The Refractive Properties of the Gluon Plasma in SU(2) Gauge Theory

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Abstract:

The deconfined phase in finite temperature SU(2) gauge theory shows clear interaction effects in the temperature range $1 \leq T/T_c \leq 2$. The interaction measure $\Delta \equiv (\epsilon - 3P)/T^4$, defined in terms of the energy density ϵ and the pressure P , peaks at some temperature T above the critical temperature T_c and is different from zero in the entire temperature interval mentioned. High statistics lattice calculations provide numerical results for the pressure P and the entropy density $s \equiv (\epsilon + P)/T$. We interpret these results in terms of an ideal gas of quasi-gluons propagating in a refractive medium. The refractive properties of the medium are parametrized in terms of a temperature-dependent effective mass $M(T)$ of the excitations, which is determined from the lattice data. It shows a quite non-trivial behaviour: $M(T)$ decreases from $T = T_c$ until $T \simeq 2 T_c$; beyond this point it starts increasing again, and at high temperatures it approaches the perturbative thermal mass $M(T) \sim gT$.

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1. Introduction

Finite temperature SU(2) gauge theory provides a transition from a low temperature confined phase to a high temperature state of deconfined gluons. The critical behaviour at the transition point has been conjectured to fall into the same universality class as that of the Ising model of the same spatial dimension [1,2]. Extensive numerical studies [3] confirm this conjecture. While the transition itself thus appears to be quite well understood, the behaviour of the plasma just above the deconfinement point remains a rather open question, although it has also been studied in quite some detail by computer simulations of the lattice formulation and continues to be the subject of considerable speculation [4–12]. Since similar problems occur as well in full QCD, where they also continue to attract much attention [13], it seems worthwhile to return to the simpler SU(2) case, for which rather high precision lattice results are available.

To introduce the problem, we show in Fig.1 the behaviour of the energy density ϵ and the pressure P as function of the temperature T , measured in units of the critical temperature T_c [6]. For an ideal gas of massless gluons, we have

$$\epsilon = 3P = (\pi^2/5)T^4, \quad (1)$$

so that the interaction measure $\Delta \equiv (\epsilon - 3P)/T^4$ vanishes. We see from Fig.1 that in the range $1 \leq T/T_c \leq 2$ this is certainly not the case, so that the system evidently is not yet ideal. In fact, we expect both non-perturbative and perturbative deviations from ideal gas behaviour. A simple bag model combined with the lowest order perturbation theory gives [14]

$$P = (\pi^2/15)T^4[1 - 10(g/4\pi)^2] - B, \\ \epsilon = 3P + 4B, \quad (2)$$

where g is the running coupling constant. This gives us for the entropy density $s \equiv (\epsilon + P)/T$ the form

$$s = (4\pi^2/15)T^3[1 - 10(g/4\pi)^2], \quad (3)$$

and for the interaction measure

$$\Delta = 4B/T^4. \quad (4)$$

In this model we thus find that Δ is a measure particularly sensitive to non-perturbative interactions, while s contains only perturbative corrections to the ideal gas form. The behaviour of eq.(4) is in Fig.2 compared to the lattice results of Fig.1. While the simple model (2) reproduces some of the qualitative features of the lattice calculation, it does not agree in details. In particular, it cannot reproduce the peak found in Δ at a temperature clearly above T_c . There is evidently more going on in the plasma than bag pressure plus perturbative corrections can account for.

It was then noted [4] that the absence of massless gluon modes of low momentum leads to a peak structure in Δ , and a low momentum cut-off dependent on $T - T_c$ was shown to reproduce the lattice results reasonably well [6]. The fate of the removed low-momentum modes is not particularly important for this agreement, so that lattice results cannot distinguish between different models for this sector (massive gluons, remnant glueball states).

In the present paper, we want to address the question of interactions above T_c in a somewhat different way. The most obvious effect of the presence of a medium is the appearance of a non-trivial dielectric tensor: the medium becomes refractive. All excitations change their dispersion relations, and in particular, photons in a plasma behave as if they were massive. This “mass” is of dynamical origin and depends both on the properties of the medium (density, temperature) and on the momentum of the constituent in question. In an isotropic medium, the dielectric tensor can be decomposed into transversal and longitudinal parts. The corresponding forms for QCD are well-known in the limit of small coupling. The basic dimensional quantity is the plasma frequency ω_o , given by

$$\omega_o^2 = g^2 T^2 \left(\frac{N_f + 2N_c}{18} \right) \quad (5)$$

with N_f and N_c for the number of flavours and of colours, respectively. Different limits of the dielectric tensor describe different physical situations. Thus the Debye screening mass is defined as the zero-frequency limit of the longitudinal part of the dielectric tensor, leading to $m_D = \sqrt{3}\omega_o$. For the thermodynamic properties of the gluon plasma, on the other hand, the most important contributions come from the high momentum limit of the transversal part of the dielectric tensor. The dispersion relation now becomes

$$k_{oT}^2 - \mathbf{k}_T^2 = \left(\frac{3}{2} \right) \omega_o^2 + O(k_T^{-2}), \quad (6)$$

where k_{oT} is the energy of the transversal excitation and $\mathbf{k}_T$ its momentum. We see that the refractive medium leads to an effective gluon mass $M = (3\omega_o^2/2)^{1/2}$. In this paper, we will try to use the concept of a refractive medium over the entire temperature range from T_c to the perturbative regime, by introducing an effective dynamical mass for the gluons.

The partition function of an ideal gas of gluons of mass M is given by

$$-\ln Z(T, V) = (3V/\pi^2) \int_0^\infty dk k^2 \ln(1 - \exp[-\sqrt{M^2 + k^2}/T]). \quad (7)$$

In this function, we shall consider the gluon mass as temperature-dependent. By calculating from eq.(7) the pressure P and the entropy density s and comparing both with the lattice results, we have two independent ways to determine the temperature dependence of $M(T)$. This amounts to parametrizing the effect of the interaction in the plasma in the form of a mass of non-interacting constituents, and we shall find that it gives us some interesting insight into the nature of the plasma above the deconfinement point.

2. The Effective Mass Representation

The first step of our analysis is to solve eq.(7) for M , using lattice results for pressure and entropy density. In principle, the entropy density is the more suitable thermodynamical quantity for lattice calculations, since it can be determined completely in terms of the difference between space-space and space-time plaquettes, while the pressure requires also plaquette averages on a symmetric lattice [15,6,7]. For this reason, the entropy density results go up to about $10 T_c$, while for the pressure they exist only below $2.5 T_c$. On the other hand, the pressure is directly connected to the mass M through eq.(7),

$$P = -\frac{T}{V} \ln Z, \quad (8)$$

whereas the entropy density

$$s = \left(\frac{\partial P}{\partial T} \right)_V \quad (9)$$

involves both the mass M and its derivative dM/dT . To obtain M also for temperatures above $2.5 T_c$, we shall therefore integrate eq.(9) numerically [5] and fix the integration constant by requiring that the “new” pressure thus obtained coincides with the directly calculated pressure at or near T_c .

Since we thus have two independent ways to obtain the pressure, our procedure serves as a self-consistency test for the lattice calculations. In Fig.1 we show the pressure and energy density calculated directly from (space-space, space-time and symmetric) plaquette averages, compared to the values obtained from integrating s over T to get P and hence also $\epsilon = sT - P$. The agreement is quite good over the whole range, which gives us some confidence in the validity of the lattice results. Note the integration of the entropy density requires a relation between the temperature and the input coupling g^2 ; we have here used the perturbative renormalisation group relation

$$a\Lambda_L = \exp\left\{-\frac{12\pi^2}{11} \frac{1}{g^2} - \frac{51}{121} \ln \frac{g^2}{24\pi^2}\right\}. \quad (10)$$

This asymptotic form is likely to be incorrect in the region just above T_c , where strong deviations are observed for SU(2) and SU(3) gauge theory [16]. The agreement found between the directly calculated pressure (which requires $a(g^2)$ only to determine the scale in T) and that gotten by integrating s (where $a(g^2)$ enters both in the T -scale and in the value of P) thus constrains the possible deviations of $a(g^2)$ from the renormalisation group form (10).

Our results can now be converted into a temperature dependent effective gluon mass M , using the relations (7) and (8). Its form is shown in Fig. 3, both from direct pressure calculations (below $2.5 T_c$) and from the numerical integration of the entropy density. Again the two sets of points coincide well in the overlap region. The effective mass is found to drop quickly just above T_c ; it has a minimum at $T \simeq 2 T_c$, and then it increases almost linearly in T . Such a behaviour is certainly not evident from the behaviour of the pressure and energy density alone, so the effective mass

representation enables us to uncover some “hidden” features of the lattice results. How can these features be interpreted?

3. The High Temperature Regime

As we have introduced it here, M is not a mass of some real excitations in the gluon plasma, but only some average measure of the interactions in this plasma. Hence we want to see if it can be related to some known aspects of the interaction in the perturbative as well as in the non-perturbative regime.

Let us consider high temperature region first. Here perturbation theory should be applicable, since the running coupling constant $g(T) \rightarrow 0$ as $T \rightarrow \infty$. The resulting form for the pressure up to first order was included in eq.(2). On the other hand, we can expand eqs.(7/8) for small M/T to get

$$P = (\pi^2/15)T^4[1 - 15(M/2\pi T)^2] \quad (11)$$

Comparing eqs.(11) and (2), we see that perturbation theory leads to

$$M^2 = T^2 g^2(T)/6 \quad (12)$$

for behaviour of the effective mass at high temperature. This agrees quite well with the nearly linear T -dependence of $M(T)$ seen in Fig.3.*

According to renormalization group theory, the running coupling constant $g(T)$ in finite temperature SU(2) gauge theory is determined by a scale parameter Λ_T ,

$$g(T)^2 = \frac{12 \pi^2}{11 \ln(T/\Lambda_T)}. \quad (13)$$

We can therefore parametrise the high temperature lattice results in the form

$$M^2 = \frac{2\pi^2 T^2}{11 [\ln(T/T_c) + \delta]}, \quad (14)$$

with

$$\delta = \ln \frac{T_c}{\Lambda_T} \simeq 1.2 \quad (15)$$

determined by the value of M at $T/T_c = 10$. This leads to the behaviour shown in Fig.4. We see that the effective mass M does appear to approach its perturbative

* The calculation of an effective gluon mass based on the lowest order self-energy graph gives $m^2 = g^2(T) T^2/3$; this differs by a factor 2 from eq.(12). The origin of this “discrepancy” is that in perturbation theory, the effect of the entire medium on one test gluon is calculated. This effect is, however, due to gluon-gluon interactions, and if we want to introduce an M^2 for all gluons, we have to partition m^2 in lowest order between two interaction partners, so that $M^2 = m^2/2$. We are grateful to E. Shuryak for pointing this out to us.

thermal form in the range between 5 - 10 T_c ; this is in accord with other studies of the “perturbative horizon” [17].

If we use $T_c \simeq 270$ MeV as the critical temperature for SU(2) gauge theory, the value of δ obtained from eqs.(14/15) implies $\Lambda_T \simeq 80$ MeV. To estimate what this means, we recall that the usual running coupling varies as $1/\ln(K/\Lambda_{\overline{MS}})$, where K is a measure of the energy, and $\Lambda_{\overline{MS}} \simeq 200 - 300$ MeV [18]. If we take K here as a measure of the gluon energy and hence assume $K^2 = 4 < k^2 > /3 = (4T)^2$ [19], we expect $\Lambda_T \simeq 50 - 80$ MeV. The order of magnitude of the Λ_T obtained from our fit to $M(T)$ is thus quite reasonable.

4. The Behaviour Near T_c

Our simple model of an ideal gas of massive gluons is therefore in good qualitative agreement with the expected high temperature behaviour. This shows that the effective mass parametrization makes some physical sense: in a wide range of temperatures above T_c the main effect of gluon-gluon interactions can be accounted for by a gap in the dispersion relation. In other words, the energy of excitation starts not at zero but at some non-zero value closely related to what we have here called an effective mass. The remaining part of interactions contributes relatively little, so that in many respects gluons indeed behave as if they were massive non-interacting transversal waves. The origin of the gap in the dispersion relation will be perturbative or non-perturbative, depending on the temperature, and the competition between the two regimes leads to the minimum in effective mass at $T \simeq 2T_c$; the purely perturbative regime appears to start for $T \gtrsim 2.5 T_c$.

At present we have no consistent model describing the dispersion relation in the non-perturbative regime, so our results can only serve as an indication that near T_c something interesting is happening to the dispersion relation. The only well-established feature in this region is the critical behaviour of the thermodynamic observables of the gluon gas at the phase transition point [2,3]. It is therefore of interest to see if a critical exponent can be extracted from our effective mass description.

The specific heat c_V diverges at the critical point of a second order phase transition,

$$c_V \sim \left(1 - \frac{T_c}{T}\right)^{-\alpha}, \quad (16)$$

with α denoting the corresponding critical exponent. The specific heat is proportional to the second derivative of the pressure with respect to the temperature, $c_V \sim (\partial^2 P / \partial T^2)$. From eqs.(7/8) one can see that the mass M depends on the temperature T only through the pressure P : $M(T) = M(P(T))$; the dependence of M on P is regular everywhere. Therefore only the second derivative of the effective mass with respect to the temperature is singular, and its critical behaviour must coincide with that of the pressure,

$$\frac{\partial^2 M}{\partial T^2} \sim \frac{\partial^2 P}{\partial T^2} \sim (T - T_c)^{-\alpha} \quad \text{for } T \rightarrow T_c \quad (17)$$

To obtain an estimate of α from the behaviour of $M(T)$ near T_c , we write $P(T)$ in the form

$$P(T) = P(T_c) + s(T_c)(T - T_c) + A(T - T_c)^{2-\alpha} + O((T - T_c)^2) \quad (18)$$

where $P(T_c)$ and $s(T_c)$ are pressure and entropy density at the critical point, respectively, and A is a numerical coefficient. It follows from eq.(18) that near T_c

$$\ln F(T) = -\alpha \ln(T - T_c) + \text{const.} \quad (19)$$

where $F(T) = [P(T) - P(T_c) - s(T_c)(T - T_c)] / (T - T_c)^2$. The behaviour of $F(T)$ is shown in Fig.5. We see that for $T \rightarrow T_c$ the slope $-\alpha$ indeed seems to approach the value $\alpha = 0.107 \pm 0.005$ obtained for the Ising model in three space dimensions [20].

We further note the relatively large magnitude of the effective mass near T_c , which leads to the mentioned gap in the dispersion relation and hence results in a reduction of the particle density in comparison to the massless case. This reduction is the stronger the closer the particle's momentum is to zero. In the presence of a finite effective mass we thus get

$$(\exp(\sqrt{M^2 + p^2}/T) - 1)^{-1} \rightarrow (\exp(M/T) - 1)^{-1} = \text{const.} \quad (20)$$

in the limit of vanishing momentum $p \rightarrow 0$, while the zero-mass case leads to the singular form

$$(\exp(p/T) - 1)^{-1} \rightarrow T/p. \quad (21)$$

This reduction is responsible for the relatively slow growth of P in the range $T_c \leq T \lesssim 2 T_c$.

We thus find that a model based on the refractive properties of the medium suppresses low momentum excitations and in this sense is similar to models with a low momentum cut-off [4,6]. But in contrast to the latter, the effective mass model implies that not only the low-momentum excitations are suppressed, but that in a wide range of momenta the potential energy of excitations dominates their kinetic energy. A suppression of this kind seems less artificial than a sharp cut-off at some point in momentum space. In our model, the gap in the dispersion relation remains present for all temperatures; the effective mass never vanishes (in fact, increases with increasing T), but rather becomes negligible at high T in comparison to the particle momentum. The validity of such a refractive picture is also supported by a comparison with the simplest case of a QED plasma.

In the region $T < T_c$, eq.(7) would have to be replaced by the partition function for an ideal gas of massive bosonic hadrons, such as glueballs, so M in this case would correspond to the mass of the lowest glueball state. But higher excited gluonium states can play a role as well, so that below T_c the degeneracy factor also becomes temperature-dependent, and relative weights become important for a correct interpretation. We can only note that we obtain here a value $M = (6.0 \pm 0.5) T_c$ in the range $T = (0.92 - 1.0) T_c$, and that this is roughly of the same order of magnitude as glueball mass predictions [21].

5. Conclusions

We have considered a model for an ideal gas of gluons in a refractive medium. We have described the refractive properties of the medium by introducing a temperature-dependent effective mass $M(T)$ for the excitations, and we have parametrized the lattice data in terms of this effective mass. The model was shown to agree with the weak coupling limit.

The temperature-dependence of the effective mass now embodies the effects of interactions in the plasma. Just above T_c , $M(T)$ drops very quickly; it has a minimum around $T \simeq 2 T_c$, and starts to increase nearly linearly at higher temperatures. Its growth for $T > 2 T_c$ agrees with the perturbative thermal mass $M(T) \sim gT$. This high temperature behaviour is otherwise not evident from the lattice calculations themselves. The behaviour of $M(T)$ near the critical point shows a singularity corresponding to the second order phase transition; the resulting critical exponent is in accord with that obtained by universality arguments from the corresponding Ising model.

Acknowledgments.

The authors are grateful to F. Karsch, V. Mitrushkin and B. Petersson for helpful discussions.

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Figure Captions

Fig. 1: Pressure (triangles) and energy density (crosses), normalised to the Stefan-Boltzmann forms, from calculations on a $18^3 \times 4$ lattice [6], compared to the forms obtained by numerical integration of the entropy (solid and dashed lines, respectively).

Fig. 2: Entropy density (crosses) and interaction measure Δ (circles) as functions of temperature, from calculations on a $18^3 \times 4$ lattice [6]. The solid line shows the bag model prediction for Δ with $B = 0.6 T_c^4$.

Fig. 3: The effective mass M as a function of temperature. Crosses show the results obtained from direct lattice calculations of the pressure, the solid line those obtained by numerical integration of the entropy density calculations.

Fig. 4: The ratio of M to its perturbative limit $gT/\sqrt{6}$, as function of T/T_c .

Fig. 5: Critical behaviour of M near the phase transition point. The constant in eq.(19) is fixed at $T_1 \equiv 1.0253T_c$, the dotted lines show the error margin of the calculation, and the dash-dotted line indicates the Ising model exponent.

Fig. 1

Fig. 2

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