CM-P00056220

CERN-TH.6539/92

RECENT SOLAR NEUTRINO OBSERVATIONS AND UNCONVENTIONAL NEUTRINO PROPERTIES

P.I. Krastev *
Theory Division
CERN, CH-1211 Geneva 23, Switzerland

and

S.T. Petcov *
Scuola Internazionale Superiore di Studi Avanzati,
and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste,
I-34014 Trieste, Italy

Abstract

The possibilities are studied that vacuum oscillations or matter-enhanced transitions of the solar neutrinos are at the origin of the solar neutrino problem. The recently published first results from the GALLEX experiment and the latest Kamiokande data, together with the data available from the Homestake and SAGE experiments, are used in the analyses. We derive the regions of values of neutrino oscillation (transition) parameters for which the hypotheses of vacuum oscillations or of matter-enhanced transitions of solar neutrinos allow the results of the indicated experiments to be reconciled. Implications for future solar neutrino experiments are also discussed.

CERN-TH.6539/92

June 1992

* Permanent address: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria.

1. INTRODUCTION

If neutrinos possess "unconventional" intrinsic properties and/or couplings (non-zero masses, non-trivial mixing, magnetic dipole moments, etc.), the electron neutrinos (ν_e) emitted by the Sun can take part in a number of physically rather different processes while they travel to the Earth. These include, as is well known, oscillations in vacuum [1, 2] and transitions in the Sun [3–9] into different weak eigenstate neutrinos (ν_μ , and/or ν_τ , and/or sterile neutrino(s), ν_s , etc.), as well as a decay into a lighter neutrino with emission of additional sufficiently light (or massless) particle(s) [10]. The transitions in the Sun can occur when the solar ν_e propagate from the central region of the Sun, where they are supposedly produced [11, 12], to its surface. They can be of several varieties, depending on the physical mechanism that generates them. If neutrinos possess non-zero masses, the transitions of interest can be induced by the presence of non-trivial neutrino mixing in vacuum and/or by flavour changing neutral current (FCNC) interactions of the ν_e with the particles forming the matter in the Sun [5, 6]. Both types of transitions can be resonantly enhanced by solar-matter effects [3–6]. Matter-enhanced transitions of the solar neutrinos induced by FCNC interactions of the ν_e can occur in the Sun, even if neutrino mixing does not exist in vacuum and the neutrinos are massless particles [5]. Further, the solar neutrinos can undergo a spin precession or spin-flavour transitions in the magnetic field of the Sun if the electron neutrinos possess a sufficiently large magnetic (electric) dipole moment or a transition moment of the magnetic (electric) dipole type [7, 8]. Matter effects can enhance resonantly both the spin precession and the spin-flavour transitions of the solar neutrinos [9].

The processes indicated above can considerably alter the spectrum and/or the composition of the initial solar neutrino flux. They have been widely discussed (see, e.g., refs. [13, 3, 5–10, 14–16] and the articles quoted therein) as a possible cause for the apparent discrepancy between the results of the Davis et al. [17], Kamiokande II [18], and SAGE [19] solar neutrino experiments and the corresponding theoretical predictions based on standard solar model calculations [11, 12].

In the present work we study the implications of the recently published first results of the GALLEX solar neutrino experiment [20] for some of the proposed neutrino physics explanations of the solar neutrino observations. We analyse in detail the possibility to describe the current solar neutrino data assuming that the solar ν_e take part in oscillations in vacuum [1, 2] or undergo matter-enhanced transitions in the Sun [3, 4]. The results of the first 220 days of data taking of the Kamiokande III collaboration [21] are also included in our analysis.

2. THE SSM PREDICTIONS VERSUS THE SOLAR NEUTRINO DATA

According to the standard solar model [11, 12], the solar ν_e flux consists of several components (pp, ${}^7\text{Be}$, ${}^8\text{B}$, pep, ${}^{13}\text{N}$, ${}^{15}\text{O}$, etc.), three of which will play a major role in our discussion: the least energetic pp neutrinos ($E_{\nu}(\text{pp}) \leq 0.42$ MeV), the intermediate energy monochromatic ${}^7\text{Be}$ neutrinos [$E_{\nu}({}^7\text{Be}) = 0.862$ MeV (90% of the ${}^7\text{Be}$ neutrinos), 0.36 MeV (10%)], and the higher energy ${}^8\text{B}$ neutrinos ($E_{\nu}({}^8\text{B}) \leq 14$ MeV). Three different methods of detection of the solar neutrinos have been exploited in the five experiments that

have provided data so far: the radiochemical Cl–Ar method, proposed by Pontecorvo in 1946 [22], the neutrino-electron elastic scattering reaction, and the Ga–Ge radiochemical method [23]. The Cl–Ar method, as is well known, is used in the pioneer experiment of Davis et al.; the ν_e – e^- elastic scattering reaction is used by the Kamiokande II and Kamiokande III collaborations, while the SAGE and GALLEX experiments are based on the Ga–Ge method. The Cl–Ar and the Ga–Ge radiochemical methods permit us to detect electron neutrinos with energies exceeding 0.816 MeV and 0.233 MeV, respectively. The Kamiokande II and III experiments can register any active neutrino (i.e. ν_e , ν_μ , ν_τ) having energy greater than 7.5 MeV.

It follows from the presently existing results based on SSM calculations [24, 12] that from 70% to 77% of the event rate in the experiment of Davis et al. should be due to the ${}^8\text{B}$ neutrinos, while the ${}^7\text{Be}$ neutrinos should generate only from 14% to 17% of the solar-neutrino-induced events. The pp neutrinos give no contribution to the event rate in the Cl–Ar detectors because they possess energies that are below the threshold energy of the ν_e -induced Cl–Ar reaction. With a threshold energy of 7.5 MeV the Kamiokande II and Kamiokande III detectors are sensitive only to the ${}^8\text{B}$ component of the solar neutrino flux. Finally, the pp, the ${}^7\text{Be}$, and the ${}^8\text{B}$ neutrinos are predicted to produce, respectively, from 54% to 57%, from 24% to 27%, and from 7.4% to 10% of the events in the Ga–Ge experiments SAGE and GALLEX. Thus, the largest fractions of the solar-neutrino-induced signals in the Davis et al. and Kamiokande detectors, and in the SAGE and GALLEX detectors are produced, according to the SSM, by very different components of the solar neutrino flux, namely by the ${}^8\text{B}$ and the pp neutrinos. Although the ${}^7\text{Be}$ neutrinos contribute to the event rates in both Cl–Ar and Ga–Ge detectors, their respective contributions, being non-negligible, do not exceed considerably, as we shall see, the experimental uncertainties in the presently existing data of the corresponding experiments. All these facts are more or less well known.

Next, we shall summarize briefly some of the solar neutrino data and the relevant SSM results, which will be used in our analysis. The average rate of production of ${}^{37}\text{Ar}$ by the solar neutrinos, $\bar{R}(\text{Ar})$, observed in the experiment of Davis et al. [17] during approximately a 20-year period of data taking, reads:

$$\bar{R}(\text{Ar}) = (2.19 \pm 0.25) \text{ SNU}. \quad (1)$$

The current SSM prediction for $\bar{R}(\text{Ar})$ vary between the recent result of Bahcall and Pinsonneault [24]

$$\bar{R}(\text{Ar})_{BP} = (8.0 \pm 3.0) \text{ SNU}, \quad (2)$$

where 3 SNU corresponds to an "effective 3 standard deviations uncertainty", and the prediction of Turck-Chièze et al. [12]

$$\bar{R}(\text{Ar})_S = (6.4 \pm 1.4) \text{ SNU}, \quad (3)$$

1.4 SNU representing an estimated 1 s.d. uncertainty.

The Kamiokande II and Kamiokande III collaborations have measured the flux of ${}^8\text{B}$ neutrinos during a total of 1040 days (in the period 1986–1990) and of 220 days (in 1991),

respectively. The combined result of the two experiments for the inferred average flux of ^8B neutrinos, $\bar{\Phi}(\text{B})$, has the form [21]

$$\bar{\Phi}(\text{B}) = (0.48 \pm 0.05 \pm 0.06) \Phi^0, \quad (4)$$

where $\Phi^0 = 5.8 \times 10^6 \nu_e \text{ cm}^{-2} \text{ sec}^{-1}$. It differs very little from the result of the Kamiokande II collaboration [18]. The values of $\bar{\Phi}(\text{B})$, calculated theoretically in refs. [24] and [12], are again to a large extent representative of the possible variations in the current SSM predictions. They are given respectively by:

$$\bar{\Phi}(\text{B})_{BP} = 0.98(1 \pm 0.43) \Phi^0 \quad (3 \text{ s.d.}), \quad (5)$$

$$\bar{\Phi}(\text{B})_S = 0.74(1 \pm 0.25) \Phi^0 \quad (1 \text{ s.d.}). \quad (6)$$

Let us note that apart from differences in the used input data (notably the cross-section of the reaction $p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$), the calculations performed in refs. [24] and [12] leading to the predictions (2), (5) and (3), (6) differ in the treatment of the diffusion of ${}^4\text{He}$ in the Sun: the latter is taken into account in the analysis of the authors of ref. [24] and is not accounted for in ref. [12]. It is interesting to note that the predictions of Bahcall and Pinsonneault [24] and those of Bahcall and Ulrich [11] concerning the different components of the solar neutrino flux are very similar, although they are based on physically rather different models.

Finally, let us quote the results of the Ga-Ge experiments. In their most physics runs (≈ 150 days (5 runs) of data taking in the period January–June 1990), the SAGE collaboration observed a rate of ${}^{71}\text{Ge}$ production, which was compatible with the background rate. So, their result is expressed as an upper limit on the rate of ${}^{71}\text{Ge}$ production due to solar neutrinos [19]:

$$\bar{R}_{\text{exp}}(\text{Ge}) < 79 \text{ SNU} \quad (90\% \text{ C.L.}). \quad (7)$$

The GALLEX collaboration, on the basis of data collected in 14 runs during 295 days of measurements (in the period 14 May 1991 to 31 March 1992) has announced recently the observation of a positive signal due to solar neutrinos [20]:

$$\bar{R}_{\text{exp}}(\text{Ge}) = (83 \pm 19 \pm 8) \text{ SNU}. \quad (8)$$

The SSM predictions for $\bar{R}(\text{Ge})$ vary very little [11, 12, 24]:

$$\bar{R}_{\text{th}}(\text{Ge}) = [(122 \text{ to } 132) \pm (5 \text{ to } 7)] \text{ SNU} \quad (1\text{s.d.}). \quad (9)$$

Two remarks are in order. Given the uncertainties in the observations, there is no strong contradiction, in our opinion, between results (7) and (8) of the SAGE and the GALLEX experiments. Secondly, the result, remarkable in many respects, of the GALLEX collaboration, if confirmed by the scheduled calibration measurements, would imply, in particular, that the pp neutrinos are indeed emitted by the Sun and that a considerable fraction of these neutrinos reaches the Earth without undergoing any changes.

Taken together, the current solar neutrino observations by no means prove that neutrinos possess "unconventional" properties and/or couplings, manifesting themselves in the existence of solar neutrino oscillations, matter-enhanced or matter-induced transitions, or in solar neutrino instability. However, the observations contain evidence in favour of this possibility. In fact, the strongest evidence in favour of the neutrino physics solution of the solar neutrino problem comes from the current results (17) of the experiment of Davis et al. and from the present SSM predictions, eqs. (2) and (3).

In what follows we shall assume that the differences between the solar neutrino data and the SSM predictions are not due to unaccounted for astrophysical effects, poorly known nuclear reaction cross-sections, etc., but rather are related to a "non-standard" behaviour (i.e. oscillations, transitions, decays) of the solar electron neutrinos on their way to the Earth. We shall try to understand and describe the specific features of the data on the basis of the indicated hypothesis. Our analyses will be primarily concerned with the possibility that the solar neutrinos take part in oscillations in vacuum while they travel to the Earth, or undergo resonant conversion in matter while they propagate from the central region to the surface of the Sun.

Let us begin with two qualitative observations. Considering only the mean values of $\bar{R}(\text{Ar})$, $\bar{\Phi}(\text{B})$ and $\bar{R}(\text{Ge})$ in eqs. (1)–(6), (8) and (9), one arrives at the conclusion that $\bar{R}_{\text{exp}}(\text{Ar})/\bar{R}(\text{Ar})_{BP(S)} < \bar{\Phi}_{\text{exp}}(\text{B})/\bar{\Phi}(\text{B})_{BP(S)} < \bar{R}_{\text{exp}}(\text{Ge})/\bar{R}(\text{Ge})_{\text{th}} < 1$. This implies that i) the ${}^7\text{Be}$ neutrino flux can be suppressed much more strongly than the ${}^8\text{B}$ neutrino flux, and ii) the possible mechanism of suppression of the signals in the Cl-Ar, ν - e^- scattering and the Ga-Ge experiments in the case under discussion should produce a strong dependence of the degree of suppression of the different energy components of the solar neutrino flux on the ν_e energy. Taking into account a 2 s.d. uncertainty in the experimental results for $\bar{R}(\text{Ar})$, $\bar{\Phi}(\text{B})$ and $\bar{R}(\text{Ge})$, eqs. (1), (4), (8), and the mean values of the corresponding theoretical predictions [eqs. (2) (3)], (5) ((6)) and (9)], it is easy to convince oneself that the existing data are compatible also with $\bar{R}_{\text{exp}}(\text{Ar})/\bar{R}(\text{Ar})_{BP(S)} \cong \bar{\Phi}_{\text{exp}}(\text{B})/\bar{\Phi}(\text{B})_{BP(S)} \cong \bar{R}_{\text{exp}}(\text{Ge})/\bar{R}_{\text{th}}(\text{Ge})$, i.e. an energy-independent suppression of the pp, ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes (by approximately a factor of 3).

3. VACUUM OSCILLATIONS OF SOLAR NEUTRINOS

Consider next the possibility that solar neutrinos take part in two-neutrino oscillations in vacuum: $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, or $\nu_e \leftrightarrow \nu_\mu$. These oscillations are characterized by two parameters: $\Delta m^2 = m_2^2 - m_1^2 > 0$ and $\sin^2 2\theta$, $m_{1,2}$ being the masses of two neutrinos with definite mass in vacuum, and θ being the neutrino mixing angle in vacuum*. In a recent study [16] we have determined the regions of values of the parameters Δm^2 and $\sin^2 2\theta$, for which the two-neutrino oscillations of ν_e in vacuum permit to reconcile (at 90% C.L.) the Davis et al., Kamiokande II and SAGE results. The method of analysis of the data we have used in ref. [16] is, in particular, especially sensitive to the dependence of the neutrino oscillation probability on the Sun–Earth distance. It utilizes the data obtained in each of the physics runs of the indicated three experiments, rather than averages of data

* For further details concerning the theory of the neutrino oscillations in vacuum, see for instance refs. [13, 25].

accumulated in several runs (for details see ref. [16]). The results we have obtained for the cases of $\nu_e \leftrightarrow \nu_\mu(\tau)$ and $\nu_e \leftrightarrow \nu_s$ oscillations using the SSM predictions of ref. [24]* (or of ref. [11], which is practically equivalent) are presented graphically as shaded (grey + black) areas in fig. 1a and fig. 1b, respectively. The values of Δm^2 and $\sin^2 2\theta$ forming the shaded areas in fig. 1a and fig. 1b lie in the intervals [16]:

$$\begin{aligned} 0.5 \times 10^{-10} \text{ eV}^2 \leq \Delta m^2 \leq 1.1 \times 10^{-10} \text{ eV}^2, \\ 0.75 \leq \sin^2 2\theta \leq 1. \end{aligned} \quad (10)$$

Note that for $\Delta m^2 \simeq 10^{-10} \text{ eV}^2$ the oscillation length in vacuum [13], $L_\nu = 4\pi p / \Delta m^2$, p being the absolute value of the neutrino momentum, is comparable, for the solar neutrinos having energies $p \simeq 10 \text{ MeV}$, with the average distance between the Sun and the Earth, $R_0 \simeq 1.49 \times 10^8 \text{ km}$.

The analysis we have performed in ref. [16] also led to the conclusion that if the solar electron neutrinos take part in two-neutrino oscillations in vacuum characterized by the values of Δm^2 and $\sin^2 2\theta$ allowed by the Davis et al. and the Kamiokande data, i) the one-year averaged rate of ^{71}Ge production in the Ga-Ge experiments, $\bar{R}_y(\text{Ge})$, should lie in the interval

$$40 \text{ SNU} \leq \bar{R}_y(\text{Ge}) < 90 \text{ SNU}, \quad (11)$$

and ii) there can be seasonal variations of $\bar{R}(\text{Ge})$, the absolute difference between the one-month average values of $\bar{R}(\text{Ge})$ for December and June being not larger than approximately 20 SNU and 15 SNU for the cases of $\nu_e \leftrightarrow \nu_\mu(\tau)$ and $\nu_e \leftrightarrow \nu_s$ oscillations, respectively. The predicted seasonal changes of $\bar{R}(\text{Ge})$ are associated with the strong dependence [for the values of Δm^2 and $\sin^2 2\theta$ belonging to the regions determined by eq. (10)] of the vacuum oscillation probability for the monochromatic ^{71}Be neutrinos on the Sun-Earth distance, which varies because of the eccentricity of the Earth's orbit around the Sun.

In figs. 1a and 1b the iso-SNU contours corresponding to values of $\bar{R}_y(\text{Ge})$ of 83 SNU, 62 SNU and 104 SNU [i.e. the mean value and the 1 s.d. lower and upper limit values of the GALLEX result (8)] are also shown. The values of Δm^2 and $\sin^2 2\theta$ forming the black areas are allowed, including the GALLEX data at 68% C.L. (1 s.d.). Even at this confidence level, the solar neutrino data can be described perfectly well on the basis of the hypothesis of two-neutrino oscillations in vacuum of the solar ν_e , the corresponding region of values of Δm^2 and $\sin^2 2\theta$ being somewhat reduced by the restrictions following from the GALLEX result. As can be shown, at 95% C.L. (i.e. 2 s.d.) the GALLEX observations

* To assess the astrophysical uncertainty in our results we have performed in ref. [16] the same analysis utilizing the SSM predictions of ref. [12]. Although the regions of values of Δm^2 and $\sin^2 2\theta$ found in this case differ somewhat from those described by eq. (10), the differences are not large (as can be seen from figs. 1a and 1b, and 2a and 2b in ref. [16]), and we do not present the results based on the model of ref. [12] here.

(8) do not lead to any new constraints on the values of Δm^2 and $\sin^2 2\theta$ allowed by the results of the other solar neutrino experiments* (grey + black areas in figs. 1a and 1b).

An analogous analysis can be performed assuming that the solar ν_e take part in three-neutrino oscillations in vacuum [26]. The latter are characterized by five parameters (see for instance [25]): two differences of the squares of the masses of the three neutrinos $\nu_{1,2,3}$ with definite masses $m_{1,2,3}$ in vacuum, $\Delta m_{21}^2 = m_2^2 - m_1^2 > 0$ and $\Delta m_{31}^2 = m_3^2 - m_1^2 > 0$, and by three lepton mixing angles in vacuum, $\theta_1, \theta_2, \theta_3$. Obviously, the solar neutrino data will restrict at least one of the two quantities Δm_{21}^2 and Δm_{31}^2 , and at least one of the three mixing angles $\theta_{1,2,3}$ to lie in intervals which are approximately the same as those determined by eq. (10); and the important predictions concerning the maximal value of $\bar{R}_y(\text{Ge})$, $\bar{R}_y(\text{Ge}) < 90 \text{ SNU}$, and the possibility of large seasonal variations of the ^{71}Be neutrino flux, remain valid also in the three-neutrino oscillation case. The vacuum oscillations of the solar neutrinos would be an unlikely solution of the solar neutrino problem if, for instance, in the Ga-Ge experiments, one observed an average rate of ^{71}Ge production per year $\bar{R}_y(\text{Ge})$ that would be definitely greater than 90 SNU.

4. THE CASE OF MATTER-ENHANCED TRANSITIONS

Let us now turn to the possibility that the solar neutrinos undergo two-neutrino transitions $\nu_e \rightarrow \nu_\mu(\tau)$ in the Sun, which are resonantly enhanced by solar matter effects [3]. At present there exists a well-developed theory and a complete, simple and very accurate analytic description of the indicated transitions of the solar neutrinos in the Sun (see, e.g., refs. [3, 25, 27]). The transitions of interest depend on the same two parameters Δm^2 and $\sin^2 2\theta$, which enter into the expression for the vacuum oscillation probabilities. For a solar ν_e with momentum $\vec{p}$, they depend also on the value of the electron number density N_e at the point of neutrino production in the Sun, $N_e(t_0)$, and on the resonance density [28, 3]

$$N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2p\sqrt{2}G_F}, \quad (12)$$

G_F being the Fermi constant, as well as on the way (relatively "fast" or "slow") N_e changes along the neutrino path in the Sun. The latter, in particular, determines the type of the solar neutrino transitions in the Sun, which can be adiabatic or non-adiabatic [3]. According to the SSM [11, 12, 24], N_e decreases approximately exponentially with a scale height $r_0 \simeq 0.1 R_\odot$, $R_\odot \simeq 6.96 \times 10^5 \text{ km}$ being the solar radius, in the radial direction from the centre to the surface of the Sun. The probability that a ν_e having momentum $\vec{p}$ and

* We did not use the data from each of the 14 runs of the GALLEX collaboration [20] in the manner this was done in ref. [16] with the data from the Cl-Ar, Kamiokande II and SAGE experiments, since the determination of the background in the data for the majority of the runs is not yet completed. Partially for the same reason, we did not utilize in our analyses more refined statistical criteria for the quality of the combined description of the data from the different solar neutrino experiments, on the basis of the vacuum oscillation and of the matter-enhanced neutrino transition hypotheses (see further). The application of such criteria would have the effect of reducing somewhat the regions of the allowed values of the parameters Δm^2 and $\sin^2 2\theta$.

produced at time t_0 in the central part of the Sun will not transform into $\nu_{\mu(r)}$ on its way to the surface of the Sun (reached at time t_s), $P(\nu_e \rightarrow \nu_e; t_0, t_s)$, is given by [29]

$$P(\nu_e \rightarrow \nu_e; t_0, t_s) = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_m(t_0) \cos 2\theta. \quad (13)$$

Here

$$P' = \frac{\exp\left[-\pi\tau_0 \frac{\Delta m^2}{2p}(1 - \cos 2\theta)\right] - \exp\left[-2\pi\tau_0 \frac{\Delta m^2}{2p}\right]}{1 - \exp\left[-2\pi\tau_0 \frac{\Delta m^2}{2p}\right]} \quad (14)$$

is [29, 30] the level-crossing probability (i.e. the analogue of the Landau-Zener probability) for exponentially varying density N_e , and $\theta_m(t_0)$ is the neutrino mixing angle in matter [4] at the point of ν_e production:

$$\tan 2\theta_m(t_0) = \frac{\tan 2\theta}{(1 - N_e(t_0)/N_e^{res})}. \quad (15)$$

For the adiabatic (non-adiabatic) $\nu_e \rightarrow \nu_{\mu(r)}$ transitions, the probability P' is negligible (non-negligible).

A few comments are in order. First, unlike the expressions for P' proposed by Parke and by Pizzochero [31], the one given by eq. (14) correctly describes the non-adiabatic transitions of the solar neutrinos for values of $\sin^2 2\theta \gtrsim 0.2$ [30, 32, 29]. Secondly, the probability $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ is not given by eqs. (13) and (14) (or by those proposed in refs. [31]) when the solar neutrinos are born in the resonance region (i.e. $|\tan 2\theta_m(t_0)| \geq 1$) and the $\nu_e \rightarrow \nu_{\mu(r)}$ transitions are non-adiabatic* [32, 29, 30]. However, the indicated possibility can be realized for the solar neutrinos only for $\sin^2 2\theta < 4 \times 10^{-3}$ [30]**, while the values of $\sin^2 2\theta$ relevant for the problem we are studying are larger, as we shall see. Third, the probability $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ contains oscillating terms [33] in addition to the expression in the right-hand side of eq. (13), which represents the average probability. As was shown in ref. [34], the oscillating terms in $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ can be important for the monochromatic ${}^7\text{Be}$ and pep neutrinos for values of $\Delta m^2 \lesssim 10^{-8} \text{ eV}^2$. We can justify neglecting the oscillating terms in $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ in the present analysis *a posteriori*: the current solar neutrino data, as shown below, suggest that $\Delta m^2 \gtrsim 10^{-7} \text{ eV}^2$. Finally, for the values of $\sin^2 2\theta \gtrsim 4 \times 10^{-3}$ we are interested in, expressions (13) and (14) describe with very good accuracy not only the transitions of the solar neutrinos born in the close (with respect to the Earth) solar hemisphere, but also those produced in the far hemisphere of the Sun [27]***.

* A simple analytic expression for $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ was derived in the indicated case and can be found in ref. [30].

** As was shown in ref. [30], for $\sin^2 2\theta < 4 \times 10^{-3}$ and $N_e^{res} > N_e(t_0)(1 - \tan 2\theta)^{-1}$ the $\nu_e \rightarrow \nu_{\mu(r)}$ transitions in the Sun are adiabatic and are described by eq. (12), in which $P' = 0$.

*** A concise summary of the analytic description of the matter-enhanced transitions of the solar neutrinos in the Sun is given in the first of the articles quoted in ref. [27].

In the calculations performed for the present study we utilize a certain procedure [30], which makes the analytic description of the matter-enhanced transitions of the solar neutrinos based on eqs. (14) and (15) remarkably precise (the precision is typically not worse than 95% in the domain of values of Δm^2 and $\sin^2 2\theta$ of interest [30]). Namely, in order to account for the possible effects of the departure from the exponential variation of the N_e in certain parts of the neutrino path in the Sun, we treat the scale height r_0 entering into the expression (15) for the probability P' as a parameter whose value is determined for given $p/\Delta m^2$ and $\sin^2 2\theta$ as

$$r_0 = -\frac{N_e^{res}}{(dN_e/dr)_{res}}, \quad (17)$$

where $(dN_e/dr)_{res}$ is the derivative of N_e in the radial direction (or in the direction of the neutrino momentum) in the corresponding resonant point in the Sun. Using the relevant tables in ref. [11], we interpolate with a standard cubic spline the values of N_e . Given the value of N_e^{res} and utilizing the spline, one can find the location of the resonant point in the Sun, calculate the value of the derivative of N_e and, correspondingly, the requisite value of r_0 , at this point*.

The probability $P(\nu_e \rightarrow \nu_e; t_0, t_s)$, calculated for given $p/\Delta m^2$ and $\sin^2 2\theta$ in the manner indicated above, is then averaged over the region of neutrino production in the Sun using the spatial distribution of the source strengths of the different components of the solar neutrino flux given in table XI of ref. [11]**. Let us mention that, for $\Delta m^2 \gtrsim 10^{-7} \text{ eV}^2$, the averaging over the region of solar neutrino production and the integration over the neutrino energy render negligible all the interference terms that appear in the probability of ν_e survival, because of the $\nu_e \leftrightarrow \nu_{\mu(r)}$ oscillations in vacuum taking place on the way of the neutrinos from the surface of the Sun to that of the Earth. Thus, the probability that ν_e will remain ν_e while it travels from the central part of the Sun to the surface of the Earth is effectively equal to $P(\nu_e \rightarrow \nu_e; t_0, t_s)$ and is given by eqs. (13) and (14).

The remaining part of the analysis is rather straightforward. The use of an analytic expression for the ν_e survival probability allows us to draw contours of equal suppression of the signals in the different solar neutrino experiments with sufficiently small steps in Δm^2 and $\sin^2 2\theta$. This is important when looking for an overlap of two narrow regions allowed by the data from different experiments.

The results of our study are presented graphically in figs. 2a and 2b where, respectively, the 68% C.L. and 95% C.L. iso-SNU and iso-suppression contours, corresponding to the data (1), (8) and (4) of the Davis et al., GALLEX and Kamiokande experiments, are shown. The iso-SNU contours associated with the GALLEX result (8) give the values of Δm^2 and $\sin^2 2\theta$ for which the average rate of ${}^{71}\text{Ge}$ production per year in the Ga-Ge experiments, $\bar{R}_y$, is within the limits $62 \text{ SNU} \leq \bar{R}_y \leq 104 \text{ SNU}$ (fig. 2a) and $41 \text{ SNU} \leq \bar{R}_y \leq 125 \text{ SNU}$ (fig. 2b). We have depicted in the same figures also the iso-SNU contours corresponding to the 68% C.L. and 90% C.L. upper limits obtained by the SAGE

* See ref. [30] for further details.

** The N_e distributions and the neutrino source strength distributions given in ref. [11] differ very little from those obtained in ref. [24].

collaboration (the 95% C.L. upper limits from the SAGE data were not available to us). The curves were derived taking the mean values of the SSM predictions (2) and (5); the value of 132 SNU [11, 24] has been used as a SSM prediction for the event rate in the Ga-Ge experiments*. As is clear from figs. 2a and 2b, the current solar neutrino data can be described rather well on the basis of the hypothesis that the solar neutrinos undergo two-neutrino matter-enhanced transitions into active neutrinos of a different flavour [i.e. $\nu_e \rightarrow \nu_{\mu(\tau)}$] in the Sun. The values of Δm^2 and $\sin^2 2\theta$, for which the matter-enhanced transitions of the solar ν_e allow us to reproduce the results of the Davis et al., Kamiokande and GALLEX experiments, are shown as shaded areas (grey + black) on figs. 2a and 2b. The implications of the SAGE results are also indicated. The restrictions following from the absence of a difference between the event rates observed during the day and during the night by the Kamiokande II experiment are taken into account in figs. 2a and 2b as well. The 95% C.L. allowed values of the parameters Δm^2 and $\sin^2 2\theta$, forming the three disconnected shaded areas on fig. 2b, lie in the intervals:

$$3.2 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 1.2 \times 10^{-5} \text{ eV}^2, \quad (18a)$$

$$5 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 1.6 \times 10^{-2}, \quad (18b)$$

$$\Delta m^2 \sin^2 2\theta \cong (4.3 \pm 1.1) \times 10^{-8} \text{ eV}^2, \quad (18c)$$

$$5.4 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 1.1 \times 10^{-4} \text{ eV}^2, \quad (19a)$$

$$0.18 \lesssim \sin^2 2\theta \lesssim 0.86, \quad (19b)$$

$$10^{-7} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 1.8 \times 10^{-6} \text{ eV}^2, \quad (20a)$$

$$0.74 \lesssim \sin^2 2\theta \lesssim 0.93. \quad (20b)$$

A few comments concerning the physical character of the $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions corresponding to the values (18), (19) and (20) of Δm^2 and $\sin^2 2\theta$ for which one obtains a solution of the solar neutrino problem, are in order. For Δm^2 and $\sin^2 2\theta$ belonging to the intervals (18a)–(18b) (the small mixing angle case) the transitions of the ^8B neutrinos are non-adiabatic, while the transitions of the pp and the ^7Be neutrinos are adiabatic. We shall call this solution "non-adiabatic". In the case of the non-adiabatic solution the

* We do not take into account the uncertainties in the theoretical predictions (2), (5) and (9), which are quite large [especially for $\bar{R}(\text{Cl})$] and, perhaps, not completely known, although it seems unlikely that they alone can account for the solar neutrino problem. Further, in a more refined statistical analysis of the experimental results, one should treat the data from the different experiments simultaneously, and thus determine the relevant allowed regions of the values of the parameters Δm^2 and $\sin^2 2\theta$ at a chosen confidence level. We believe that the results one would obtain using the more consistent approach, taking care of the indicated inaccuracies, would not be very different from the one derived by us, as the effects on these results from the improvements of the analysis along these two points tend to mutually cancel.

probability of ν_e survival in the Sun strongly depends on the energy of the solar ν_e : it is very small ($\sim 10^{-3}$) for the ^7Be neutrinos, close to 1 for the pp neutrinos, and varies with the energy for the ^8B neutrinos. The large mixing angle solutions corresponding to values of Δm^2 and $\sin^2 2\theta$ from (19a)–(19b) and (20a)–(20b) are associated with adiabatic transitions of the ^8B , ^7Be and pp neutrinos. We shall use the term "adiabatic" for these solutions. For the values of Δm^2 and $\sin^2 2\theta$ from the intervals (20a)–(20b), for instance, the survival probability for the ^8B , ^7Be and pp electron neutrinos is roughly the same and is approximately equal to $\sin^2 \theta$. The same is true for the allowed values of Δm^2 and $\sin^2 2\theta$ from the intervals (19a)–(19b) when $\sin^2 2\theta \gtrsim 0.65$. For $0.18 \lesssim \sin^2 2\theta \lesssim 0.65$ and the corresponding values of Δm^2 from (19a) the survival probability for the ^8B , ^7Be , and pp neutrinos can differ substantially: for example, for $\sin^2 2\theta = 0.2$ and $\Delta m^2 = 10^{-4} \text{ eV}^2$ it is approximately equal to 1 for the ^7Be and the pp neutrinos, and exhibits very strong energy dependence in the case of ^8B neutrinos with an average value equal to roughly 0.2 for the neutrinos with energy in the interval $7 \text{ MeV} \lesssim E \lesssim 10 \text{ MeV}$.

How will the solutions we have described above change if the solar neutrinos undergo transitions into sterile neutrinos, $\nu_e \rightarrow \nu_s$, in the Sun? The iso-suppression contours on figs. 2a and 2b following from the Kamiokande data will have slightly different positions in this case: while the "horizontal" sections of the contours will practically not change, the "diagonal" and the "vertical" (large mixing angle) sections will be located more "outwards", i.e. for given Δm^2 the corresponding values of $\sin^2 2\theta$ on these two sections of the contours will be respectively smaller and larger than the values on the Kamiokande contours shown in figs. 2a and 2b. This implies that the 1 s.d. solutions (see fig. 2a) can disappear, while the two regions of fig. 2b that are associated with eqs. (18) and (19) will "shrink" somewhat in $\sin^2 2\theta$: the first towards the smaller values of $\sin^2 2\theta$ from the interval (18b), and the second towards the larger values of $\sin^2 2\theta$ from the interval (19b).

5. IMPLICATIONS FOR FUTURE SOLAR NEUTRINO EXPERIMENTS

Finally, we would like to consider some of the implications of our results for the future solar neutrino experiments SNO [35], Super-Kamiokande [36], BOREXINO [37] and ICARUS [38] (the first two are fully approved and are under construction; the other two are at the stage of prototype construction and/or testing). All these are planned to be high-statistics experiments with real time event detection. Except for the BOREXINO, they will be sensitive only to the ^8B component of the solar ν_e flux. Approximately 90% of the signal induced by the solar neutrinos in the BOREXINO detector (~ 50 events/day according to the SSM) is predicted to be due to the ^7Be neutrinos. The detection of the solar neutrinos in the Super-Kamiokande and the BOREXINO experiments will be via the $\nu_e\text{-e}^-$ elastic scattering reaction; in SNO and ICARUS experiments this reaction will also be used (in addition to other processes) for registration of the solar neutrinos. In the SNO, Super-Kamiokande and ICARUS experiments, which will detect only the ^8B neutrinos, the energy of the final-state electrons from the $\nu_e\text{-e}^-$ elastic scattering will be measured, although with different "thresholds": 5 MeV, 7.5 MeV, and 3 MeV, respectively. The question we would like to address is whether the indicated experiments will be capable to distinguish between the different solutions of the solar neutrino problem discussed above.

The designed characteristics of the BOREXINO detector [37] and the remarkably narrow width of the ${}^7\text{Be}$ neutrino 0.86 MeV line make the BOREXINO experiment appear best adapted to test the vacuum oscillation solution of the solar neutrino problem [16] and to distinguish between it and the matter-enhanced transitions (or MSW) solutions. In the case of the vacuum oscillation solution [see eq. (10)], a rather strong monthly and/or seasonal variation of the flux of the ${}^7\text{Be}$ electron neutrinos at the Earth surface is predicted to take place (in addition to the expected standard $\sim 7\%$ seasonal variation), as the probability of the ${}^7\text{Be}$ neutrino survival, $P({}^7\text{Be})$, exhibits typically a strong oscillatory dependence on the Sun-Earth distance. For instance, for $\Delta m^2 = 7.9 \times 10^{-11} \text{ eV}^2$, $6.3 \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta = 0.8$ ($\sin^2 2\theta = 0.9$), the one month average values of $P({}^7\text{Be})$ for December and June read, respectively: 0.19 and 0.35; 0.35 and 0.19 (0.10 and 0.26; 0.28 and 0.10). Let us recall that the suppression factor F of the signal in the BOREXINO experiment is approximately equal to the suppression factor $F({}^7\text{Be})$ of the 0.86 MeV ${}^7\text{Be}$ neutrino flux, $F \approx F({}^7\text{Be})$, which in turn is given by $F({}^7\text{Be}) \approx P({}^7\text{Be}) + 0.16(1 - P({}^7\text{Be}))$ in the case of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations ($\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions), and coincides with $P({}^7\text{Be})$, $F({}^7\text{Be}) = P({}^7\text{Be})$, if $\nu_e \leftrightarrow \nu_s$ oscillations ($\nu_e \rightarrow \nu_s$ transitions) take place. It should be noted that the seasonal variations of the flux of the ${}^8\text{B}$ electron neutrinos (and therefore of the event rates in the SNO, Super-Kamiokande and ICARUS detectors), predicted to take place because of the vacuum oscillations only, are not larger than 15%. For some of the allowed values of Δm^2 and $\sin^2 2\theta$ from (10) (e.g. for $\Delta m^2 \sim 10^{-10} \text{ eV}^2$) they can compensate the standard 7% seasonal change of the flux of ${}^8\text{B}$ neutrinos reaching the Earth, associated simply with the change of the Sun-Earth distance. It should also be mentioned that for some of the marginally allowed values of Δm^2 in the interval given in eq. (10) (e.g. for $\Delta m^2 \approx 0.5 \times 10^{-10} \text{ eV}^2$, 10^{-10} eV^2 , see figs. 1a and 1b), the seasonal variation of $P({}^7\text{Be})$ is not dramatic [being $\sim (15-25)\%$] and the difference between the monthly average values of $P({}^7\text{Be})$ is maximal for December and March (September), the values for December and June being approximately equal.

In contrast with the behaviour of $P({}^7\text{Be})$ in the vacuum oscillation solution case, the averaged probability of ν_e survival in the Sun (and therefore, the corresponding suppression factors) does not exhibit, in the case of the MSW solutions, monthly and/or seasonal variations, being independent on the Sun-Earth distance for any of the components (${}^8\text{B}$, ${}^7\text{Be}$, pp, etc.) of the solar ν_e flux. Thus, a constant value of $F({}^7\text{Be})$ all through the year will be an evidence in favour of the MSW solutions of the solar neutrino problem. Would it be possible to distinguish between the different MSW solutions on the basis of the BOREXINO data? Values of $F({}^7\text{Be}) \approx 0.16$ and $F({}^7\text{Be}) \sim 10^{-3}$ are predicted in the case of the non-adiabatic solution: the first corresponds to $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions, and the second to $\nu_e \rightarrow \nu_s$ transitions. The suppression factor $F({}^7\text{Be})$ should lie in the interval $0.20 \lesssim F({}^7\text{Be}) \lesssim 0.60$ ($0.60 < F({}^7\text{Be}) \leq 1$) if the adiabatic solution is valid and $\sin^2 2\theta \geq 0.65$ ($0.18 \leq \sin^2 2\theta < 0.65$).

The SNO, Super-Kamiokande and ICARUS experiments will provide, in particular, information about the spectrum of the final-state electrons in the $\nu_e e^-$ elastic scattering reaction. This information could be used to try to distinguish between the different solutions of the solar neutrino problem. The predicted spectra for several typical values of Δm^2 and $\sin^2 2\theta$ from (10), (18), (19) and (20), and ν_e oscillations or transitions in

$\nu_{\mu(\tau)}$ are shown in figs. 3a and 3b. As follows from these figures, it will be difficult, but perhaps not impossible, to determine the type of the solution on the basis only of the data about the e^- spectrum. It is also obvious from figs. 3a and 3b that the experiments using detectors with lower "thresholds" will have greater capabilities to discriminate between the different solutions.

To conclude, we have shown that the vacuum oscillations and the matter-enhanced transitions of the solar neutrinos continue to be viable and aesthetically very appealing solutions of the solar neutrino problem. Future real-time, high-statistics solar neutrino experiments (SNO, Super-Kamiokande, BOREXINO, ICARUS) will be able to test further and possibly discriminate between these solutions.

Acknowledgements

We would like to thank Y. Suzuki for providing us with the latest data of the Kamiokande III experiment. The work of S.T.P. was supported in part by the Bulgarian National Science Foundation via grant PH-16.

Note Added. At the International Conference on High Energy Physics, held in August in Dallas (Texas), the SAGE collaboration announced a new result based on data collected during 1991 [39]: $\bar{R}_{\text{SAGE}}(\text{Ge}) = (85^{+22}_{-32} \pm 20)$ SNU, which is in very good agreement with that obtained by the GALLEX collaboration during approximately the same period, eq. (8). Combining their 1990 and 1991 data, the SAGE collaboration finds [39] $\bar{R}_{\text{SAGE}}(\text{Ge}) = (58^{+17}_{-24} \pm 14)$ SNU. As can be shown, the latest SAGE result (unlike their first one) practically does not lead (at 68% C.L. and 95% C.L.) to new constraints on the parameters Δm^2 and $\sin^2 2\theta$, in addition to those following from the Davis et al., Kamiokande and GALLEX data, and shown graphically in figs. 1a and 1b for the case of vacuum oscillations, and in figs. 2a and 2b for the case of matter-enhanced transitions of the solar ν_e . Thus, eqs. (10) and (18)–(20) determine the values of Δm^2 and $\sin^2 2\theta$ for which the current solar neutrino observations, including the latest SAGE data, can be explained assuming that the solar neutrinos take part in two-neutrino oscillations in vacuum ($\nu_e \leftrightarrow \nu_{\mu(\tau)}$ or $\nu_e \leftrightarrow \nu_s$) when they travel from the surface of the Sun to the surface of the Earth, or undergo two-neutrino matter-enhanced transitions $\nu_e \rightarrow \nu_{\mu(\tau)}$ while propagating from the central part of the Sun to its surface.

REFERENCES

1. B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33 (1957) 549.
2. P. Pontecorvo, Zh. Eksp. Teor. Fiz. 34 (1958) 247; 53 (1967) 1717.
3. S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42 (1985) 1441.
4. L. Wolfenstein, Phys. Rev. D17 (1978) 2369.
5. M.M. Guzzo, A. Masiero and S.T. Petcov, Phys. Lett. 260B (1991) 154.
6. M.M. Guzzo and S.T. Petcov, Phys. Lett. 271B (1991) 172; E. Roulet, Phys. Rev. D44 (1991) R935.
7. A. Cisneros, Astrophys. & Space Sci. 10 (1970) 87.
8. L.B. Okun, M.B. Voloshin and M.I. Vysotski, Yad. Fiz. 44 (1986) 677.
9. C.-S. Lim and W. Marciano, Phys. Rev. D37 (1988) 1368; E.Kh. Akhmedov, Phys. Lett. 213B (1988) 64.
10. J.N. Bahcall, N. Cabibbo and A. Yahil, Phys. Rev. Lett. 28 (1972) 316; J.N. Bahcall et al., Phys. Lett. 181B (1986) 369; Z.G. Berezhiani et al., Z. Phys. C54 (1992) 581.
11. J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. 60 (1988) 298.
12. S. Turck-Chièze et al., Astrophys. J. 335 (1988) 425; S. Turck-Chièze, Talk given at the "Neutrino '92" Int. Conf., Granada, June 7-12, 1992.
13. S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41 (1978) 225.
14. P.I. Krastev, S.P. Mikheyev and A.Yu. Smirnov, INR preprint P-695, Moscow, 1991; J.N. Bahcall and W.C. Haxton, Phys. Rev. D40 (1989) 931; A.J. Baltz and J. Weneser, Phys. Rev. D37 (1988) 3364; J. Bouchez et al., Z. Phys. C32 (1986) 499; S.P. Rosen and J.M. Gelb, Phys. Rev. D34 (1989) 969; S.J. Parke and T.P. Walker, Phys. Rev. Lett. 57 (1986) 2322.
15. V. Barger, R.J.N. Phillips and K. Whisnant, Phys. Rev. D43 (1991) 1110; A. Acker, S. Pakvasa and J. Pantaleone, Phys. Rev. D43 (1991) 1754.
16. P.I. Krastev and S.T. Petcov, Phys. Lett. 285B (1992) 85.
17. R. Davis et al., XXI Int. Cosmic Ray Conf., January 6-19, 1990, Adelaide, Australia, Conference papers, vol. 7, p. 155, 1990.
18. K.S. Hirata et al., Phys. Rev. Lett. 63 (1989) 16, and 65 (1990) 1297 and 1301.
19. A.I. Abazov et al. (SAGE collaboration), Phys. Rev. Lett. 67 (1991) 332; T.J. Bowles, Talk given at the "Neutrino '92" Int. Conf., Granada, June 7-12, 1992.
20. P. Anselmann et al. (GALLEX collaboration), Gran Sasso Nat. Lab. report GX1-1992 (LNGS-92/30), June 1, 1992.
21. Y. Suzuki, ICRP-Report-92-15; Y. Totsuka, Talk given at the "Neutrino '92" Int. Conf., Granada, June 7-12, 1992.
22. B. Pontecorvo, Chalk River Lab. Report PD-205 (1946).
23. V.A. Kuzmin, Sov. Phys. JETP 22 (1966) 1050.
24. J.N. Bahcall and M. Pinsonneault, Princeton preprint IASSNS-AST 92/10. This paper contains a review of the results obtained by other authors and references to their work.
25. S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.
26. A. De Rújula et al., Nucl. Phys. B168 (1980) 54.
27. S.T. Petcov, Nucl. Phys. B (Suppl.) 13 (1990) 527; Proc. "Neutrino '88" Int. Conf., Boston, June 1988, eds. J. Scheps, T. Kafka and P. Nath (World Scientific, Singapore, 1988), p. 663.
28. V. Barger et al., Phys. Rev. D22 (1980) 2718.
29. S.T. Petcov, Phys. Lett. 200B (1988) 373.
30. P.I. Krastev and S.T. Petcov, Phys. Lett. 207B (1988) 64; (E): *ibid.* 214B (1988) 661; Proc. Moriond Workshop on "5th Force Neutrino Physics", eds. O. Fackler and J. Tran Thanh Van (Frontières, Gif-sur-Yvette, 1988), p. 179.
31. S.J. Parke, Phys. Rev. Lett. 57 (1986) 1275; P. Pizzochero, Phys. Rev. D36 (1987) 2293.
32. S.T. Petcov, Phys. Lett. 191B (1987) 299.
33. S.T. Petcov, Phys. Lett. 214B (1988) 139.
34. J. Rich and S.T. Petcov, Phys. Lett. 224B (1989) 426.
35. G. Ewan et al., Sudbury Neutrino Observatory Proposal, SNO-87-12, 1987.
36. T. Kajita, Physics with the Super-Kamiokande Detector, Tokyo preprint ICRR 185-89-2, 1989; Y. Totsuka, Tokyo preprint ICRR 227-90-20, 1990.
37. C. Arpesella et al., BOREXINO proposal, eds. G. Bellini, R. Ragghavan et al. (Univ. of Milan, Milan, 1992), vols. 1 and 2.
38. J.N. Bahcall, M. Baldo-Ceolin, D. Cline and C. Rubbia, Phys. Lett. 178B (1986) 324; ICARUS I: An optimized, real-time detector of solar neutrinos, by the ICARUS collaboration, Frascati report LNF-89/005(R), 1989.
39. V. Gavrin, Talk given at the XXVth Int. Conf. on High Energy Physics, Dallas (USA), August 6-12, 1992.

Figure captions

Fig. 1a. Regions of values of the parameters Δm^2 and $\sin^2 2\theta$ allowed by the Homestake, Kamiokande II and SAGE 1990 individual run data (grey+black area), and by the 68% C.L. (black area) and 95% C.L. (grey+black area) GALLEX results in the case of solar neutrino oscillations into active neutrinos: $\nu_e \leftrightarrow \nu_{\mu(\tau)}$. Shown are also the iso-SNU contours corresponding to an average rate of ^{71}Ge production in the Ga-Ge experiments of 83 SNU (dash-dotted line), 104 SNU and 63 SNU (solid lines). The dash-dotted line determines approximately the restrictions following from the SAGE upper limit, eq. (7).

Fig. 1b. The same as in fig. 1a, but for solar neutrino oscillations into sterile neutrinos: $\nu_e \leftrightarrow \nu_s$.

Fig. 2a. Regions of values of Δm^2 and $\sin^2 2\theta$ for which the matter-enhanced two-neutrino transitions $\nu_e \rightarrow \nu_{\mu(\tau)}$ provide a solution to the solar neutrino problem (shaded areas): the iso-SNU and the iso-suppression contours correspond to the 68% C.L. Homestake (solid lines), Kamiokande II and III (dashed lines), SAGE 1990 (dotted line), and GALLEX (dash-dotted lines) results. The region allowed by the Homestake data is located between the two solid-line contours, etc.

Fig. 2b. The same as in fig. 2a, but for the 95% C.L. Homestake, Kamiokande II and III, and GALLEX results. The dotted line corresponds to the SAGE 90% C.L. upper limit, eq. (7).

Fig. 3a. The spectrum of the final state electrons from the $\nu_e e^-$ elastic scattering reaction in the case of vacuum oscillations $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ of solar neutrinos, characterized by values of Δm^2 and $\sin^2 2\theta$ from (10) for which one obtains a description of the solar neutrino data. The spectrum is normalized to the standard one (for $\nu_e e^-$ elastic scattering) in the absence of oscillations. The values of Δm^2 (in eV^2) and $\sin^2 2\theta$, for which the curves have been obtained, are given in brackets by the curves.

Fig. 3b. The same as in fig. 3a, but for the case of matter-enhanced $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions of solar neutrinos. The dashed curve corresponds to non-adiabatic transitions, while the solid curves correspond to adiabatic ones.

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b