current is given by:
\[ j_B^\mu = \frac{1}{3} \sum_q \bar{q}_L \gamma^\mu q_L = \frac{1}{3} \sum_q (\bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R), \]
where \( L(R) \) stands for the left-(right-)handed part of the quark field. For clarity, we have suppressed color and flavor indices in \( q \).

A similar expression can be written for the lepton-number current \( j_L^\mu \). Since weak \( SU(2) \) only couples to left-handed fields, the conservation of \( j_B^\mu, j_L^\mu \) may be afflicted with quantum anomalies in analogy with the anomaly effect in the explanation of \( \pi^0 \)-decay\(^1\). To test for the conservation of \( j_B^\mu \) we consider the triangle graph in Fig. 1:

\[ \begin{align*}
\text{SU}(2)_L & \quad \text{SU}(2)_L \\
\text{SU}(2)_L & \quad \text{SU}(2)_L
\end{align*} \]

Figure 1. Triangle graph producing an anomaly in the conservation of the baryon-number current.

Since the \( SU(2)_L \)-currents are coupled to gauge fields, current conservation plus Bose symmetry on the two \( SU(2)_L \) vertices completely fix the diagram in Fig. 1. If \( E^a, B^a, a = 1, 2, 3 \) are the \( SU(2)_L \) electric and magnetic fields, the conservation equation \( \partial_\mu j_B^\mu \) is modified to:
\[ \partial_\mu j_B^\mu = \frac{\alpha_w}{4\pi} \vec{E}^a \cdot \vec{B}^a. \]

The integrated anomaly for three families becomes:
\[ \Delta B = \Delta L = -3 \frac{\alpha_w}{4\pi} \int_{-\infty}^{+\infty} dt \int d^3 \vec{x} \quad \vec{E}^a \cdot \vec{B}^a = -3Q \]

The quantity \( Q \) is an integer, and it is a number characterizing the topological properties of the gauge field configuration. The argument of the integral in (3) is a total derivative in spacetime, and it does not receive contributions to any order of perturbation theory. There are only non-perturbative contributions to B- and L-violating processes in the Standard Model. The configurations responsible for these processes are known as instantons. In a sector of topological charge \( Q \), and in the simplest situation where we ignore contributions from the Higgs field, the gauge field action is bounded below by the instanton action:
\[ S \geq S_{\text{int}} = \frac{2\pi}{\alpha_w} |Q| \]

This can be interpreted by saying that there is a huge barrier between initial and final configurations differing by B and L quantum numbers. There is a tunnelling suppression in the amplitude of the order of \( \exp \left( -\frac{2\pi^2}{\alpha_w} \right) \approx 10^{-78} \). Needless to say, this is unobservable. At extreme conditions, however, such as high temperature or densities\(^3\), (B+L)-violating processes may be unsuppressed. If we plot the potential energy for gauge fields, an oversimplified picture looks like Fig. 2:

\[ \begin{align*}
\text{V} & \quad \Delta B = 0 \\
-1 & \quad \text{Tunnelling} \\
0 & \quad 1 \\
1 & \quad 2
\end{align*} \]

Figure 2. Schematic portrayal of the potential energy for gauge fields.

The minimal height of the barrier between two absolute minima is a saddle-point with a single negative eigenvalue, and it corresponds to a classical finite energy solution to the gauge field-Higgs systems known as the sphaleron\(^20\). Depending on details which are irrelevant for the present discussion, the sphaleron scale is
\[ E_{sp} \sim \frac{\pi M_W}{\alpha_w} (7 - 14 TeV) \]

where \( M_W \) is the W-mass. At finite temperature we can go over the barrier by thermal
fluctuations. The Boltzmann factor is small, and the tunnelling contribution is hardly relevant. We can estimate the (B+L)-violation amplitude as:

$$\Pi[T] \sim e^{-F_{sp}[T]/T} \sim e^{-M_W(T)/T}$$

(6)

where $F_{sp}[T]$ is the sphaleron free energy, and $M_W(T)$ is the temperature-dependent W-mass. Since at high $T$ the low-energy broken symmetries are restored, $M_W(T)$ is small for large $T$. The sphaleron solution can be used to generate the baryon asymmetry on the Universe.

Although at finite temperature it is reasonable to expect (B+L)-violation, it is more surprising that this phenomenon could also be observed at supercollider energies. The processes envisaged include $qq$ or $q\bar{q}$ collisions with very high multiplicity in the final state. Thus we have in the final state the minimum number of leptons and quarks to saturate the minimal selection rule $\Delta B = \Delta L = -3$, together with a large number of vector mesons and Higgs particles (Fig. 3).

![Schematic (B+L)-violating process with high multiplicity in the final state.](image-url)

The only known way of computing this amplitude is to evaluate the instanton-mediated scattering amplitude and its radiative corrections. The claim is that the inclusion of all high-multiplicity final states leads to an exponentially growing (in energy) contribution which offsets the exponential suppression factor due to instantons. The delicate and controversial issue is how complete this cancellation is. For a process with $n_W$ W’s and $n_H$ Higgses, the (B+L)-violating amplitude has the general form

$$A_{n_W,n_H}^{\Delta(B+L)} \sim \frac{(n_W + n_H)!}{M_W^{n_W+n_H}} \frac{(\alpha_W)^{(n_W+n_H)/2}}{\epsilon^{2\pi/\alpha_W}}$$

The total cross-section for (B+L)-violation can be written as:

$$\sigma_{tot}^{\Delta(B+L)} \sim \epsilon \exp \frac{4\pi}{\alpha_W} F\left(\frac{\sqrt{s}}{E_{sp}}\right)$$

(7)

Figure 4. The solid line represents the most optimistic estimate of the functions $F$ in (7). The broken line could represent the expectations of the less enthusiastic.

$\sqrt{s}$ represents the centre-of-mass energy, and $E_{sp}$ is given in (5). The difficult part of the problem is the computation of the function $F$. In Ref. 1, the computations of many groups are collected, and the first few orders of $F$ are given by:

$$F\left[\frac{\sqrt{s}}{E_{sp}}\right] = -1 + 0.34 \left(\frac{\sqrt{s}}{E_{sp}}\right)^{1/2} - 0.09 \left(\frac{\sqrt{s}}{E_{sp}}\right)^2 + 0 \left(\frac{\sqrt{s}}{E_{sp}}\right)^3 (1 + \ell n \frac{\sqrt{s}}{E_{sp}})$$

(8)

There are also problems with violation of s-wave unitarity. Being extremely optimistic, one might be tempted to conclude that $F$ looks qualitatively like the solid line in Fig. 3. A more pessimistic, although probably more realistic assessment of the situation is that after unitarization and higher-order effects are
NEW COMPUTATIONAL RULES IN QCD FROM STRING THEORY

Perturbative theory is one of our basic tools for the understanding of short-distance physics. In QCD specially, perturbative computations are crucial in present and future hadron colliders to understand the Standard Model, and to help discern what lies beyond. It is notorious, however, that traditional Feynman graph computations become rather cumbersome, even for tree-level processes. For example, the tree-level scattering amplitude for two to six gluons involves more than 30 000 graphs. It is obviously prohibitive to evaluate such an amplitude with conventional techniques, and it is quite clear that new computational tools are required. Work by many people has made a number of multijet computations possible. If one wants to include one-loop contributions, the computations are prohibitively difficult. Bern and Kosower have proposed a general method to compute one-loop corrections using string theory. They are able to derive a set of simple rules whose result is to effectively combine all diagrams for a given process, perform all cancellations, Lorentz algebra and one-loop momentum integral. The final answer is just the Feynman parameter integrals left at the end of any one-loop computation. The starting point of their method is to consider the $n$-gluon scattering amplitude in string theory. This amplitude is rather simple to write down, and it automatically sums up all Feynman graphs, organizes them in colour decompositions and leaves us with the Feynman parameter integrals. These simplifications were already known for tree diagrams. What is important in the Bern-Kosower work is that they are also able to derive simple rules for one-loop corrections, and it is not unreasonable to expect that they (or someone else) should be able to extend their rules to higher-loop graphs. In string theories without tachyons, in the limit as the string tension goes to infinity, one is left with the massless excitations of the theory. When we consider loops, all massive states will circulate around the loop, and therefore one has to exercise a certain amount of care to be sure that the only contributions left are those coming from the modes wanted. In particular, the particles in the gravity sector have to be treated separately. At the one-loop level this is not a difficult problem. To control the infinite string tension limit properly, Bern and Kosower choose a consistent four-dimensional heterotic string theory. A flavour of the type of string computations involved is now presented. To compute an $n$-gluon amplitude at one loop in string theory, we first note that in this case the world-sheet of the string is a torus (Fig. 5).