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SOME RESULTS FOR STRINGS AT $D > 1$ ¹

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SOME RESULTS FOR STRINGS AT $D > 1$

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In this talk I would like to report on joint work done in collaboration with J.L.F. Barbón and Č. Crnković¹, aiming at finding an exact and tractable formulation of non-critical string theory in space-time dimensions $D > 1$. The coupling of $D \leq 1$ conformal field theories to two-dimensional gravity has been successfully studied using continuum methods^{2,3,4,5} and matrix models^{6,7,8,9}. For $D > 1$, apart from the original papers^{6,7} and the work of Boulatov et al.¹⁰, little work has been done analytically to understand these systems. An interesting exception is Ref. 11. In our own work, we use techniques of large- N Quantum Field Theory to reduce the exact partition function for strings in $D > 1$ (but $D < 6$) to a single Hermitian one-matrix problem. The reduced model techniques of Eguchi-Kawai¹² (EK) and the Twisted Eguchi-Kawai construction of Gonzalez-Arroyo and Okawa^{13,14} allow us to obtain an exact correspondence with certain types of one-matrix models. By analyzing the effective actions derived from these matrix models, we identify a family of critical points whose string susceptibilities are characterized by two integers (n, m) and such that

$$\gamma_{st} = \frac{p}{n + m + 1} \quad 0 \leq p \leq n \quad (1)$$

We find a second-order phase transition where both gravity and some polymer-like matter become critical simultaneously. It is at this multicritical point that we take the continuum limit yielding the exponents (1). We have also computed the planar correlation functions

of a collection of macroscopic loop operators. When $n = 1, m \geq 2$, the general structure of these correlators seems to indicate that our approximations to the exact model do capture some of the basic features of tachyonic states.

The basic problem in describing the motion of strings in D -dimensional space-time is the evaluation of the functional integral:

$$\begin{aligned} \mathcal{Z} = & \int \mathcal{D}g_{ab} \mathcal{D}X_\mu \\ & \exp \left(-\frac{1}{2} \int_\Sigma d^2\sigma \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X_\mu - \right. \\ & \left. - \mu_0 \int_\Sigma \sqrt{g} + \dots \right) \end{aligned} \quad (2)$$

where X^μ is the field representing the embedding of the string world-sheet into space-time, and g_{ab} is the two-dimensional metric. The partition function (2) is quite formal, and one can regularize it^{6,7,8,9} by replacing the sum over two-dimensional geometries by a sum over triangulations. The result is:

$$\begin{aligned} \mathcal{Z} = & \sum_T \frac{e^{-\mu|T|}}{n(T)} \int \prod_{i \in T_0} \sigma_i^\alpha d^D X_i \\ & \cdot \prod_{\langle ij \rangle \in T_1} G(X_i - X_j) \end{aligned} \quad (3)$$

In (3) μ is the bare two-dimensional cosmological constant, T is a generic triangulation, T_0 (resp. T_1) represents the set of vertices (resp. edges) of T , $\sigma_i = q_i/3$, where q_i is the coordination number of site i , and $G(X_i - X_j)$ is a propagator factor. To reproduce (2), we take $G(X)$ to be a Gaussian. Using large- N

techniques, it is shown in Refs. 8 and 9 that $\mathcal{Z}$ is equivalent to the large- N limit of a $N \times N$ Φ^3 matrix field theory:

$$\begin{aligned} \mathcal{Z} &= \lim_{N \rightarrow \infty} \log \int \mathcal{D}\Phi \\ &\exp \left(-N \int d^D X \text{Tr} \left(\frac{1}{2} \phi G^{-1} \phi + \right. \right. \\ &\quad \left. \left. + \frac{1}{3} g \Phi^3 \right) \right); g = e^{-\mu} \end{aligned} \quad (4)$$

Since for $D < 6$ the Φ^3 field theory is super-renormalizable, the critical properties of (4) should not depend on whether we replace the Gaussian by the Feynman propagator in (4). We will assume this replacement from now on. Writing $\mathcal{Z}$ in terms of the free energy F , $\mathcal{Z} = \exp N^2 F$, we are interested in finding the critical values g_c for the cosmological constant. Near these values we find the scaling behaviour:

$$\chi \sim \frac{\partial^2 F}{\partial g^2} \sim (g - g_c)^{-\gamma_{\text{ct}}} \quad (5)$$

To prove the equivalence of (4) to a one-matrix model, we first take a lattice approximation for the D -dimensional target space-time with periodic boundary conditions. This lattice will have L^D sites. The next step is to note that for $SU(N)$ we can construct a collection of D twist matrices $\Gamma_\mu, \mu = 1, 2, \dots, D$ commuting up to an element of the centre of $SU(N)$, Z_N :

$$\Gamma_\mu \Gamma_\nu = Z_{\nu\mu} \Gamma_\mu \Gamma_\nu \quad (6)$$

$$Z_{\mu\nu} = e^{Z_{Mi} n_{\mu\nu} / N} \quad (7)$$

$n_{\mu\nu} = -n_{\nu\mu}$ is a matrix of integers. One of the basic ideas of the TEK reduction¹³ is to require the Γ_μ 's to provide a faithful projective representation of the L^D -lattice translation group. This implies some constraints on $n_{\mu\nu}$, among them $N = L^{D/2}$. Then in the large- N limit, following Refs. 12 and 13, we can show that the planar Schwinger-Dyson equation for (4) coincides with those of the reduced theory obtained from (4) by replacing:

$$\Phi(x) \rightarrow D(x) \phi D(x)^{-1} \quad (8)$$

$$D(x) = \prod_{\mu=1}^D \Gamma_\mu^{x_\mu} \quad (9)$$

The x_μ 's are measured in integer units of the lattice spacing. The reduced action is therefore:

$$S = \frac{1}{2} \sum_\mu \text{Tr}(\Gamma_\mu \Phi \Gamma_\mu^{-1} - \Phi)^2 + \frac{1}{3} g \text{tr} \Phi^3 \quad (10)$$

where now Φ is a constant matrix. One important property of (10) is that all its planar graphs coincide graph by graph with those of the original theory. We can go further and simply take (10) as our definition of the D -dimensional non-critical bosonic string. As constructed, (10) only works in $D = 2, 4$. However, some simple modifications allow us to construct theories in $D = 3, 5$ (see Ref. 1 for details).

The obvious thing one would like to do with (10) is to write Φ in terms of a unitary matrix U and a real diagonal matrix γ :

$$\Phi = U \begin{pmatrix} \gamma_1 & & \\ & \ddots & \\ & & \gamma_N \end{pmatrix} U^+, \quad (11)$$

and to perform subsequently the angular integral (integration over U). The result should be an effective one-matrix model of the form:

$$\mathcal{Z}_{\text{eff}} = \int D\Phi e^{-N^2 \Gamma_{\text{eff}}[\Phi]} \quad (12)$$

$$\Gamma_{\text{eff}}[\Phi] = V(x_1, x_2, \dots, x_n, \dots) + \sum_{i \geq 1} g_i x_i \quad (13)$$

$$x_n \equiv \frac{1}{N} \text{Tr} \phi^n \quad (14)$$

where V is a function at least quadratic in the trace variables x_n . Note further that the vertices $g_n x_n$ with standard large- N techniques generate simplicial approximations to smooth surfaces. On the other hand, couplings of the form $x_{n_1} x_{n_2}, \dots, x_{n_p}$ represent p triangulated surfaces touching at one point, where the number of double lines contributed to the diagram by x_{n_i} equals n_i . Couplings of the form x_n^2 were first studied in Ref. 15. These couplings, if

they dominate, tend to make the surface look like a collection of branched polymers. From the point of view of a continuum formulation, these couplings can be interpreted in terms of singular configurations of the Liouville field. Inclusion of these singularities in the continuum quantization of Liouville theory is yet to be done. The singularities would correspond to making the metric vanish along some random gas of closed curves. In the simplest case of x_2^2 square interactions, these curves of vanishing metric should be self-avoiding. In Ref. 1 we have been able to solve exactly (13) in the planar limit by assuming that V is an analytic function of the x_i 's and with the added simplification that V depends on an arbitrary, although finite, number of x_i 's. Furthermore, and always under these assumptions, we have determined under what conditions there is “enough room” to open macroscopic loops on the surface. In those cases where this is possible, we have computed all planar multiloop correlation functions. We now briefly outline the salient features of our analysis. We find three types of phases:

1. $\gamma_{st} < 0$. The gravity couplings become critical, and the polymer couplings are non-critical. This is essentially the same as the pure gravity case.
2. $\gamma_{st} > 0$. Gravity is not critical, but matter becomes critical. This case describes a phase akin to branched polymers.
3. $\gamma_{st} > 0$. An interesting subset of item two, both gravity and matter become critical simultaneously. This is the multicritical point in which we are interested. In this case γ_{st} takes the values given in (1) depending on details. The phase transition is second order, and it makes sense to go to the continuum limit. In the pair of integers (n, m) determining the string susceptibility in (1), n gives the order of criticality in the polymer couplings and m gives it in the pure gravity couplings. $n = 1$ corresponds to tuning only the quadratic part of V in (13).

4. We have computed the correlation functions of macroscopic loops for the (n, m) -critical points. We find that only for $n = 1$ can we open them. For other cases ($n > 1$), we could say naively that the polymerization is so complete that there is hardly any room to open up those loops on the surface. For $(1, m)$ theories, the two-loop correlator is given by:

$$\langle w(\ell_1)w(\ell_2) \rangle = \frac{e^{-(\ell_1+\ell_2)/u}}{\ell_1 + \ell_2} + \frac{u}{\ell_1 \ell_2} e^{-(\ell_1+\ell_2)/n}, \quad (15)$$

and $u \propto \mu^{-\frac{1}{2}}$, where μ is the renormalized cosmological constant in the simple case. Note that the second term on the right-hand side can also be written as:

$$\langle w(\ell_1)P \rangle \frac{1}{\langle PP \rangle} \langle Pw(\ell_2) \rangle, \quad (16)$$

where P is the puncture operator. This contribution is reminiscent of the tachyon contribution in critical string theories. Similar results are obtained for multiloop correlators. The fact that the puncture operator P behaves in some sense as a tachyon state gives some credence to the approximation of V we have taken. Obviously more work is required before we can definitely conclude that our truncated effective action captures the universal features of the original model (10).

5. We have computed the $1/N^2$ corrections to our model and they are in agreement with the genus dependence found by David⁴ and Distler and Kawai⁵.

We feel that features 1 - 5 provide encouraging evidence that our approximate study of the original model (10) captures some of its universal features. More work is certainly necessary to determine the critical properties of (10). Further details can be found in Ref. 1.

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