& \quad + \sigma \sin\alpha \mathcal{V}_d(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \Big) \Big] \\
& + \frac{1}{2s_W^2} \Big[\frac{1}{\sin\beta} \Big(\kappa \sin\alpha \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \\
& \quad - \sigma \cos\alpha \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \Big) \\
& \quad + \frac{1}{\cos\beta} \Big(\kappa \cos\alpha \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \\
& \quad + \sigma \sin\alpha \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \Big) \\
& \quad + \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_Z, M_W) + \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_Z, M_W) \Big] \\
& + \frac{1}{2s_W^2} \Big\{ \frac{1}{\tan\beta \sin\beta} \Big(\sigma \sin\alpha \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_\Phi) \\
& \quad + \kappa \cos\alpha \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_\Phi) \Big) \\
& \quad + \frac{\tan\beta}{\cos\beta} \Big(-\sigma \cos\alpha \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_\Phi) \\
& \quad + \kappa \sin\alpha \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_\Phi) \Big) \\
& \quad + \frac{1}{\tan^2\beta} \mathcal{V}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_3}, M_\Phi) \\
& \quad + \tan^2\beta \mathcal{V}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_3}, M_\Phi) \Big] \Big\} \quad ,
\end{aligned}$$

$$\begin{aligned}
\delta_{v,H}^{(1)} &= \left(\frac{\alpha_{em}}{4\pi} \right) 2 \times \\
& \left\{ \frac{1}{4s_W^2} \left[\sum_{h=\pm} \left(\frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \mathcal{W}_c^h(m_t^2, M_W^2, m_b^2, M_{H_1}, m_t, m_b) \right. \right. \right. \\
& \quad \left. \left. - \frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \mathcal{W}_c^h(m_t^2, M_W^2, m_b^2, M_{H_2}, m_t, m_b) \right) \right. \\
& \quad \left. + \mathcal{W}_c^-(m_t^2, M_W^2, m_b^2, M_{H_3}, m_t, m_b) - \mathcal{W}_c^-(m_t^2, M_W^2, m_b^2, M_Z, m_t, m_b) \right] \\
& + \frac{1}{2s_W^2} \left[\frac{1}{\sin\beta} \Big(\kappa \sin\alpha \mathcal{W}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right. \\
& \quad \left. - \sigma \cos\alpha \mathcal{W}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \Big) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\cos\beta} \left(\kappa \cos\alpha \mathcal{W}_d(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \right. \\
& \quad \left. + \sigma \sin\alpha \mathcal{W}_d(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \right) \Big] \\
& + \frac{1}{2s_W^2} \left[\frac{1}{\sin\beta} \left(\kappa \sin\alpha \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right. \right. \\
& \quad \left. \left. - \sigma \cos\alpha \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \right) \right. \\
& \quad + \frac{1}{\cos\beta} \left(\kappa \cos\alpha \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \right. \\
& \quad \left. + \sigma \sin\alpha \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \right) \\
& \quad \left. - \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_Z, M_W) - \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_Z, M_W) \right] \\
& - \frac{1}{2s_W^2} \left[\frac{\tan\beta}{\sin\beta} \left(\sigma \sin\alpha \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_\Phi) \right. \right. \\
& \quad \left. \left. + \kappa \cos\alpha \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_\Phi) \right) \right. \\
& \quad + \frac{1}{\tan\beta \cos\beta} \left(-\sigma \cos\alpha \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_\Phi) \right. \\
& \quad \left. + \kappa \sin\alpha \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_\Phi) \right) \\
& \quad \left. - \mathcal{W}_e(m_t^2, M_W^2, m_b^2, m_t, M_{H_3}, M_\Phi) - \mathcal{W}_e(m_b^2, M_W^2, m_t^2, m_b, M_{H_3}, M_\Phi) \right] \Big\} ,
\end{aligned}$$

$$\begin{aligned}
\delta_{v,H}^{(2)} &= \left(\frac{\alpha_{em}}{4\pi} \right) 2 \times \\
& \left\{ \frac{1}{4s_W^2} \left[\sum_{h=\pm} \left(\frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \mathcal{X}_c^h(m_t^2, M_W^2, m_b^2, M_{H_1}, m_t, m_b) \right. \right. \right. \\
& \quad \left. \left. - \frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \mathcal{X}^h(m_t^2, M_W^2, m_b^2, M_{H_2}, m_t, m_b) \right) \right. \\
& \quad \left. + \mathcal{X}_c^-(m_t^2, M_W^2, m_b^2, M_{H_3}, m_t, m_b) - \mathcal{X}_c^-(m_t^2, M_W^2, m_b^2, M_Z, m_t, m_b) \right] \\
& + \frac{1}{2s_W^2} \left[\frac{1}{\sin\beta} \left(\kappa \sin\alpha \mathcal{X}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right. \right. \\
& \quad \left. \left. - \sigma \cos\alpha \mathcal{X}_d(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \right) \right] \\
& - \frac{1}{2s_W^2} \sum_{h=\pm} \left[h \left[\frac{1}{\sin\beta} \left(\kappa \sin\alpha \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right. \right. \right. \\
& \quad \left. \left. - \sigma \cos\alpha \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \right) \right] \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\cos\beta} \left(\kappa \cos\alpha \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \right. \\
& \quad \left. + \sigma \sin\alpha \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \right) \Big] \\
& - \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_Z, M_W) - \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_Z, M_W) \Big] \\
& - \frac{1}{2s_W^2} \frac{m_b^2}{M_W^2} \left[\frac{1}{\sin\beta} \left(\kappa \sin\alpha \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_W) \right. \right. \\
& \quad \left. \left. - \sigma \cos\alpha \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_W) \right) \right. \\
& - \frac{1}{\cos\beta} \left(\kappa \cos\alpha \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_W) \right. \\
& \quad \left. + \sigma \sin\alpha \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_W) \right) \\
& \left. - \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_Z, M_W) - \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_Z, M_W) \right] \\
& - \frac{1}{2s_W^2} \sum_{h=\pm} \left[h \left[\frac{1}{\tan\beta \sin\beta} \left(\sigma \sin\alpha \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_\Phi) \right. \right. \right. \\
& \quad \left. \left. + \kappa \cos\alpha \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_\Phi) \right) \right. \\
& + \frac{1}{\tan\beta \cos\beta} \left(-\sigma \cos\alpha \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_\Phi) \right. \\
& \quad \left. \left. + \kappa \sin\alpha \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_\Phi) \right) \right] \\
& - \frac{1}{\tan^2\beta} \mathcal{X}_e^h(m_t^2, M_W^2, m_b^2, m_t, M_{H_3}, M_\Phi) \\
& \left. + \mathcal{X}_e^h(m_b^2, M_W^2, m_t^2, m_b, M_{H_3}, M_\Phi) \right] \\
& + \frac{1}{2s_W^2} \frac{m_b^2}{M_W^2} \left[\frac{\tan\beta}{\sin\beta} \left(\sigma \sin\alpha \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_{H_1}, M_\Phi) \right. \right. \\
& \quad \left. \left. + \kappa \cos\alpha \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_{H_2}, M_\Phi) \right) \right. \\
& + \frac{\tan\beta}{\cos\beta} \left(-\sigma \cos\alpha \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_{H_1}, M_\Phi) \right. \\
& \quad \left. \left. + \kappa \sin\alpha \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_{H_2}, M_\Phi) \right) \right. \\
& - \mathcal{X}_e^0(m_t^2, M_W^2, m_b^2, m_t, M_{H_3}, M_\Phi) \\
& \left. - \tan^2\beta \mathcal{X}_e^0(m_b^2, M_W^2, m_t^2, m_b, M_{H_3}, M_\Phi) \right] \Big\} .
\end{aligned}$$

All the functions $\mathcal{V}_i, \mathcal{W}_i, \mathcal{X}_i$ are defined and calculated in ref. [4] where also the non-Higgs contributions can be found. The coefficient $\delta_{v,H}^{(3)}$ can be easily obtained

from $\delta_{v,H}^{(2)}$ by the substitutions $m_t \leftrightarrow m_b$, $\sin\alpha \leftrightarrow \cos\alpha$, $\sin\beta \leftrightarrow \cos\beta$, $\tan\beta \leftrightarrow 1/\tan\beta$ and $\sigma \leftrightarrow -\sigma$.

We emphasize that the above formulae are also applicable to the virtual electroweak correction of all on-shell $ff'W$ -vertices in the context of the THDM.

4 Results and Discussion

In the on-shell scheme used above the Born width (4) is parametrized by the fine structure constant and the sine of the Weinberg angle ($\Gamma_0(\alpha_{\text{em}})$). In ref. [5] it has been shown that the dominant part of the MSM electroweak corrections can be accounted for by parametrizing the lowest order width in terms of G_F and M_W ($\Gamma_0(G_F)$). Therefore we present the corrections always with respect to

$$\Gamma_0(G_F) = \frac{\sqrt{2}G_F s_W^2 M_W^2}{\pi\alpha_{\text{em}}} \Gamma_0(\alpha_{\text{em}}) = \frac{\Gamma_0(\alpha_{\text{em}})}{1 - \Delta r}. \quad (13)$$

The corresponding relative corrections are obtained from the ones in the on-shell parametrization $\delta^{\text{ew}}(\alpha_{\text{em}})$ given in the preceding sections as

$$\delta^{\text{ew}}(G_F) = \delta^{\text{ew}}(\alpha_{\text{em}}) - \Delta r. \quad (14)$$

The large contributions owing to the running of α_{em} and the ρ -parameter, i.e. the dominant self energy corrections, cancel in this difference. Moreover for fixed M_W it is independent of the light fermion masses.

Since in this paper we are only interested in the non standard corrections we split all corrections in a Standard Model part and a non standard (NS) part. The latter is always defined as the difference of the relevant quantity in the THDM and the MSM, e.g.

$$\Delta r_{\text{NS}} = \Delta r_{\text{THDM}} - \Delta r_{\text{MSM}}. \quad (15)$$

In the following we discuss the non standard corrections to $\Gamma_0(G_F)$, i.e.

$$\begin{aligned} \delta_{\text{NS}}^{\text{ew}}(G_F) &= \delta_{\text{NS}}^{\text{ew}}(\alpha_{\text{em}}) - \Delta r_{\text{NS}} \\ &= \left(\delta_{\text{THDM}}^{\text{ew}}(\alpha_{\text{em}}) - \delta_{\text{MSM}}^{\text{ew}}(\alpha_{\text{em}}) \right) - \left(\Delta r_{\text{THDM}} - \Delta r_{\text{MSM}} \right). \end{aligned} \quad (16)$$

Consequently corrections involving no virtual Higgs states, e.g. all QCD and photonic corrections need not to be taken into account. A detailed discussion of the MSM case is given in [5, 6].

For the numerical evaluation we have to fix the input parameters. The values for the MSM parameters are taken from [13, 14]:

$$\begin{aligned} \alpha_{\text{em}} &= 1/137.03599, & G_F &= 11.6637 (TeV)^{-2}, \\ M_Z &= 91.175 GeV, & |V_{tb}| &= 0.999, \\ m_u &= 0.041 GeV, & m_d &= 0.041 GeV, & m_s &= 0.15 GeV, \\ m_c &= 1.5 GeV, & m_b &= 4.5 GeV. \end{aligned}$$

The light quark masses are effective parameters consistent with the experimentally measured hadronic photon vacuum polarization [15]. Instead of the relatively inaccurately measured W boson mass M_W we use the accurately known Fermi constant G_F as numerical input. The W mass is calculated via

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha_{\text{em}}}{\sqrt{2} G_F} \frac{1}{1 - \Delta r_{\text{THDM}}}, \quad (17)$$

where Δr , the weak corrections to the muon decay, depends on all parameters of the model, particularly on M_W and the unknown Higgs boson and top quark masses.

The Higgs sector of the THDM involves the following additional parameters

$$M_{H_1}, M_{H_2}, M_{H_3}, M_\Phi, \beta, \alpha. \quad (18)$$

In general none of the two scalar Higgs bosons H_1, H_2 can be identified with the MSM Higgs. However, for the appropriate choice of the mixing angles $\alpha = \beta$ the Higgs boson H_1 has the same couplings to non spin 0 particles as the MSM Higgs boson and H_2, H_3, Φ appear as additional non standard fields. In order to evaluate a meaningful non standard correction we always choose $\alpha = \beta$ and $M_{H_1} = M_H$, where M_H is the mass of the MSM Higgs boson for which we take 100 GeV . Different values for $\alpha (\neq \beta)$ do not change the numerical results considerably.

Since the presence of the second Higgs affects the value of M_W determined from (17) we consistently use the THDM value in all quantities in (16), i.e. also in the MSM quantities. Otherwise we would get additional non standard corrections from all diagrams due to different values of the W -mass in both models. The change of the relative correction owing to this origin is of order α_{em}^2 and neglected here.

For $\alpha = \beta$ a very small M_{H_2} is still allowed by LEP data if $M_{H_3} > M_Z/2$ [16]. For very light H_2 we could formally obtain gigantic corrections, which are related to an IR-divergence for zero Higgs mass. In this region the tree level decay $t \rightarrow bW H_2$ yields sizable contributions which cancel the large virtual corrections to the decay $t \rightarrow bW$. Therefore we restrict ourselves to neutral Higgs masses of at least 5 GeV . Recent examinations of data from B- and K-physics and CP-violation [17] as well as unitarity and perturbativity considerations [18] indicate for $\alpha = \beta$ and $M_{H_1} = 100 \text{ GeV}$ the following allowed parameter region

$$0.5 \leq \tan\beta \leq 100, \quad 0 < M_{H_2}, M_{H_3} \leq 1 \text{ TeV}, \quad M_\Phi \geq \frac{1}{2} M_Z. \quad (19)$$

So far the top quark mass is not directly measured. Apart from the model independent lower limit [19] $m_t \geq 54 \text{ GeV}$ it is restricted from radiative corrections to precision experiments. Since there is no global analysis for the THDM we take

$\tan\beta = 0.7, \quad M_{H_1} = 100 \text{ GeV}$								
M_{H_2}	M_{H_3}	M_Φ	$m_{t,\max}$	$\delta_{\text{NS}}^{\text{ew}}(G_F)$	m_t	$\delta_{\text{NS}}^{\text{ew}}(G_F)$	$m_{t,\min}$	$\delta_{\text{NS}}^{\text{ew}}(G_F)$
5	600	360	233	-12.7	200	-8.9	161	-5.4
5	1000	550	328	-28.2	300	-22.7	270	-17.7
50	600	380	230	-3.8	200	-2.7	158	-1.5
600	600	600	168	-0.03	130	-0.01	85	0.00
1000	1000	1000	169	-0.01	130	0.00	85	0.00
5	200	150	175	-6.5	120	-2.6	88	-1.3
200	200	150	166	-0.7	120	-0.1	85	0.0
5	50	140	148	-4.5	110	-2.1	85	-1.1
600	50	210	218	-1.4	180	-0.4	143	-0.2
225	50	65	173	+0.7	130	+0.3	86	0.0
200	200	50	143	+0.3	110	+0.1	85	0.0
600	200	50	88	-0.1			85	-0.1

Table 1: Non standard virtual electroweak corrections for $\tan\beta = 0.7$. All masses are given in GeV and the values of $\delta_{\text{NS}}^{\text{ew}}(G_F)$ in %.

this fact into account by considering the experimental limits on the W -mass only. More precisely for fixed values of the Higgs masses we allow only those values of m_t which yield via (17) values for M_W in agreement with the experimental value $M_W = 80.14 \pm 0.27 \text{ GeV}$ from UA2 and CDF [19]. The allowed values for m_t may differ considerably from the MSM limits. Even if the present experimental bounds on $\Gamma(Z \rightarrow b\bar{b})$ are taken into account, the top quark mass may reach 300 GeV for Higgs masses in the $0.5 - 1 \text{ TeV}$ range [20].

We are in particular interested in large non standard corrections. Since these involve factors $\tan\beta$ or $\cot\beta$ we are led to consider extreme values of $\tan\beta$. For our numerical analysis we have chosen $\tan\beta = 0.7$ and $\tan\beta = 70$ well within the limits listed above.

In table 1 and 2 we give the non standard radiative corrections $\delta_{\text{NS}}^{\text{ew}}(G_F)$ for selected interesting Higgs mass constellations. Thereby the values of m_t have been chosen such that the whole allowed range is covered. More specific the given extreme values of $m_{t,\max}$ and $m_{t,\min}$ roughly correspond to values for M_W which are 1σ away from the experimental mean value. As can be seen the sensitivity to m_t is strong for $\tan\beta < 1$ but weak for $\tan\beta \gg 1$. This is due to contributions $\propto m_t^2/(M_W^2 \tan^2\beta)$ which are enhanced in the former case but suppressed in the latter.

We emphasize that $\delta_{\text{NS}}^{\text{ew}}(G_F)$ is negative for most of the allowed parameter

$\tan\beta = 70$, $M_{H_1} = 100 \text{ GeV}$								
M_{H_2}	M_{H_3}	M_Φ	$m_{t,\text{max}}$	$\delta_{\text{NS}}^{\text{ew}}(G_F)$	m_t	$\delta_{\text{NS}}^{\text{ew}}(G_F)$	$m_{t,\text{min}}$	$\delta_{\text{NS}}^{\text{ew}}(G_F)$
10	600	360	232	-2.8	200	-2.8	161	-2.7
10	1000	550	328	-3.2	300	-3.2	270	-3.2
50	600	380	230	-1.1	200	-1.1	158	-1.1
600	600	600	168	-0.01	130	-0.01	85	0.00
1000	1000	1000	168	0.00	120	0.00	85	0.00
10	200	150	175	-1.6	120	-1.8	88	-1.8
200	200	150	166	-1.0	120	-0.3	85	-0.2
10	50	140	148	-1.6	110	-1.9	85	-1.9
600	50	210	218	-1.4	180	-1.1	143	-1.0
225	50	65	173	+1.2	130	+1.0	86	-0.6
200	200	50	143	+1.0	110	+0.1	85	-1.0
600	200	50	88	-2.0			85	-2.1
100	105	50	166	+2.1	130	+2.0	85	+0.5

Table 2: Non standard virtual electroweak corrections for $\tan\beta = 70$. All masses are given in GeV and the values of $\delta_{\text{NS}}^{\text{ew}}(G_F)$ in %.

space. So the positive MSM-corrections [5, 6] are in general partially cancelled or even turned into negative ones. But it is also possible to get a positive $\delta_{\text{NS}}^{\text{ew}}(G_F)$ up to 2% for $M_{H_2}, M_{H_3} > M_\Phi$ and $\tan\beta = 70$.

Large corrections correspond to large mass splittings within the Higgs sector. Extreme values can be obtained for $M_{H_{1/2}} < M_\Phi < M_{H_3}$, i.e. $\delta_{\text{NS}}^{\text{ew}}(G_F) \approx -28\%$ for $M_{H_1} = 5 \text{ GeV}$, $M_{H_3} = 1000 \text{ GeV}$ and $M_\Phi = 550 \text{ GeV}$. In order to be consistent with current experimental data on M_W these values of the Higgs masses enforce a large top quark mass [20] which in turn enhances the top-Yukawa couplings. The size of the corrections grows with the mass of the pseudoscalar Higgs. The case $M_{H_3} < M_\Phi < M_{H_{1/2}}$ does not allow for extraordinary large corrections. If the masses of all additional Higgs bosons are large and approximately equal the corrections become very small. This is due to the decoupling of the additional Higgs doublet in this limit. It is an interesting fact that the MSM is the limiting case of the THDM for $\alpha = \beta$ and a heavy degenerate extended Higgs sector.

We will now discuss in detail some special Higgs mass constellations. The non standard corrections become particularly large if the scalar Higgs boson is light (see, however, the remark before eq. (19)). The strong rise of $\delta_{\text{NS}}^{\text{ew}}(G_F)$ with decreasing M_{H_2} is shown in fig. 3 for $M_{H_3} = 600 \text{ GeV}$, $M_\Phi = 370 \text{ GeV}$ and different m_t . (For $\tan\beta = 70$ we take 10 GeV as the lower bound for M_{H_2} in

$\tan\beta$	M_Φ/GeV	$m_t = 90\text{ GeV}$	$m_t = 120\text{ GeV}$	$m_t = 160\text{ GeV}$
0.7	100	+0.2	-0.2	-0.3
	200	+0.3	+0.2	-0.4
	600	+0.4	+0.3	-0.1
2	100	+0.2	+0.2	+0.2
	200	+0.2	+0.3	+0.2
	600	+0.3	+0.5	+0.6
10	100	+0.1	+0.1	+0.1
	200	+0.1	+0.1	+0.2
	600	+0.2	+0.4	+0.5
70	100	-0.2	0.0	+1.5
	200	+0.1	+0.1	+0.0
	600	+0.2	+0.4	+0.5

Table 3: Non standard virtual electroweak corrections for supersymmetric parameter relations.

order to avoid conflict with Υ -decay considerations [21].) The parts of the curves which are in agreement with the experimental value of M_W are indicated by solid lines, the (1σ) -excluded parts by dashed lines. For $\tan\beta = 0.7$ the non standard corrections exceed -10% if M_2 is small, for $\tan\beta = 70$ they reach up to -3% .

For $\tan\beta = 0.7$ large corrections are directly related to large top masses which are allowed for $M_{H_{1,2}} < M_\Phi < M_{H_3}$. As illustrated in fig. 4 for $M_{H_2} = 50\text{ GeV}$ and $M_{H_3} = 600\text{ GeV}$ the corrections exhibit an additional enhancement near the threshold $M_\Phi = m_t - m_b$. Exactly on this threshold the $\mathcal{O}(\alpha_{\text{em}})$ corrections diverge. This singularity is smeared out when the finite width of the top quark is taken into account. In this context we will not further discuss this problem. For $\tan\beta = 70$ the corrections hardly depend on m_t above that threshold. For $M_\Phi \approx m_t - m_b$ the largest effects occur even for small top mass. In contrast for $\tan\beta = 0.7$ the corrections rise quadratically with m_t . This is illustrated in fig. 5.

A particularly interesting scenario is the supersymmetric version of the THDM. There only two parameters out of M_{H_2} , M_{H_3} , M_Φ , β and α are independent [1]. The supersymmetric constraints² do not allow for large corrections. As illustrated in table 3 they are generally well below 1%. The only exception is the corner of parameter space where M_Φ is small and $\tan\beta$ and m_t are large. We want to stress that our analysis does not yield the full MSSM corrections but only the corresponding Higgs contributions. However, for large masses of the superparticles the

²Note that in this case $\alpha \neq \beta$.

THM may be viewed as a low energy approximation of the MSSM.

Our results are in good agreement with the figures presented in [7]. Note, however, that the value of $\tan \beta$ used there is 0.6 instead of 0.7 as stated in the paper. For the supersymmetric case we agree within 0.2% with respect to the lowest order.

A final remark is in order. For $M_\Phi + m_b < m_t$ the tree decay $t \rightarrow b\Phi$ is possible and yields important contributions to the total top width. This case has not been considered here.

5 Summary

We have discussed the additional electroweak corrections to the decay $t \rightarrow bW$ owing to a second Higgs doublet. Whereas the MSM corrections are positive and below 2% the non standard corrections are generally negative and acquire substantial values for small $\tan \beta$, large top mass and a small mass of the additional scalar Higgs boson. For the considered values of $\tan \beta$ the corrections are typically of the order of a few percent and thus dominate the MSM corrections. Imposing the supersymmetric parameter relations the additional corrections are at most $\sim 1\%$ and usually much smaller.

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Figure 3: Non standard virtual electroweak corrections over M_{H_2} for $M_{H_1} = 100$ GeV, $M_{H_3} = 600$ GeV and $M_{\Phi} = 370$ GeV.

Figure 4: Non standard virtual electroweak corrections over M_ϕ for $M_{H_1} = 100 \text{ GeV}$, $M_{H_2} = 50 \text{ GeV}$ and $M_{H_3} = 600 \text{ GeV}$.

Figure 5: Non standard virtual electroweak corrections over m_t for $M_{H_1} = 100 \text{ GeV}$, $M_{H_2} = 50 \text{ GeV}$ and $M_{H_3} = 600 \text{ GeV}$.