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RECENT PROGRESS IN QCD JET PHYSICS IN e^+e^- COLLISIONS^{*)}**B.R. Webber⁺⁾** Theoretical Physics Division, CERN
CH - 1211 Geneva 23**ABSTRACT**

Recent progress is reviewed in the following areas of e^+e^- jet physics: exponentiation of large higher-order logarithms and its application to improve determinations of the strong coupling constant from event shape distributions; jet clustering algorithms and studies of jet multiplicities using the new $k_\perp$ -algorithm; studies of two-particle correlations in jet fragmentation.

^{*)} Invited talk at XXVI International Conference on High Energy Physics, Dallas, Texas, August 1992.

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Abstract

Recent progress is reviewed in the following areas of e^+e^- jet physics: exponentiation of large higher-order logarithms and its application to improve determinations of the strong coupling constant from event shape distributions; jet clustering algorithms and studies of jet multiplicities using the new k_T -algorithm; studies of two-particle correlations in jet fragmentation.

INTRODUCTION

Over the past two years, the great wealth of new data on hadronic final states in Z^0 decay has prompted a strong revival of activity in QCD jet physics. A better understanding of jet properties is important, not only to test QCD itself and to determine its fundamental parameter α_S , but also to be able to use jet signatures for other kinds of physics. In this brief review, I concentrate on some of the recent theoretical progress. Comparisons between these theoretical ideas and experiment are presented in other contributions to this session, and in the plenary talk on QCD.

LARGE LOGARITHMS

Many of the most detailed current tests of QCD depend on comparing e^+e^- annihilation data with predictions based on perturbation theory. For totally inclusive quantities, such as the hadronic cross section, the perturbative prediction is a well-behaved power series

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have a 'semi-inclusive' quantity, which is finite in perturbation theory but has up to two large logarithms for each power of α_S .

Representing the large logarithm by L , a semi-inclusive quantity $F(L)$ thus has a perturbative expansion of the general form

$$\begin{aligned} F(L) &\sim 1 + \\ &+ \alpha_S(A_{12}L^2 + A_{11}L + A_{10} + \dots) \\ &+ \alpha_S^2(A_{24}L^4 + A_{23}L^3 + A_{22}L^2 + \dots)(1) \\ &+ \alpha_S^3(A_{36}L^6 + A_{35}L^5 + A_{34}L^4 + \dots) \end{aligned}$$

Now suppose we make a change of scale in the definition of α_S :

$$\alpha_S(\mu^2) \rightarrow \alpha_S(f\mu^2) = \alpha_S - \beta_0\alpha_S^2 \ln f + \dots \quad (2)$$

There must be a corresponding change in the coefficients A_{ij} in order to cancel the scale dependence order by order

$$\begin{aligned} A_{22} &\rightarrow A_{22} + A_{12}\beta_0 \ln f + \dots \\ A_{34} &\rightarrow A_{34} + A_{24}\beta_0 \ln f + \dots \end{aligned} \quad (3)$$

It follows that in order to measure α_S (or equivalently the value of the QCD scale Λ in a particular renormalization scheme) from a semi-inclusive distribution one needs to know the values of the *next-to-next-to-leading* coefficients $A_{n,n-2}$ to all orders.

EXPONENTIATION

For some quantities there are factorization theorems [1] that imply *exponentiation* of logarithms in the following sense:

$$\begin{aligned} F(L) &\sim (1 + C_1\alpha_S + C_2\alpha_S^2 + \dots) \\ &\times \exp[\alpha_S(G_{12}L^2 + G_{11}L)] \quad (4) \\ &+ \alpha_S^2(G_{23}L^3 + G_{22}L^2 + G_{21}L) + \dots \end{aligned}$$

The corrections to this result vanish as $L \rightarrow \infty$. Notice that in the exponent the coefficients are such that $G_{n,m>n+1} = 0$, that is, one finds only $n+1$ instead of $2n$ large logarithms in

the expansion of $\ln F$ to order α_S^3 . A change of scale in α_S changes the *next-to-leading* coefficients in the exponent, G_{22} etc. Thus when exponentiation can be proven it suffices to know the leading and next-to-leading logarithmic coefficients in the exponent in order to determine α_S .

In QED, factorization of matrix elements for multiple soft photon emission, and exponentiation of the corresponding logarithms, follow from the effective independence of successive emissions. In QCD the gauge boson is itself charged and therefore the situation is more complicated. It turns out, however, that for sufficiently inclusive quantities there is a cancellation between leading non-Abelian logarithms from real and virtual contributions, and remaining next-to-leading logarithms can be resummed by replacing α_S by a running coupling with a suitable argument. In this way one obtains a generalized form of exponentiation for the non-Abelian case [1, 2].

Factorization of matrix elements alone is not sufficient for the exponentiation of large logarithms in the distribution of an observable quantity. Factorization of the phase space in the relevant region is also needed. As discussed below, this can be proven for certain e^+e^- event shape measures near the two-jet configuration, leading to the possibility of measuring α_S from event shape distributions in this semi-inclusive region.

EVENT SHAPES

The fitting of hadronic event shape distributions in e^+e^- annihilation is by now a well established method for α_S determination. A number of infrared-safe event shape variables have been defined and computed in full to second order in α_S [3]. The perturbative predictions can be compared with experimental data after correction for the non-perturbative hadronization process. However, there remain

some serious uncertainties about how best to perform such comparisons:

i) *Range to be fitted.* Near the kinematic limits, the perturbative expansions for event shapes involve large double logarithms of the type described above. In comparing data with fixed-order calculations one must therefore use only a restricted region well away from phase space boundaries, whereas most of the data are close to the limit corresponding to a two-jet configuration.

ii) *Hadronization corrections.* The most successful hadronization models are those incorporated in parton-shower event simulation programs [4], which are based on the assumption that hadronization is a local process in phase space, involving a rather large number of final-state partons at a low momentum transfer scale. Such models are not suitable for estimating corrections to fixed-order perturbative calculations, which involve at most four partons in second order.

iii) *Renormalization scale dependence.* The result obtained for α_S depends on the renormalization scale μ used in the perturbative calculation. This dependence represents a residual uncertainty due to orders higher than those computed. Intuitively, one would expect $\mu \sim Q$ to be the natural scale, but in many cases $\mu \ll Q$ appears to give a better fit, with a somewhat smaller value of $\alpha_S(Q^2)$. The strong scale dependence, combined with the preference for such small scales, has been the biggest source of uncertainty in α_S determinations from event shapes at LEP [5].

In a series of recent papers [6]–[10], it was shown that all of the above types of uncertainty can be reduced by resummation of large logarithms in event shape variables that satisfy the conditions for exponentiation: i) Resummation allows analyses to be extended closer to the two-jet kinematic limit, giving better statistical precision; ii) It corresponds

closely to what is done, at a less precise level, in parton shower simulations, and so it can be combined more reliably with hadronization corrections estimated from models based on parton showers; iii) It reduces the renormalization scale dependence of the prediction, by taking into account an important class of higher-order corrections. Most importantly, it reproduces, at a scale $\mu \sim Q$, the effect of using a scale $\mu \ll Q$ in the second-order calculation, and thus it removes the motivation for using a small scale.

The event shape variables considered in detail in Refs. [6]–[10] are the thrust, the heavy jet mass, the energy-energy correlation, and two new quantities related to the transverse broadening of jets. In all cases, some trans-formation of variables is required before factorization occurs. For example, in the thrust distribution double logarithms of $1 - T$ occur in the high thrust region, $T \rightarrow 1$. In this region we have

$$T \sim 1 - (k_1^2 + k_2^2)/Q^2 \quad (5)$$

where k_1 and k_2 are the invariant masses of the two final-state jets and Q is the c.m. energy. Thus the cross section for thrust $T > 1 - \tau$ is given at small τ by

$$\sigma(1 - T < \tau) \sim \int dk_1 dk_2 J(Q, k_1) \times J(Q, k_2) \Theta(\tau Q^2 - k_1^2 - k_2^2) \quad (6)$$

where $J(Q, k)$, the jet mass distribution at scale Q , factorizes and exponentiates to next-to-leading order [11, 12]. In order to factorize the phase space, we perform a Laplace transformation to obtain

$$\sigma(1 - T < \tau) \sim \frac{1}{2\pi i} \int \frac{dN}{N} \exp(N\tau) [J_N(Q)]^2 \quad (7)$$

where

$$J_N(Q) = \int_0^\infty dk J(Q, k) \exp(-Nk^2/Q^2). \quad (8)$$

Thus the Laplace transform of $\sigma(1 - T < \tau)$ factorizes to next-to-leading order. Inverting the transform gives a prediction of the distribution in the high-thrust region, which can be combined with the fixed-order prediction for lower T after subtracting all the terms up to order α_S^2 that have been exponentiated. The resulting prediction [6] is more stable with respect to hadronization corrections and renormalization scale dependence. The corresponding predictions of other event shapes show similar good features [7, 8, 9].

By now, several experimental studies have been performed comparing these predictions with data [13], and more are in progress. Many of the results are reviewed in other contributions to these Proceedings, so only a few general remarks will be made here.

The most important general feature of the comparisons is that they no longer show a preference for small values of the renormalization scale μ : indeed a fit usually cannot be obtained for $\mu \ll Q$. Since the principal large logarithms have been explicitly resummed, there is no dynamical argument for using a scale much different from Q , and most resummed analyses quote a theoretical uncertainty corresponding to a rather small range of scales around $\mu \sim Q$. The resulting values of α_S are slightly larger than those obtained from earlier fixed-order analyses: the small scales preferred by the latter tend to be correlated with a smaller value of α_S .

The resummed predictions of event shape distributions still have their limitations, which should not be overlooked. At very high thrust, where the value of $\alpha_S \ln(1 - T)$ is large, the predictions still become unreliable. At present energies the estimated hadronization corrections also become uncomfortably large in the same region. At low thrust, the resummed terms are not enhanced but instead the non-logarithmic contributions become very small. Thus the

terms of order α_S^3 and beyond generated by resummation, which are reliable only at high thrust, may significantly distort the prediction in the low-thrust region. This effect may be assessed and controlled by varying the lower limit of the fitted region, and by comparing different schemes for matching the resummed and fixed-order predictions [13].

JET ALGORITHMS

Fixed-order analyses of multijet fractions in e^+e^- annihilation are another popular way of measuring α_S , and so one would like to apply the exponentiation technique to these quantities. It is also vital to understand QCD multijet final states when searching for new physics in hadronic channels. As in the case of event shapes, the QCD matrix elements have the required factorization properties, but factorization of the phase space is also necessary in order for generalized exponentiation to occur.

The most common way of defining jets in e^+e^- collisions has been by a jet clustering algorithm of the JADE type [14]. One defines a resolution parameter y_{cut} and combines the pair of particles (ij) of lowest invariant mass M_{ij} into a cluster if $y_{ij} \equiv M_{ij}^2/Q^2 < y_{\text{cut}}$. This procedure is iterated for the particles and clusters until no more combination is possible, at which stage the remaining clusters are called jets. The n -jet fraction of events is a function of the jet resolution, $R_n(y_{\text{cut}})$, whose perturbative expansion involves double logarithms of y_{cut} .

It turns out that, for the JADE algorithm as outlined above, even the leading logarithms of y_{cut} do not exponentiate, because the phase space defined by the invariant mass cut does not factorize [15]. Neglecting the particle (or cluster) masses, the resolution variable is

$$y_{ij} = 2E_i E_j (1 - \cos \theta_{ij})/Q^2. \quad (9)$$

The factor of $E_i E_j$ builds in an attractive kinematic correlation between soft gluons, which for example enhances the 3-jet fraction at the expense of those for 2- and 4-jets. As a result, the resummation of logarithms of y_{cut} for the unmodified JADE algorithm appears a hopeless task.

The lack of exponentiation in the JADE algorithm has led to the introduction of similar algorithms with different resolution variables to overcome this problem. The best choice seems to be the $k_{\perp}$ - or 'Durham' algorithm [16]-[19], in which Eq. (9) is replaced by

$$y_{ij} = 2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij}) / Q^2. \quad (10)$$

The decoupling of E_i and E_j leads to exponentiation of leading and next-to-leading logarithms for the 2-jet fraction and to generalized exponentiation for higher multijets [17, 18]. As expected, the hadronization corrections also appear to be under better control [19]. The issue of renormalization scale dependence, which as we discussed concerns next-to-next-to-leading logarithms, is still under investigation.

The $k_{\perp}$ -algorithm can be generalized to lepton-hadron and hadron-hadron collisions in a way that leads to universal factorization of initial-state mass singularities [20].

JET MULTIPLICITIES

The exponentiation properties of the jet fractions defined according to the $k_{\perp}$ -algorithm (10) make it possible to predict the average multiplicity of jets using this algorithm [21], either as a function of c.m. energy Q at fixed resolution y_{cut} , or as a function of y_{cut} at fixed Q . The latter predictions are more useful since they can be tested with great precision at $Q = m_Z$. Very good agreement has been found [13], although the central value obtained for $\alpha_S(m_Z)$ is a little higher than in the event shape measurements.

Predictions in which one first selects a 2- or 3-jet sample at a given resolution $y_{\text{cut}} = y_1$ and then studies the 'sub-jet' multiplicity in each jet as a function of $y_{\text{cut}} < y_1$ have also been made [22]. Since the first two jets are initiated by quarks while the third is due to a gluon, one may hope to learn about the fragmentation of gluon jets from this analysis. The naive expectation that the gluon jet has a higher multiplicity in proportion to its colour charge squared, i.e. $C_A/C_F = 9/4$ times that of a quark jet, is not fulfilled at present energies, due to QCD coherence effects which strongly suppress the fragmentation of the gluon jet in the 3-jet sample. Hence one typically finds that the predicted multiplicity in the 3-jet sample is less than $3/2$ times that in the 2-jet events. The experimental results [23] are in agreement with these predictions.

Because of the good theoretical properties of the $k_{\perp}$ -algorithm, much more detailed predictions can be made concerning the distributions of sub-jet within jets. Comparisons with experiment, possibly using events with tagged quark jets, should cast much light on the properties of gluon jets in the near future.

TWO-PARTICLE CORRELATIONS

New information on jet fragmentation in e^+e^- collisions has been provided by measurement of the two-particle momentum distribution [24]. Defining

$$\xi = \ln \left(\frac{1}{x} \right) \quad (11)$$

where $x = 2p/Q$ is the momentum fraction, let $D(\xi)$ and $D(\xi_1, \xi_2)$ represent the single-particle and two-particle inclusive ξ -distributions, respectively. Then the two-particle correlation function,

$$R(\xi_1, \xi_2) = D(\xi_1, \xi_2) / D(\xi_1) D(\xi_2) \quad (12)$$

is predicted to have the form [25]

$$R = c_1 + c_2(\xi_1 + \xi_2) + c_3(\xi_1 - \xi_2)^2 + \dots \quad (13)$$

where the coefficients c_n have power series expansions in $\alpha \equiv \sqrt{2\beta_0\alpha_S}$:

$$\begin{aligned} c_1 &= 1.375 - 1.262\alpha + d_1\alpha^2 + \dots \\ c_2 &= 0.877\alpha^3 + d_2\alpha^4 + \dots \\ c_3 &= -1.125\alpha^4 + d_3\alpha^5 + \dots \end{aligned} \quad (14)$$

Here d_n represent unknown higher-order coefficients. Table 1 shows the values of the coefficients obtained from experiment [24], together with the predictions (14), taking $\alpha = 0.4$ at $Q = m_Z$. The last column shows the values of the unknown higher-order coefficients d_n that would be required for perfect agreement with experiment. We see that they are all of order unity or smaller, showing that the agreement is as good as should be expected, given the rather large value of the expansion parameter α . This agreement lends support to the idea that these two-particle correlations are coming primarily from the perturbative phase of jet evolution, rather than from non-perturbative processes such as resonance decays.

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Table 1. Coefficients in Two-Particle Correlation Function (13).

n	c_n (OPAL)	c_n (QCD)	d_n (fit)
1	0.928(2)	$0.870 + 0.160d_1$	0.36
2	0.025(3)	$0.056 + 0.026d_2$	-1.21
3	-0.021(3)	$-0.029 + 0.010d_3$	0.78

REFERENCES

1. For a review and earlier references, see J.C. Collins and D.E. Soper, Ann. Rev. Nucl. Part. Sci. 37 (1987) 383.
2. G. Sterman, Nucl. Phys. B281 (1987) 310; S. Catani, E. d'Emilio and L. Trentadue, Phys. Lett. 211B (1988) 335; S. Catani and L. Trentadue, Phys. Lett. 217B (1989) 539; Nucl. Phys. B327 (1989) 353; Nucl. Phys. B353 (1991) 183.
3. Z. Kunszt, P. Nason, G. Marchesini and B.R. Webber, in 'Z Physics at LEP 1', CERN 89-08, vol. 1, p. 373.
4. For a review see B.R. Webber, Ann. Rev. Nucl. Part. Sci. 36 (1986) 253.
5. ALEPH Collaboration, D. Decamp et al., Phys. Lett. 234B (1990) 399, Phys. Lett. 255B (1991) 623, Phys. Lett. 257B (1991) 479; DELPHI Collaboration, P. Aarnio et al., Phys. Lett. 240B (1990) 271, Phys. Lett. 247B (1990) 167, P. Abreu et al., Zeit. Phys. C54 (1992) 55; L3 Collaboration, B. Adeva et al., Phys. Lett. 237B (1990) 136, Phys. Lett. 248B (1990) 464; OPAL Collaboration, M.Z. Akrawy et al., Phys. Lett. 235B (1990) 389, Phys. Lett. 252B (1990) 159, Zeit. Phys. C47 (1990) 505, Zeit. Phys. C49 (1991) 375; SLD Collaboration, K. Abe et al., Stanford preprints SLAC-PUB-5904, 5905.

6. S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Phys. Lett. 263B (1991) 491.
7. S. Catani, G. Turnock and B.R. Webber, Phys. Lett. 272B (1991) 368.
8. G. Turnock, Cambridge preprint Cavendish-HEP-91/3.
9. S. Catani, G. Turnock and B.R. Webber, CERN preprint TH.6570/92, to be published in Phys. Lett. B.
10. S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Cambridge preprint Cavendish-HEP-91/11, CERN preprint TH.6640/92.
11. S. Catani and L. Trentadue, Phys. Lett. 217B (1989) 539; Nucl. Phys. B327 (1989) 353; Nucl. Phys. B353 (1991) 183.
12. S. Catani, G. Marchesini and B.R. Webber, Nucl. Phys. B349 (1991) 635.
13. OPAL Collaboration, P.D. Acton et al., Zeit. Phys. C55 (1992) 1; ALEPH Collaboration, D. Decamp et al., Phys. Lett. 284B (1992) 163; L3 Collaboration, O. Adriani et al., Phys. Lett. 284B (1992) 471; DELPHI Collaboration, P. Abreu et al., contributed paper at this Conference.
14. JADE Collaboration, S. Bethke et al., Phys. Lett. 213B (1988) 235.
15. N. Brown and W.J. Stirling, Phys. Lett. 252B (1990) 657; Zeit. Phys. C53 (1992) 629.
16. Report of Hard QCD Working Group, in Proc. Durham Workshop on Jet Studies at LEP and HERA, J. Phys. G 17 (1991) 1537.
17. S. Catani, Yu.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Phys. Lett. 269B (1991) 432.
18. S. Catani, in 'QCD at 200 TeV', Proc. 17th INFN Elosatron Project Workshop, Erice, Italy, June 1991, ed. L. Cifarelli and Yu.L. Dokshitzer (Plenum Press, New York); S. Catani, Yu.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Cambridge preprint Cavendish-HEP-91/12, in preparation.
19. S. Bethke, Z. Kunszt, D.E. Soper and W.J. Stirling, Nucl. Phys. B370 (1992) 310.
20. S. Catani, Yu.L. Dokshitzer and B.R. Webber, Phys. Lett. 285B (1992) 291; S. Catani, Yu.L. Dokshitzer, M.H. Seymour and B.R. Webber, CERN preprint in preparation.
21. S. Catani, Yu.L. Dokshitzer, F. Fiorani and B.R. Webber, Nucl. Phys. B377 (1992) 445.
22. S. Catani, Yu.L. Dokshitzer, F. Fiorani and B.R. Webber, Nucl. Phys. B383 (1992) 419.
23. ALEPH and L3 Collaborations, contributed papers at this Conference.
24. OPAL Collaboration, P.D. Acton et al., Phys. Lett. 287B (1992) 401.
25. C.P. Fong and B.R. Webber, Phys. Lett. 241B (1990) 255; Nucl. Phys. B355 (1991) 54.