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CHIRAL LATTICE THEORIES WITHOUT REDUNDANT VARIABLES

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Abstract - In this note we discuss a lattice regularization of chiral gauge theories which does not make use of unphysical auxiliary degrees of freedom. Chiral fermions are described by Weyl spinors and the fermion doubling problem is solved by a Wilson term which explicitly breaks fermion number. The resulting anomalous baryon number non-conservation, in the continuum limit, is discussed.

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The subject of lattice discretized chiral gauge theories has attracted considerable attention in the course of the last years. As well known, the problem with chiral theories is the impossibility of regularizing the fermionic matter in a chiral invariant way. This is not just a technical subtlety, but implies spectacular physical consequences such as the non vanishing of the $\pi^0 \rightarrow \gamma\gamma$ decay, even in the chiral limit, the $\eta - \eta'$ mass splitting and an interdependence between quark and lepton degrees of freedom required in order to cancel anomalies in the Weinberg-Salam model.

Once in possess of a consistent discretization scheme, it will be possible to study non-perturbative phenomena, along the lines already explored in QCD, in particular the structure of the physical spectrum, in order to elucidate the dynamics of possible symmetry breaking. Another interesting field of application would be the study of baryon number non-conservation at zero or finite temperature.

Several approaches to the problem of lattice quantization of chiral theories, have been proposed in the literature[1][2]. We will refer to the approach discussed by us in [1] as the Roma Approach (RA). In ref.[3] we discussed the relation between the RA and the other approaches and showed under which conditions the existing schemes can be regarded as equivalent. We also argued that, in fact, the RA is the most economical scheme to implement.

Lattice approaches to chiral gauge theories have been recently criticized in ref.[4] on the ground that anomalies related to global simmetries seem not to be correctly reproduced. In particular, interesting non-perturbative phenomena, such as anomalous baryon number non-conservation[5] could be absent. As we shall see later, this observation does not apply to the RA. In addition, these criticisms loose impact if we recall that, from a general field theoretical point of view, the lattice is nothing but a particular regularization scheme. This means that it must be possible to discretize **any** formal continuum theory without being forced to introduce additional degrees of freedom, unless for practical or aesthetic reasons. The only acceptable physical restrictions are those necessary in any regularization scheme, as, for example, the cancellation of anomalies.

The approaches formulated thus far, including the RA (although in a mild form), require the introduction of some, more or less fictitious, degree of freedom. In the present note we discuss the definition of a chiral gauge theory without the introduction of redundant degrees of freedom. As we will show, this formulation is of great help in clarifying questions related to possible non-perturbative phenomena, including the problem of anomalous baryon number non-conservation.

Let us just recall that **any** construction of a field theoretical model starts with a formal (non-regularized) target theory (TT), whose role is to summarize the general properties (typically symmetries) we would like to see realized in the true, non formal, quantized theory.

In the present case we choose as an example of TT the simple chiral theory already discussed in ref.[1], namely a chiral gauge theory based on the group SU(2) (this restriction is only of a technical nature, in order to avoid the introduction of several fermionic species needed for anomaly cancellation):

$$\begin{aligned}
 S &= S_G + S_{gf} \\
 S_G &= \int d^4x \left(\bar{\chi}_{\dot{\alpha}} \mathcal{D}^{\dot{\alpha}\beta} \chi_{\beta} + \frac{1}{4} W_{\mu\nu}^a W_{\mu\nu}^a \right) \\
 S_{gf} &= \int d^4x \left\{ \left(\frac{1}{2\alpha_0} \right) (\partial_{\mu} W_{\mu}^a) (\partial_{\nu} W_{\nu}^a) + \bar{c} \partial_{\mu} D_{\mu} c \right\}
 \end{aligned} \tag{1}$$

In eq. (1), we have adopted the usual SL(2,C) notation for Weyl spinors. In Minkowsky space-time, the bar over the Weyl spinor would denote complex

conjugation. In the Euclidean formulation, employed here, the functional integration is to be performed on the two groups of Grassmann variables, χ and $\bar{\chi}$, independently. Thus in complete analogy with the Dirac case we define:

$$\langle O \rangle = \int \delta A \delta \bar{c} \delta c \delta \bar{\chi} \delta \chi \exp[-S] O \quad (2)$$

where the functional measure is understood to be normalized to 1.

When the formal theory described by eqs. (1) and (2) is regularized, the problems related to the existence of anomalies come into play: it will not be possible to have, at the same time, (i) a regularized spectrum corresponding to that of the formal theory (1) and (ii) chiral symmetry at the cutoff level. In particular, when the theory is discretized, fermion doublers appear, which can be removed by the introduction of an appropriate Wilson term. The scheme discussed in ref.[1] is different from the one presented here, because the former required the introduction of a fictitious right-handed multiplet, ψ_R , together with the physical left-handed one, ψ_L . The fictitious fermions, ψ_R , would decouple in the continuum and were only used to write down a Wilson term in the Dirac form¹:

$$S'_W = a \int d^4 x (\bar{\psi}_L \partial_\mu \partial_\mu \psi_R + h.c.) \quad (3)$$

where a denotes the lattice spacing.

Within the approach described by eq. (1), doublers will still be present and can be removed by a Wilson-like term of the Weyl form[6][7]:

$$S_W = a \int d^4 x (\chi^\alpha \partial_\mu \partial_\mu \chi_\alpha + \bar{\chi}_{\dot{\alpha}} \partial_\mu \partial_\mu \bar{\chi}^{\dot{\alpha}}) \quad (4)$$

In this way we are led to consider a discretized action of the form:

$$\begin{aligned} S &= -\frac{1}{2} \sum_{x\mu} [\bar{\chi}_{\dot{\alpha}}(x+\mu) (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} V_\mu^+(x) \chi_\alpha(x) - \bar{\chi}_{\dot{\alpha}}(x) (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} V_\mu(x) \chi_\alpha(x+\mu)] + S_W \\ S_W &\equiv -r \sum_{x\mu} [(\chi^\alpha(x+\mu) - \chi^\alpha(x)) \chi_\alpha(x) + \bar{\chi}_{\dot{\alpha}}(x) (\bar{\chi}^{\dot{\alpha}}(x+\mu) - \bar{\chi}^{\dot{\alpha}}(x))] \end{aligned} \quad (5)$$

Working with Weyl fields we must consider three different kinds of propagators which, in the free field limit, read as follows:

$$\begin{aligned} \int d^4 x e^{ipx} \langle \chi_\alpha(x) \chi_\beta(0) \rangle &= \frac{-\varepsilon_{\alpha\beta}}{D(p) + W(p)^2} W(p) \\ \int d^4 x e^{ipx} \langle \bar{\chi}^{\dot{\alpha}}(x) \chi^\alpha(0) \rangle &= \frac{-i}{D(p) + W(p)^2} \sum_\nu \sin p_\nu (\bar{\sigma}_\nu)^{\dot{\alpha}\alpha} \\ \int d^4 x e^{ipx} \langle \bar{\chi}_{\dot{\alpha}}(x) \bar{\chi}_{\dot{\beta}}(0) \rangle &= \frac{-\varepsilon_{\dot{\alpha}\dot{\beta}}}{D(p) + W(p)^2} W(p) \end{aligned} \quad (6a)$$

¹We will use, whenever possible, the continuum notation.

where:

$$\begin{aligned} W(p) &= 2r \sum_{\mu} (1 - \cos p_{\mu}) \\ D(p) &= \sum_{\mu} (\sin p_{\mu})^2 \end{aligned} \quad (6b)$$

Eqs. (6) exhibit the doubling phenomenon for $r=0$, the removal of the doubling through the Wilson term and, most important, the violation of fermion number induced by the Wilson term (4). As in the RA, the Wilson term is an irrelevant coupling and can be compensated by finite, or power divergent, non-gauge invariant (and fermion number-violating) counterterms with dimension ≤ 4 . Perturbation theory within this framework is being explored[8] and looks fine.

We want to remark here that, as usual, fermion integration can be performed exactly, both in perturbative and non-perturbative settings, being of a gaussian nature. It leaves us with a functional determinant which, at first sight is not well defined from a geometrical point of view, since it involves an operator which transforms undotted indices into dotted ones. We recall however that, due to the normalization of the functional measure, only ratios of functional determinants are needed. In the present case we can factorize from the left (or the right) a term such as $\det(\not{D}^{\dot{\alpha}\beta})$ and the remaining functional determinant will involve an integral operator with only undotted (dotted) indices.

In order to be able to discuss the problem of anomalous current non-conservation, which is a phenomenon already visible at the perturbative level, we must describe first the construction of observables within the framework of a gauge non-invariant regularization. Such a construction can be performed similarly, both in the RA and in the scheme discussed in the present paper. The basic idea is quite simple. Once a complete set of renormalized composite operators is defined, along the lines usually considered in field theory, we take suitable linear combinations of them:

$$O \equiv \sum_i c_i O_i \quad (7)$$

and fix the coefficients c_i in such a way that the formal BRST identities:

$$\langle O(x) \delta_{BRST} \{ \Phi_1(x_1) \dots \Phi_n(x_n) \} \rangle = 0 \quad (8)$$

be satisfied up to terms of order a . In eq.(8) $\Phi_i(x)$ denotes any one of the elementary fields of the theory. Equation (8) simply states the BRST invariance of $O(x)$ in the continuum limit. When applied to the current describing the flow of the fermion number, eq. (8) forces us to add to it an operator proportional to:

$$K_{\mu}(x) = \varepsilon_{\mu\nu\rho\sigma} (A_{\nu}^a \partial_{\rho} A_{\sigma}^a + \frac{1}{3} f^{abc} A_{\nu}^a A_{\rho}^b A_{\sigma}^c) \quad (9)$$

It is in this way that the anomaly of the fermion current is reproduced.

Finally we consider the question of fermion number non-conservation. Its explicit violation in the lattice action (5) is welcome for the following reason. In the RA we had, at the lattice level, an exactly conserved fermion number (left + right). The corresponding conserved current is not gauge invariant. The gauge invariant current, constructed along the lines discussed above, has, however, the expected anomaly. This means that fermion number is not necessarily conserved in physical processes.

In the RA, only fermion number conserving Green's functions can be different from zero at the lattice level, and we could get the physically interesting, fermion number violating amplitude, only by cluster factorization. For example, let us denote by $O_{\Delta F=1}(x)$ a product of local operators clustered around x , describing a transition with $\Delta F = 1$ and by $O_{\Delta F=-1}(y)$ a similar product clustered around another point y , with $\Delta F = -1$. Then the lattice Green's function $\langle O_{\Delta F=1}(x) O_{\Delta F=-1}(y) \rangle$ will be different from zero and we could define fermion number violating Green's functions by:

$$\lim_{x-y \rightarrow \infty} \langle O_{\Delta F=1}(x) O_{\Delta F=-1}(y) \rangle = \langle O_{\Delta F=1}(x) \rangle \langle O_{\Delta F=-1}(y) \rangle \quad (10)$$

Of course it is an open, dynamical, question if the Green's functions in the rhs of eq. (10) will be actually different from zero.

One of the interesting features of the scheme based on (5), is the possibility to write down Green's functions corresponding to processes in which the fermion number is violated, already at the lattice level. This fact, by itself, does not imply fermion number non-conservation in physical, continuum processes. It could very well happen that such Green's functions vanish as $a \rightarrow 0$, as is the case in perturbation theory. However, in presence of appropriate non-perturbative configurations (instantons?) they could survive the continuum limit and describe observable physical phenomena.

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