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THE DECAYS $K^+ \rightarrow e^+ \nu_e \gamma$ and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$, AND TENSORIAL COUPLINGS IN THE EFFECTIVE WEAK HAMILTONIAN

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ABSTRACT

A tensorial coupling in the effective weak Hamiltonian has been observed in a recent experiment on the decay $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$. The coupling constant for this interaction is about 10^6 times the one predicted by the standard model. If this effect really exists in the $\Delta S = 0$ effective weak Hamiltonian, we suggest that an analogous kind of interaction may exist in the $\Delta S = 1$ sector. The decays $K^+ \rightarrow e^+ \nu_e \gamma$ and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ are revised in the light of a possible tensorial coupling in the $\Delta S = 1$ effective Hamiltonian. If this new effect exists also in the $\Delta S = 1$ sector, with the same coupling as in the $\Delta S = 0$ sector, and if it is suppressed by the Cabibbo angle, then it can be easily observed at the Daphne laboratory in the μ decay of the K^+ . If this effect is not suppressed by the Cabibbo angle, then also the electron channel decay is very good for detecting it.

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1 Introduction

The weak decays in the sector of charged currents with $\Delta S = 0, 1$ are described very well by the effective weak Hamiltonian

$$H_{eff}^{V-A}(x) = \frac{G_F}{\sqrt{2}} [J_\mu^{1W}(x) J_\mu^{1W}(x)], \quad (1)$$

where G_F is the Fermi coupling and J_μ^{1W} is the total charged weak current. From the $V-A$ and universality weak interaction hypothesis, by considering only the effect of the three light quarks, the weak current J_μ^{1W} is given by

$$\begin{aligned} J_\mu^{1W}(x) &= J_\mu^{1W}(x) + J_\mu^{1W}(x), \\ J_\mu^{1W} &= \cos \theta_c [V_\mu^{\Delta S=0} - A_\mu^{\Delta S=0}] + \sin \theta_c [V_\mu^{\Delta S=1} - A_\mu^{\Delta S=1}], \\ J_\mu^{1W} &= \sum_i [\bar{\psi}_i \gamma_\mu (1 - \gamma_5) \psi_i], \end{aligned} \quad (2)$$

where J_μ^{1W} is the total weak lepton current and the sum is over the three lepton families, $V^{\Delta S=0,1}$ and $A^{\Delta S=0,1}$ are the vector and axial weak hadronic currents respectively, and θ_c is the Cabibbo angle. All the experiments confirm the phenomenological implications of the standard effective Hamiltonian in eq. (1), and very small limits have been found on other possible kinds of interaction. These limits come from the data analysis of beta [1] and muon [2] decays.

In a recent experiment [3] it has been observed that in the π^- decay in the channel

$$\pi^- \rightarrow e^- \bar{\nu}_e \gamma \quad (3)$$

the theoretical branching ratio, calculated with the standard $V-A$ weak interaction theory, differs from the measured one by more than 3 standard deviations. In particular, a branching ratio $B_R^{exp.} = (1.6 \pm 0.23) \times 10^{-7}$ has been measured while the theoretical value is $B_R^{th} = (2.41 \pm 0.07) \times 10^{-7}$. This discrepancy may be avoided by adding to the standard matrix elements of the process (3) the amplitude of a tensorial interaction given by [3],[4]:

$$M_T = i \frac{e G_F}{\sqrt{2}} \cos \theta_c F_T^0 e^\mu q^\nu [\bar{u}(p_e) \sigma_{\mu\nu}^L v(p_\nu)], \quad (4)$$

where $\sigma_{\mu\nu}^{R/L}$ is defined as $\sigma_{\mu\nu}^{R/L} = 1/2[\gamma_\mu, \gamma_\nu](1 \pm \gamma_5)$, $e = \sqrt{\alpha}/2\pi$ is the electric charge, p_e and p_ν are the four-momenta of the electron and antineutrino respectively, and F_T^0 is a constant form factor (the superscript 0, 1 indexing F_T and f_T refer to $\Delta S=0, 1$).

In ref. [3] the authors measure the form factor F_T in the following way. The amplitude M_T interferes destructively with the standard matrix elements and if a $F_T^0 \simeq 5 \times 10^{-3}$ value is used then B_R^{th} has the same value of the measured branching ratio $B_R^{exp.}$. The phenomenological matrix element of eq. (4) can be reproduced [4] by using an effective Hamiltonian $(H_{eff}^T)^{\Delta S=0}$ of the four-fermion interaction of lepton and quark as follows:

$$(H_{eff}^T)^{\Delta S=0} = \frac{f_T^0 G_F}{\sqrt{2}} \cos \theta_c [\bar{\psi}_e \sigma_{\mu\nu}^L \psi_d] [\bar{\psi}_e \sigma_{\mu\nu}^L \psi_u] + h.c., \quad (5)$$

where f_T^0 is an adimensional coupling. Using the PCAC hypothesis, it is possible to relate f_T^0 to the measured form factor F_T^0 . The result is the following [6]:

$$F_T^0 = \frac{2}{3} \frac{\chi(\mu)}{f_\pi} f_T^0 \quad (6)$$

where $\langle \mu \rangle \equiv \langle 0 | \bar{q} q | 0 \rangle$ is the well-known vacuum expectation value for the quark condensate and χ is defined by [7]

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | \gamma(k) \rangle = e_q \chi(\mu) F_{\mu\nu}; \quad (7)$$

$F_{\mu\nu} = i(k_\mu \epsilon_\nu - k_\nu \epsilon_\mu)$ and e_q is the electric charge of the quark q . Assuming the values $\langle \mu \rangle = -(0.24 \text{ GeV})^3$, $\chi = -5.7 \pm 0.6 \text{ GeV}^{-2}$ and using formula (6), we obtain [6]

$$f_T^0 \simeq (1.4 \pm 0.4) \times 10^{-2}. \quad (8)$$

This result is consistent with the limit $|f_T^0| < 0.095$ (at 68% of confidence level), which comes from beta decay [4].

The tensorial coupling of eq. (5) can be obtained from the standard model [6], using the effective field theory techniques. This gives $f_T^0 \simeq G_F m_\pi^2 (m_c/m_q) \ln(M_W/m_c) \simeq 10^{-8} - 10^{-9}$, that is $10^{-6} - 10^{-7}$ times the value of f_T^0 in eq. (8). The same techniques can be applied to the minimal supersymmetric version of the standard model and the predicted value is then $f_T^0 \simeq 10^{-4}$ [4].

In a recent theoretical work [9] an upper limit of $|f_T^0| < 10^{-4}$ has been found, which is in contradiction with the f_T^0 of (8). In this work it was pointed out that the electromagnetic radiative corrections to the effective Hamiltonian of eq. (5) induce the interaction term H_{eff}^{SP} given by

$$H_{eff}^{SP} = \frac{G_F \lambda_{SP}}{2\sqrt{2}} [\bar{\psi}_e (1 + \gamma_5) \psi_d] [\bar{\psi}_e (1 + \gamma_5) \psi_u] + h.c., \quad (9)$$

where the coupling λ_{SP} depends on f_T^0 . From the limits on the coupling λ_{SP} given by the decay $\pi^- \rightarrow \bar{\nu}_e e^-$, one obtains $|f_T^0| < 10^{-4}$. There are no arguments to avoid this upper bound on f_T^0 , except by assuming a rather fine tuning with a bare term or by invoking a new symmetry principle.

So it is important to make further experimental studies on the decay $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$ to either confirm or infirm the results of the experiment in ref. [3]. If this effect really exists, then it could be a signal of new physics.

In this paper we suggest an analogous tensorial coupling for the processes with changing strangeness with the effective Hamiltonian $(H_{eff}^T)^{\Delta S=1}$ given by

$$(H_{eff}^T)^{\Delta S=1} = \frac{f_T^1 G_F}{2\sqrt{2}} \sin \theta_c [\bar{\psi}_e \sigma_{\mu\nu}^L \psi_s] [\bar{\psi}_l \sigma_{\mu\nu}^L \psi_u] + h.c., \quad (10)$$

where in principle f_T^1 may be different from the value of f_T^0 . The index l stands for any lepton family assuming lepton universality. The aim of this paper is to explore the tensorial interaction (10) in the leptonic decays of the K^+ . In order to get a feeling of the interesting range for the coupling f_T^1 , we make two different reasonable hypotheses:

1. We impose the universality for the coupling between the quark tensorial currents $J_{\mu\nu}^d, J_{\mu\nu}^s$ given by

$$\begin{aligned} J_{\mu\nu}^d &= \cos \theta_c J_{\mu\nu}^d = \cos \theta_c [\bar{\psi}_u \sigma_{\mu\nu}^L \psi_d] \\ J_{\mu\nu}^s &= \sin \theta_c J_{\mu\nu}^s = \sin \theta_c [\bar{\psi}_u \sigma_{\mu\nu}^L \psi_s] \end{aligned} \quad (11)$$

and the leptonic tensorial current $J_{\mu\nu}^l = \bar{\psi}_l \sigma_{\mu\nu}^L \psi_l$. Then the couplings f_T^1 and f_T^0 must be equal.

2. Universality can be assumed for the coupling of the currents J_{μ}^d and J_{μ}^s (excluding this time the Cabibbo angle mixing) with J_{μ}^l .

In this case one has $f_T^d \simeq f_T^s \cos \theta_c / \sin \theta_c$ and the tensorial coupling effect should be amplified for the processes with changing strangeness, because of the suppression of the standard $V - A$ theory by a factor $\sin \theta_c$.

These two hypotheses suggest to explore the following range for f_T^l

$$0 < |f_T^l| < f_T^d / \tan \theta_c \simeq 6.2 \times 10^{-2}. \quad (12)$$

The best way to study the effects of the effective Hamiltonian of eq. (10) is through the decays

$$K^+ \rightarrow e^+ \nu_e \gamma \quad \text{and} \quad K^+ \rightarrow \mu^+ \nu_\mu \gamma \quad (13)$$

In fact, on the one hand, matrix elements of the tensorial interaction of eq. (10) are not suppressed by PCAC. On the other hand, these decays are particularly sensitive to the tensorial coupling in the range (12). Moreover, it is easy to relate the tensorial form factor F_T^l to the tensorial coupling f_T^l , as in the case of the decays $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$. On the contrary, the decay $K^+ \rightarrow \pi^0 e^+ \nu_e$, which has a large branching ratio and could be interesting for detecting the tensorial interaction of eq. (10), has the matrix element corresponding to the tensorial interaction of eq. (10) suppressed by PCAC. Indeed, using the PCAC approximation and the well-known reduction formula for the matrix element $\langle \pi^0(p_\pi) | B_{\mu\nu}(0) | K^+(p_K) \rangle$, where $B_{\mu\nu}(x) \equiv \bar{\psi}_s(x) \sigma_{\mu\nu}^R \psi_u(x)$, we obtain

$$\langle \pi^0(p_\pi) | B_{\mu\nu}(0) | K^+(p_K) \rangle \simeq \frac{1}{f_\pi} \int d^3x \langle 0 | A_0^0(x), B_{\mu\nu}(0) |_{x=0} | K^+(p_K) \rangle = 0, \quad (14)$$

where A_0^0 is the axial weak hadronic neutral current which annihilates the final pion state $\langle \pi^0 |$. The result (14) holds because it is not possible to construct an antisymmetric tensor with only one momentum. The matrix element (10) will be different from zero, only at higher orders in the chiral perturbation theory.

2 Analytical and numerical results for the decays $K^+ \rightarrow l^+ \nu_l \gamma$

We start with the calculation of the amplitude for the process

$$K^+(p) \rightarrow l^+(p_l) \nu_l(p_\nu) \gamma(q), \quad (15)$$

where p , p_l , p_ν , and q are the corresponding four momenta and the index l stands for a muon or an electron. For the decays (15) we calculate the total branching ratio and their sensitivity to the variations of the coupling f_T in the range (12).

The Feynman diagrams at tree level are given in figure 1. The first two diagrams 1a, 1b are the so-called inner bremsstrahlung (IB) diagrams, because the photon γ is emitted by the external lines. The other diagram is the so-called "structure-dependent" (SD) diagram, because the photon is emitted by an intermediate hadronic state (different from the K) and its matrix element will depend on form factors related to the low-energy effect of the strong interactions. The form factors which appear in the SD diagram are well measured, see for example ref. [11].

Assuming the standard theory $V - A$, the matrix element M_1 for the diagram of fig. 1a is given by:

$$M_1 = \frac{ieG_F}{\sqrt{2}} \sin \theta_c f_K \epsilon^{\dagger \mu} p^\beta \left[\bar{u}(p_\nu) \gamma_\beta (1 - \gamma_5) \left(\frac{\not{q} + \not{p} - m_l}{(q + p)^2 - m_l^2} \right) \gamma_\mu v(p_l) \right], \quad (16)$$

where ϵ_μ is the photon polarization vector, m_l is the mass of the lepton l and f_K is the K decay constant defined as

$$\langle 0 | A_4^+ - i A_5^+ | K^+(p) \rangle = i f_K p^4. \quad (17)$$

The indices 4, 5 are the usual indices of flavour $SU(3)_F$ axial currents. The matrix element M_2 obtained by summing the diagrams 1b and 1c is the following

$$M_2 = -\frac{ieG_F}{\sqrt{2}} \sin \theta_c \epsilon^{\dagger \mu} \left[\bar{u}(p_\nu) \gamma^\beta (1 - \gamma_5) v(p_l) \right] \left[V_{\mu\beta}(p, q) - A_{\mu\beta}(p, q) \right] \quad (18)$$

where the tensors $V_{\mu\beta}(p, q)$ and $A_{\mu\beta}(p, q)$ are defined through

$$I^{\mu\beta}(p, q) = \int d^4x e^{i q x} \langle 0 | T V_{em}^\mu(x) (I_4^\beta(0) - i I_5^\beta(0)) | K^+(p) \rangle, \quad I = V, A, \quad (19)$$

and V_{em}^μ is the hadronic electromagnetic current. In the quark model, the currents used in the test are given by

$$\begin{aligned} V_4^\mu - i V_5^\mu &= \bar{\psi}_s \gamma^\mu \psi_u \\ A_4^\mu - i A_5^\mu &= \bar{\psi}_s \gamma^\mu \gamma_5 \psi_u \\ V_{em}^\mu &= \frac{2}{3} \bar{\psi}_u \gamma^\mu \psi_u - \frac{1}{3} \bar{\psi}_s \gamma^\mu \psi_s - \frac{1}{3} \bar{\psi}_s \gamma^\mu \psi_s. \end{aligned} \quad (20)$$

By the electromagnetic gauge invariance, the tensor $I_{\mu\nu}$ satisfies the following Ward identities

$$q^\mu A_{\mu\beta} = -f_K p_\beta \quad \text{and} \quad q^\mu V_{\mu\beta} = 0. \quad (21)$$

Using the Lorentz covariance and the electromagnetic gauge invariance given by eq. (21), it follows

$$\begin{aligned} V_{\mu\nu} &= i \frac{V}{M_K} \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta \\ A_{\mu\nu} &= (pq) \frac{A}{M_K} \left[g_{\mu\nu} - \frac{p_\mu q_\nu}{(pq)} \right] - f_K \left[g_{\mu\nu} + \frac{p_\mu q_\nu}{(pq)} \right], \end{aligned} \quad (22)$$

where M_K is the mass of the K , the four-momentum $Q_\mu = q_\mu - p_\mu$ and A , V are the axial, vector form factor respectively. For CP invariance V and A are real. In general these form factors will depend weakly on Q^2 , as can be shown by using chiral perturbation theory. At the one-loop level in chiral perturbation theory, they are constant and the result is the following [11]:

$$V + A = -0.137 \quad \text{and} \quad V - A = -0.052 \quad (23)$$

The result in eq. (23) agrees [11] with experimental data. In the following we consider V and A as constant form factors and we will use for numerical calculations the values given in eq. (23).

For the diagram 1d, which contains the matrix element of the new interaction (10), we have

$$M_T = \frac{f_T^l}{\sqrt{2}} G_F \sin \theta_c T^{\mu\nu} \left[\bar{u}(p_\nu) \sigma_{\mu\nu}^R v(p_l) \right], \quad (24)$$

where the tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = \langle \gamma(q) | \bar{\psi}_s(0) \sigma^{\mu\nu} \gamma_5 \psi_u(0) | K^+(p) \rangle = i e \frac{F_T^1}{2f_1^1} (\epsilon^{\dagger\mu} q^\nu - \epsilon^{\dagger\nu} q^\mu), \quad (25)$$

and F_T^1 is a form factor. Substituting eq. (22) in the expression of M_2 in eq. (18), summing with M_1 in eq. (16) and with M_T in eq. (24), we find the following result for the total matrix element M :

$$M = M_{IB} + M_{SD} + M_T, \quad (26)$$

where M_{IB} , M_{SD} , M_T are given by

$$\begin{aligned} M_{IB} &= \frac{ieG_F}{\sqrt{2}} \sin \theta_c m_l f_K \epsilon_\mu^\dagger(q) \left[\bar{u}(p_\nu)(1 + \gamma_5) \left(\frac{p^\mu}{(pq)} - \frac{\not{q} \gamma^\mu + 2p^\mu}{2(pq)} \right) v(p_l) \right], \\ M_{SD} &= -\frac{ieG_F}{\sqrt{2}} \sin \theta_c \epsilon_\mu^\dagger H^{\mu\nu} L_\nu, \\ M_T &= \frac{ieG_F}{\sqrt{2}} \sin \theta_c \epsilon^{\dagger\mu} q^\nu F_T^1 \left[\bar{u}(p_\nu) \sigma_{\mu\nu}^R v(p_l) \right], \\ H_{\mu\nu} &\equiv (pq) \frac{A}{M_K} \left[-g_{\mu\nu} + \frac{p_\mu q_\nu}{(pq)} \right] + i \epsilon_{\mu\nu\alpha\beta} \frac{V}{M_K} q^\alpha p^\beta, \\ L^\mu &\equiv \bar{u}(p_\nu) \gamma_\mu (1 - \gamma_5) v(p_l). \end{aligned} \quad (27)$$

The first term, M_{IB} , is the sum of the two Feynman diagrams in figs. 1a, 1b (with the photon emitted by the external lines) and the second term, M_{SD} , is the pure SD contribution represented by the diagram in fig. 1c. The third term, M_T , is the tensorial contribution and it is given by the diagram in fig. 1d.

The relation between the constants F_T^1 and f_T^1 is the same as in eq. (6) for F_T^0 and f_T^0 , and the range of F_T^1 to be explored is thus the following:

$$0 < |F_T^1| < |F_T^0| / \tan \theta_c \simeq 2.2 \times 10^{-2}, \quad (28)$$

where we have used the range in eq. (12) and eq. (6) with $F_T^0 = 5 \times 10^{-3}$ and $\tan \theta_c = 0.225$. After calculating the square modulus $|M|^2$ summed over polarizations and integrating over the phase space, we obtain for the differential decay width Γ^l in the leptonic mode l the following result

$$\frac{d^2\Gamma^l}{dx d\lambda} = \frac{M_K}{256\pi^3} |\overline{M}|^2 = A_{SD} \rho(x, \lambda), \quad (29)$$

where

$$A_{SD} = \frac{\alpha}{32\pi^2} G_F^2 M_K^5 \sin^2 \theta_c. \quad (30)$$

The kinematical variables x and λ are defined as $x = 2pq/M_K^2$ and $\lambda = (x + y - 1 - r_l)/x$ with $y = 2pp_l/M_K^2$. In the K rest frame the variables x and λ are given by

$$x = \frac{2E_\gamma}{M_K}, \quad \lambda = \frac{E_l}{M_K} \left[1 - \sqrt{1 - \frac{r_l M_K^2}{E_l^2} \cos \theta_{\gamma l}} \right] \quad (31)$$

where E_γ and E_l are the energy of photon and lepton l respectively and $\theta_{\gamma l}$ is the angle between their three-momenta. Here $r_l \equiv (m_l/M_K)^2$ and $\rho(x, \lambda)$ is given by

$$\rho(x, \lambda) = \rho_{IB}(x, \lambda) + \rho_{SD}(x, \lambda) + \rho_{IBSD}(x, \lambda) + \rho_\tau(x, \lambda) + \rho_{IB\tau}(x, \lambda) + \rho_{SD\tau}(x, \lambda), \quad (32)$$

where

$$\begin{aligned} \rho_{IB}(x, \lambda) &= 2r_l \left(\frac{f_K}{M_K} \right)^2 f_{IB}(x, \lambda), \\ \rho_{SD}(x, \lambda) &= \frac{1}{2} \left[(V + A)^2 f_{SD+}(x, \lambda) + (V - A)^2 f_{SD-}(x, \lambda) \right], \\ \rho_{IBSD}(x, \lambda) &= 2r_l \frac{f_K}{M_K} \left[(V + A) f_+(x, \lambda) + (V - A) f_-(x, \lambda) \right], \\ \rho_\tau(x, \lambda) &= 2(F_T^1)^2 f_{\tau\tau}(x, \lambda), \\ \rho_{IB\tau}(x, \lambda) &= 4\sqrt{r_l} \frac{f_K}{M_K} F_T^1 f_{IB\tau}(x, \lambda), \\ \rho_{SD\tau}(x, \lambda) &= 2\sqrt{r_l} F_T^1 (V - A) f_{SD\tau}(x, \lambda). \end{aligned} \quad (33)$$

The expressions for the functions $f_i(x, \lambda)$ are given in the appendix. The tensorial effect is contained only in the last three terms of (33).

To obtain the differential branching ratio (B_R^l) we divide the differential width $d^2\Gamma^l/dx d\lambda$ by the total width Γ_T

$$\begin{aligned} \frac{d^2 B_R^l}{dx d\lambda} &= \frac{1}{\Gamma_T} \frac{d^2 \Gamma^l}{dx d\lambda} \\ \Gamma_T &= A_{SD} / (8.348 \times 10^{-2}) \end{aligned} \quad (34)$$

The total B_R^l is obtained by integrating eq. (34) in the full kinematical range, as follows:

$$B_R^l = \int dx d\lambda \frac{d^2 B_R^l}{dx d\lambda} \quad (35)$$

$$0 \leq x \leq 1 - r_l, \quad \frac{r_l}{1 - x} \leq \lambda \leq 1. \quad (36)$$

In the following, it is useful, for studying B_R^l in eq. (35), cutting the range in eq. (36) by arbitrary minima $\lambda^{\min}$, $x^{\min}$ for λ and x respectively, as follows:

$$x^{\min} \leq x \leq 1 - r_l, \quad \max \left\{ \lambda^{\min}, \frac{r_l}{1 - x} \right\} \leq \lambda \leq 1 \quad (37)$$

By varying these cuts, one can select in the phase space the kinematical condition where the sensitivity of the branching ratio to the tensorial coupling is enhanced. For the differential branching ratios dB_R^l/dx and $dB_R^l/d\lambda$, we have

$$\frac{dB_R^l}{dx} = \int_{\lambda^*}^1 d\lambda \frac{d^2 B_R^l}{dx d\lambda}, \quad \lambda^* = \max \left\{ \lambda^{\min}, \frac{r_l}{1 - x} \right\}, \quad 0 \leq x \leq 1 - r_l, \quad (38)$$

$$\frac{dB_R^l}{d\lambda} = \int_{x^{\min}}^{x^{\max}} dx \frac{d^2 B_R^l}{dx d\lambda}, \quad x^{\max} = 1 - \frac{r_l}{\lambda}, \quad \frac{r_l}{1 - x^{\min}} \leq \lambda \leq 1. \quad (39)$$

Now we discuss the numerical results for the total and differential branching ratios obtained using the following set of values

$$\alpha = 1/137, \quad m_e = 0.511 \times 10^{-3} \text{ GeV}, \quad m_\mu = 105.66 \times 10^{-3} \text{ GeV}, \quad \sin \theta_e = 0.2196 \\ M_K = 493.6 \times 10^{-3} \text{ GeV}, \quad G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad f_K = 160.7 \times 10^{-3} \text{ GeV}.$$

In figs. 2, 3 the results of the branching-ratio distributions dB_R^{μ}/λ and dB_R^e/λ are shown respectively for different values of the form factor F_T^1 obtained by integrating the (39) for different choices of $x^{\min}$.

The maximum sensitivity of $dB_R^{\mu}/d\lambda$ to F_T^1 is in the region of high λ , while for $dB_R^e/d\lambda$ is at small λ . The reason is the following: the IB term is peaked in the region of small λ and it is proportional to r_e . The most important tensorial contributions, that is IBT and SDT terms, have their maximum at $\lambda \simeq 0$ and are proportional to $\sqrt{r_e}$. In the muon channel the IB term dominates over all the other terms in the region of small λ . In fact r_μ is not as small as r_e and so is not suppressed as much as in the electronic case. Therefore we observe for $dB_R^{\mu}/d\lambda$ weak effects of tensorial contributions only at high λ , where IB can be neglected. At $\lambda = 1$ the tensorial contributions are zero, and so $dB_R^{\mu}/d\lambda$ is independent from F_T^1 . For the electron case the r_e is much smaller than r_μ and so the IB term at small λ is comparable with the tensorial contributions IBT and SDT. Therefore for $dB_R^e/d\lambda$, we have the maximum sensitivity to tensorial coupling at small λ (cf. fig. 3).

In figs. 4, 5 we show respectively the x distributions dB_R^{μ}/dx and dB_R^e/dx obtained by integrating eq. (38) for different values of $\lambda^{\min}$ and for different values of F_T^1 . In this case the best region where to explore the sensitivity of dB_R^{μ}/dx and dB_R^e/dx to F_T^1 is at high x . The reason for this is that the IBT and SDT contributions to dB_R^{μ}/dx are peaked at $x^{\max}$, where IB term can be neglected. In particular in the electronic case the distribution dB_R^e/dx is mainly sensitive to the modulus of F_T^1 .

In figs. 6, 7 we give the curves respectively for B_R^{μ} and B_R^e as functions of F_T^1 and for different ranges of integration. In tables 1 and 2 we report their corresponding numerical values. From the numerical results in tables 1 and 2 and from figs. 6 and 7 we observe that the decay $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ (in the kinematical range $\Lambda_3 = \{\lambda_3^{\min} \leq \lambda \leq 1, 0.7 \leq x \leq 1 - r_l\}$ with $\lambda_3^{\min} = \max\{r/(1-x), 0.7\}$) is the better channel for searching the tensor interaction of the first hypothesis, that is $f_T^1 = f_T^0$. In this range Λ_3 the total branching ratios B_R^{μ} and B_R^e calculated at $F_T^1 = F_T^0$ differ from the pure $V - A$ theory by about 7% and (1-1.5)% respectively. In the last case this difference is comparable with the radiative corrections, which are of the order of 1%.

In the second hypothesis, where in eq. (10) the coupling $f_T^1 = f_T^0/\tan \theta_e$, the percentage differences between B_R^{μ} and B_R^e from the standard $V - A$ theory are about (14-20)% and (9-28)% respectively. Therefore in this case both e and μ channel decays are good for detecting the tensorial coupling.

3 Conclusion

We assume a tensorial coupling in the $\Delta S = 1$ sector of the effective weak Hamiltonian analogous to the one introduced for explaining a recent experiment on the decay $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$ and suggest to test it in the decays $K^+ \rightarrow e^+ \nu_e \gamma$ and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$. We study two hypotheses for this tensorial coupling. One is to impose universality for the coupling in the $\Delta S = 0$ and $\Delta S = 1$ sector, including the Cabibbo angle mixing as in the standard $V - A$ theory. The other is to impose the same universality, but without

the Cabibbo mixing. For the first hypothesis the best way to observe this effect is by the K^+ decay in the μ channel: in this case it can be seen as a 7% deviation from the $V - A$ standard theory, when the standard model branching ratio is of order 10^{-5} . For the second hypothesis, both the decays are good for detecting this tensorial coupling. In this case the percentage difference from $V - A$ standard theory in the explored kinematical range is (14-20)% and (9-28)% for the electron and the muon channels respectively, when the branching ratio is in the range $(10^{-5}-10^{-6})$ and $(10^{-4}-10^{-5})$ for the electron and muon channels respectively.

The processes $K^+ \rightarrow e^+ \nu_e \gamma$ and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ can be studied very well at the Daphne laboratory, where high statistics on K^+ (about 10^{10} K^+ a year) will be reached. Therefore Daphne will be an ideal place to detect a possible tensorial coupling in the effective weak Hamiltonian via the decays $K^+ \rightarrow e^+ \nu_e \gamma$ and $K^+ \rightarrow \mu^+ \nu_\mu \gamma$.

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5 Appendix

In this appendix we give the analytical results for the functions f_i which appear in formula (33):

$$\begin{aligned}
f_{fB}(x, \lambda) &= \frac{(1-\lambda)}{\lambda x} \left[x^2 + 2(1-x)(1-r_l) - \frac{2r_l(1-r_l)}{\lambda} \right] \\
f_{SD^+}(x, \lambda) &= x^2 \lambda [(\lambda x + r_l)(1-x) - r_l] \\
f_{SD^-}(x, \lambda) &= x^2(1-\lambda) [(x-1)(r_l + x(\lambda-1)) + r_l] \\
f_+(x, \lambda) &= \frac{(1-\lambda)}{\lambda} [(x-1)(x\lambda + r_l) + r_l] \\
f_-(x, \lambda) &= \frac{(1-\lambda)}{\lambda} [x^2 + (1-x)(x\lambda + r_l) - r_l] \\
f_{\tau\tau}(x, \lambda) &= \lambda x^3(1-\lambda) \\
f_{fB\tau}(x, \lambda) &= x(1+r_l-\lambda-\frac{r_l}{\lambda}) \\
f_{SD\tau}(x, \lambda) &= x^3(1-\lambda)
\end{aligned} \tag{40}$$

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F_T^1	$(B_R^5)_{\Lambda_1} \times 10^5$	$(B_R^5)_{\Lambda_2} \times 10^5$	$(B_R^5)_{\Lambda_3} \times 10^6$
$-F_T^0/\tan\theta_c = -2.2 \times 10^{-2}$	1.55	1.11	4.64
$-F_T^0 = -5 \times 10^{-3}$	1.31	9.56×10^{-1}	4.10
0	1.30	9.50×10^{-1}	4.08
$F_T^0 = 5 \times 10^{-3}$	1.32	9.61×10^{-1}	4.12
$F_T^0/\tan\theta_c = 2.2 \times 10^{-2}$	1.60	1.13	4.70

Table 1: B_R^5 as a function of the tensorial form factor F_T^1 , and for three different ranges (Λ_i) of integration in $dxd\lambda$, where $\Lambda_i = \lambda_i^{\min} \leq \lambda \leq 1$, $x_i^{\min} \leq x \leq 1 - r_l$ with $\lambda_i^{\min} = \max\{r/(1-x), x_i^{\min}\}$ and $x_1^{\min} = 0.3$, $x_2^{\min} = 0.5$, $x_3^{\min} = 0.7$

F_T^1	$(B_R^4)_{\Lambda_1} \times 10^4$	$(B_R^4)_{\Lambda_2} \times 10^5$	$(B_R^4)_{\Lambda_3} \times 10^5$
$-F_T^0/\tan\theta_c = -2.2 \times 10^{-2}$	4.07	7.73	1.03
$-F_T^0 = -5 \times 10^{-3}$	4.37	9.00	1.31
0	4.46	9.40	1.40
$F_T^0 = 5 \times 10^{-3}$	4.55	9.79	1.50
$F_T^0/\tan\theta_c = 2.2 \times 10^{-2}$	4.89	11.3	1.86

Table 2: B_R^4 as a function of the tensorial form factor F_T^1 , and for three different ranges of integration (Λ_i) in $dxd\lambda$, where $\Lambda_i = \lambda_i^{\min} \leq \lambda \leq 1$, $x_i^{\min} \leq x \leq 1 - r_l$ with $\lambda_i^{\min} = \max\{r/(1-x), x_i^{\min}\}$ and $x_1^{\min} = 0.3$, $x_2^{\min} = 0.5$, $x_3^{\min} = 0.7$

6 Figure captions

Fig.1 The Feynman diagrams at tree level in weak and electromagnetic coupling constants for the process $K^+ \rightarrow l^+ \nu_l \gamma$. The diagram d corresponds to the matrix element of the new tensorial coupling. The diagrams c,d contain the non-perturbative strong interaction effects.

Fig.2 The differential branching ratio $dB_R^4/d\lambda$ for the process $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ as a function of λ and for some values of $x^{\min}$ and F_T^1 .

Fig.3 The differential branching ratio $dB_R^5/d\lambda$ for the process $K^+ \rightarrow e^+ \nu_e \gamma$ as a function of λ and for some values of $x^{\min}$ and F_T^1 .

Fig.4 The differential branching ratio dB_R^4/dx for the process $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ as a function of x and for some values of $\lambda^{\min}$ and F_T^1 .

Fig.5 The differential branching ratio dB_R^5/dx for the process $K^+ \rightarrow e^+ \nu_e \gamma$ as a function of x and for some values of $\lambda^{\min}$ and F_T^1 .

Fig.6 The total branching ratio B_R^4 for the process $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ as a function of F_T^1 and for some values of $\lambda^{\min}$ and $x^{\min}$.

Fig.7 The total branching ratio B_R^5 for the process $K^+ \rightarrow e^+ \nu_e \gamma$ as a function of F_T^1 and for some values of $\lambda^{\min}$ and $x^{\min}$.

Fig. 2

Fig. 1

Fig. 4

Fig. 3

Fig. 6

Fig. 5

Fig. 7