Monte Carlo program BHLUMI 2.01
for Bhabha scattering at low angles
with Yennie-Frautschi-Suura exponentiation*

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ABSTRACT

We discuss the Monte Carlo program BHLUMI 2.01 for small angle Bhabha scattering with an overall precision of .25%. The QED calculation at this precision level is essential for the luminosity measurement at LEP/SLC experiments. We summarize the theoretical development and implementation of our Yennie-Frautschi-Suura $\mathcal{O}(\alpha)$ exponentiated low angle Bhabha scattering event generator and discuss our outlook for the future.

In what follows, we present the current status of the theoretical development and implementation of the Monte Carlo program BHLUMI 2.01 [1-5], which was recently implemented at LEP/SLC experiments for calculating the QED expectations for the luminosity process $e^+e^- \rightarrow e^+e^- + n(\gamma)$ at .25% precision. BHLUMI 2.01 is the end product of a series of calculations [1-4] in which we have made new calculations of the vari-

ous QED radiative corrections to small angle Bhabha scattering at Z^0 energies. For reference, we note that the respective luminosity process measurement is important for all other experimental measurements in Z^0 physics [6,7]. Indeed, for illustration, we recall that the luminosity is a strongly varying function of time, so that the typical measurement procedure in which all data are averaged over long time intervals means that any un-

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certainty on the luminosity $\mathcal{L}$ will propagate into all measured observables such as cross sections, asymmetries, etc. For some observables, the error on $\mathcal{L}$ enters directly into the error of their measurement: for the error on N_ν , the number of massless neutrino generations as determined from the Z^0 invisible width, we have $\Delta N_\nu \sim 7\Delta\mathcal{L}/\mathcal{L}$. Hence, it is an important theoretical issue to keep the theoretical uncertainty in the expectation for $\mathcal{L}$ significantly below the attendant experimental uncertainty.

In early 1991, this pure experimental error went down below the level of 1% so that the theoretical uncertainty on the QED radiative corrections to Bhabha scattering at small angles became an essential or even the dominant component of the total error on the luminosity measurement. We emphasize that, in the luminosity measurement, the t-channel dominated QED calculation is regarded as known and calculable from the QED Lagrangian density to arbitrary accuracy. In practice, at a certain precision level, effects due to real and virtual photon radiation enter and such effects have to be calculated to an appropriate precision level. In fact, we must be definitive about what we mean when we speak about the precision level of a theoretical calculation. As defined in Ref. S, we divide the uncertainty on a theoretical calculation into two components: a physical component which refers to contributions from higher perturbative orders and a tech-

nical component which refers to biases due to numerical instabilities, programming bugs, calculational approximations, etc. Both of these contributions to the theoretical precision have to be calculated.

We recall that the radiative corrections to small angle Bhabha scattering were calculated for the PETRA/PEP experiments in Ref. 9 using standard perturbative techniques at $\mathcal{O}(\alpha)$ with respect to the Born approximation. The result of this calculation was encoded in the Monte Carlo (MC) event generator OLDBAB, so that implementation of arbitrary detector cuts and experimental efficiencies in the calculation was readily achieved. During the LEP1 Workshop [10], OLDBAB was extended in Ref. 11 to the complete $\mathcal{O}(\alpha)$ MC event generator BABAMC in which the pure weak corrections to low angle Bhabha scattering at $\mathcal{O}(\alpha)$ were also included. The precision tag of OLDBAB (and BABAMC) was around 1% and was definitely insufficient for the 1991 LEP data.

The task of reducing QED uncertainty in the luminosity measurement in papers [1-3] was divided into three corresponding steps:

- calculation of the technical uncertainty of the older $\mathcal{O}(\alpha)$ calculations,
- new calculation of the higher order effects beyond $\mathcal{O}(\alpha)$ keeping strict control on technical precision,
- determination of the total (technical + physical) precision of the multiphoton $\mathcal{O}(\alpha)$ exponentiated Monte Carlo generator BH-

Note that the event generator BHLUMI existed much earlier [12] but its development was not continued because at the time it was not obvious what kind of experimental error in the luminosity measurement would be reached in LEP experiments.^{#1}

In the first step of Ref. 1 an effort was made to get control on the technical precision of the existing $\mathcal{O}(\alpha)$ calculation, as implemented in the OLDBAB Monte Carlo program. In this non-trivial task we have used the following method: We have calculated analytically the integrated $\mathcal{O}(\alpha)$ cross section and several basic inclusive cross sections in the presence of semi realistic cuts allowing for hard bremsstrahlung. The most difficult part of the calculation was the precise analytical integration over the hard photon bremsstrahlung phase space. The agreement of the MC with the analytical results at the unprecedented level of 0.03% (for the total cross section) provided solid proof that we control technical precision of the combined (MC and analytical) calculation at this level for a wide range of the parameters defining the experimental trigger. In the process of the above tests we have made a series of modifications and improvements on the OLDBAB program. The new version we have named OLDBIS to avoid confusion with the original

^{#1} The other reason was that we could not test and improve on BHLUMI before the new higher order calculation of ref. [2] with technical precision below 0.1% existed.

code.

This first step of Ref. 1 was immediately profitable because with the new results (tables in Rref. 1) all LEP experiments (or anybody else) could check whether they are using a correct version of the $\mathcal{O}(\alpha)$ QED calculation for small angle Bhabha down to 0.03% precision. In particular, LEP collaborations could check immediately if the BABAMC program used to analyse 1990/91 data was compatible with these new $\mathcal{O}(\alpha)$ results/tests. On the other hand the results of Ref. 1 could be treated as a necessary starting point (baseline calculation) for study of the higher order corrections beyond the $\mathcal{O}(\alpha)$. One should keep in mind that it is impossible to talk about higher order corrections without knowing very precisely $\mathcal{O}(\alpha)$ corrections simply because they are defined as a difference between the given higher calculation and the $\mathcal{O}(\alpha)$ calculation! Note that in Ref. 1 we have found a very useful feature of the QED corrections to small angle Bhabha scattering, i.e. we have shown that the QED interference among emission from the upper electron line and lower positron line can be neglected. This phenomenon is very similar to the smallness of the QED interference among the initial and final state bremsstrahlung at the narrow resonance.

In the next step described in Ref. 2, we took aim at the biggest correction beyond $\mathcal{O}(\alpha)$, i.e., $\mathcal{O}(\alpha^2 L^2)$, the second order

leading-logarithmic (LL) correction where $L = \ln(|t|/m_e^2)$. This effect is for typical experimental cuts about 0.5% and it was calculated using two independent MC calculations. The very small difference between two MC results being 0.03% was taken as the technical precision of the result.^{#2} The LL calculation was also implemented in the form of the MC event generator LUMLOG such that arbitrary cuts could be imposed. Note that the MC program LUMLOG calculates the difference $\mathcal{O}(\alpha^2 L^2) - \mathcal{O}(\alpha L)$. In the lower part of Fig. 1 we show this difference as a function of the energy cut, see Refs. 2,3 for more details. This difference was used (for given cuts) to upgrade the existing $\mathcal{O}(\alpha)$ calculation (BABAMC) already used in the LEP 1990/91 data at the level of the total cross section (not for differential distributions). The other smaller perturbative sub-leading contribution $\mathcal{O}(\alpha^2 L)$ was estimated in several different ways and was found to be at the level of 0.2%. The $\mathcal{O}(\alpha^3 L^3)$ correction was directly calculated. The total QED uncertainty was estimated/calculated at the level of 0.3%. Note that in the above calculations we have used the experimental trigger of the calorimetric type which does not distinguish the electron and photon and measures their total energy. This kind of measurement eliminates corrections due to final state QED bremsstrahlung, in the LL approximation.

The OLDBIS+LUMLOG (or BABAMC

^{#2} Note that technical precision was in this case calculated and not merely estimated.

Fig. 1. Second and higher order correction for asymmetric calorimetric trigger Ξ_{NW} in BHLUMI and LUMLOG for varying energy cut z_{min} . Plotted are $\sigma^{B,L} = (\sigma_1^{B,L} - \sigma_2^{B,L})/\sigma_{Born}$, where σ_{Born} is calculated for the Narrow-Narrow angular range. $\sigma_1^B = \sigma\{\mathcal{O}(\alpha^1)_{exp}\}$ is from BHLUMI and unexponentiated $\sigma_2^B = \sigma\{\mathcal{O}(\alpha^1)\}$ is from OLDBIS [8,1]. $\sigma_1^L = \sigma\{\mathcal{O}(\alpha^3)_{exp}\}$ and $(\sigma_2^L = \sigma\{\mathcal{O}(\alpha^1)\})$ are both from LUMLOG [2]. We plot $\delta^{B,L}$ separately. Their difference represents the missing $\mathcal{O}(\sigma^2 L^2)$ bremsstrahlung correction in BHLUMI (together with its technical precision). It is less than 0.15% as quoted in Table 2b of Ref. 2. Vacuum polarization, Z and s channel γ are switched off. MC statistical errors correspond to $2.5 \cdot 10^7$ of weighted events from BHLUMI and 10^8 events from OLDBIS and LUMLOG.

+LUMLOG) solution of Ref. 2 was a temporary solution, not very convenient because it was not implemented in the form of the single MC event generator. It was limited to the total cross section (although for a wide class of

cuts) and not really applicable for the differential cross sections. In the third step of Refs. 3,4 we have taken the multiphoton MC generator of Ref. 12 which is based on the Yennie-Frautschi-Suura $\mathcal{O}(\alpha)$ exponentiation and we have proven, using the OLDBIS+LUMLOG calculation, that for semi-realistic experimental cuts close to the typical LEP experimental trigger the total cross section from BHLUMI stays very close, to within 0.15%, from the $\mathcal{O}(\alpha) + \mathcal{O}(\alpha^2 L^2)$ result. We have also improved on many aspects of the BHLUMI program with respect to the original version of Ref. 12 (numerical instabilities due to extreme smallness of the electron mass). The essence of the above test/exercise is well depicted in Fig. 1. The great advantage of BHLUMI, important for the data analysis, is that it simulates multiphoton events in a very realistic way and it is able to calculate reliably many inclusive distributions. This is in fact very helpful in reducing/understanding many subtle apparatus effects.

The new version 2.01 of the BHLUMI program was published in Ref. 4 and is now in use for 1991 LEP data analysis. It also contains as a Monte Carlo sub-generators the programs OLDBIS and LUMLOG such that the patient user is able to reproduce all important tests from Refs. 1,2 and 3 for himself.

Here, we should note that the overall precision of BHLUMI 2.01 was quoted conservatively in Ref. 3 as .25%. One contribution to this uncertainty, that from higher order

corrections to the $\gamma - Z^0$ interference effect, we estimated as .03% at $\sqrt{s} = M_{Z^0}$ whereas our more recent $\mathcal{O}(\alpha)$ exponentiated result for this effect is $\sim .1\%$, 2σ higher than the original estimate. Others [6] have recently estimated this effect at $\lesssim .2\%$, 5σ higher than the original estimate. We are currently implementing this higher order correction precisely into BHLUMI 2.01 and, in the interim period, we agree with the previous speaker that the overall precision of BHLUMI 2.01 is conservatively .3% if one considers non-QED effects also.

In this short note we concentrate only on the recent work of the present authors on the problem of QED corrections to small angle Bhabha. We refer the reader to Ref. 5 for a complete list of references to recent works in this subject.

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