EFFECTS OF DIFFUSION ON SOLAR MODELS
AND THEIR OSCILLATION FREQUENCIES

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Abstract
Settling and diffusion of helium have significant effects on the structure of solar models and their oscillation frequencies, with bearing on the interpretation of the observed frequencies of solar oscillations. We examine these effects in considerably more detail than has been done before, and compare the computed frequencies with an extensive set of observed frequencies. We find that inclusion of diffusion results in a significant improvement in the agreement between theory and observations.

I. INTRODUCTION

The effects of helium diffusion on the structure of star models were first studied by


II. OBSERVATIONS

The observed abundances of elements such as helium and carbon in the atmospheres of stars suggest that diffusion plays a significant role in the evolution of these stars. These observations are consistent with theoretical models that include diffusion as a key process. However, the details of the diffusion process are not yet fully understood.

III. THEORETICAL MODELS

Using the tools of stellar evolution theory, we have developed a comprehensive model that includes the effects of diffusion. This model reproduces the observed abundances and other properties of stars with high accuracy.

IV. CONCLUSION

Our model provides a consistent explanation for the observed abundances and other properties of stars. Further refinements are required, but the overall agreement between theory and observation is encouraging.
depletion by a factor of 25 to 100. Solar main-sequence Be depletion is apparently about a factor of 2. If the mixing below the convection zone implied by main-sequence Li depletion is included, the amount of surface He settling might be reduced by up to 50% (see PM). The exact abundance profile below the convection zone depends on the temporal and spatial dependence of the turbulence.

Evolution models that use the latest opacity tables of Rogers & Iglesias (1992) and include Coulomb effects in the equation of state need a $Y_e$ of 0.27 - 0.28 (e.g. Bahcall & Pinsonneault 1992). Helioseismic estimates of $Y_e$ (Kosovichev et al. 1992 and references therein) depend on the assumed equation of state, but generally give $Y_e = 0.23 - 0.25$ (see also Guzik & Cox 1992). This agrees with the expected amount of settling, and may favor models with little extra mixing.

Cox, Guzik, & Kidman (1989) and Guzik & Cox (1992) examined the effects of diffusion on solar oscillations. They relied on simple comparisons between the observed and calculated frequencies; hence it was difficult to separate the effects of diffusion from other uncertainties in the model, particularly those associated with the superficial layers of the Sun. Here we avoid much of this ambiguity by carrying out an inverse analysis based on the asymptotic properties of the acoustic modes; this allows us to probe the consequences of diffusion just below the base of the convection zone where they are largest.

2 COMPUTATION OF MODELS AND FREQUENCIES

Evolutionary model sequences were calculated starting from chemically homogeneous zero-age main-sequence models. Apart from the inclusion of diffusion, we carried out a "standard" solar model calculation, essentially as described by Christensen-Dalsgaard (1982, 1991). For each model the mixing-length parameter and $Y_e$ were adjusted to match the solar radius ($6.9599 \times 10^{10}$ cm) and luminosity ($3.846 \times 10^{33}$ erg sec$^{-1}$) to within one part in $10^6$ at the assumed solar age of 4.6 Gyr. Adiabatic oscillation frequencies for the models were computed as described by Christensen-Dalsgaard & Berthomieu (1991), for the modes in the compilation of Libbrecht, Woodard & Kaufmann (1990).

2.1 Equation of State and Opacity

We used the CEFF equation of state (see Christensen-Dalsgaard 1991; Christensen-Dalsgaard & Däppen 1992), where Coulomb corrections, in the Debye-Hückel formulation, have been added to the Eggleton, Faulkner & Flannery (1973) treatment. The interior opacities used were from the most recent OPAL tables (Rogers & Iglesias, 1992; Iglesias, Rogers & Wilson 1992), and were matched smoothly, near $T = 10^4$ K, to the low-$T$ opacities of Kurucz (1992). The heavy element abundance by mass was $Z = 0.02$. Relative heavy element composition follows the recent Grevesse (1991; unpublished) mixture (cf. Iglesias et al. 1992).

2.2 The Treatment of Diffusion and Turbulence

Michaud & Proffitt (1992) presented simplified formula for diffusion rates that were calibrated using solutions to the system of equations proposed by Burgers (1969) and the numerically calculated values for collision integrals given by Paquette et al. (1986). We adopt equation (17) of Michaud & Proffitt for the diffusion velocity of H:

$$V_H = \frac{15}{16} \left( \frac{2m_a}{5\pi} \right)^{1/2} \frac{(KT)^{1/2}}{c_P \ln \Lambda (0.7 + 0.3X)}$$

$$\times \left[ \left( \frac{5}{4} + \frac{9}{8} \frac{2/2}{P} \frac{3 + X}{1 + X} (3 + 5X) \frac{d\ln X}{dr} \right) \right],$$

(1)
By a linear least-squares fit (Fisk et al. 1999) to data on a function of the form:

\[ y = A e^{-x} + B e^{x} \]

we can determine the parameters \( A \) and \( B \) which minimize the sum of the squared residuals. The residuals are calculated as the difference between the observed and predicted values.

In this case, the residuals are defined as:

\[ r_i = y_i - (A e^{-x_i} + B e^{x_i}) \]

where \( y_i \) is the observed value, and \( r_i \) is the residual.

The parameters \( A \) and \( B \) are then determined by minimizing the sum of the squared residuals:

\[ S = \sum (r_i)^2 \]

This is done using the method of least squares, which involves setting the partial derivatives of \( S \) with respect to \( A \) and \( B \) to zero and solving the resulting system of equations.

The solutions for \( A \) and \( B \) are given by:

\[ A = \frac{\sum y_i e^{-x_i}}{\sum e^{-2x_i}} \]

\[ B = \frac{\sum y_i e^{x_i}}{\sum e^{2x_i}} \]

These values can then be used to predict the response of the system.

The results of the regression analysis show a strong linear correlation between the observed and predicted values, indicating that the model is a good fit for the data. The coefficient of determination, \( R^2 \), is calculated to be 0.98, which indicates that 98% of the variability in the observed values is accounted for by the model.

In conclusion, the linear least-squares fit provides a robust method for determining the parameters of the model, and the results demonstrate the effectiveness of the model in accurately predicting the behavior of the system.
region produces an abrupt decrease in $X$ just below the convection zone. The increase of the envelope abundance $X_e$ over the initial value $X_0$ is $\sim 0.3$. Including turbulent diffusion in the TD1 formulation (Model 3), reduces both the step and the increase in $X_e$. Using formulation TD2 (Model 4), which puts the largest turbulence at the base of the convection zone, eliminates the step and leads to an even smaller increase in $X_e$. In the core $X$ is smaller by about 0.01 in the models with settling. Note also from Table 1 that inclusion of diffusion increases the depth of the convection zone, to a value that is close to the value of $0.287 \pm 0.003 R$ inferred by Christensen-Dalsgaard, Gough & Thompson (1991) from helioseismic inversion; this was also found by Bahcall & Pinsonneault (1992).

The differences in sound speed between Models 2 - 4 (with diffusion) and the non-diffusive Model 1, (heavy lines in Figure 2a), are dominated by the deeper convection zones of the diffusive models, which leads to a sharp increase in temperature in those models relative to Model 1. In Model 2 this effect is partially offset by the step down in $X$, and the resulting increase in the mean molecular weight beneath the convection zone. Figure 2b shows scaled frequency differences between Model 2 and Model 1, plotted against $\nu/L$. Their general behavior reflects the asymptotic expression (5). The dependence of $S\delta \nu/\nu$ on $\nu/L$ can be understood from the sound-speed difference shown in Figure 2a: for $\nu/L \lesssim 100 \mu$Hz the modes are entirely trapped in the convection zone, and the frequency difference is dominated by the term $H_2(\nu)$ arising from differences near the surface, particularly the difference in $X_e$. In contrast, modes with $\nu/L > 100 \mu$Hz sense the substantial positive $\delta c$ just beneath the convection zone, and hence display a positive frequency difference; the transition occurs quite abruptly as the modes begin to penetrate beyond the convection zone.

As discussed in § 2.3, the sound-speed differences can be estimated from the scaled frequency differences by fitting them to equation (5) and inverting $H_1(\nu/L)$. The result of this procedure for Model 2, applied to the results in Figure 2b, is shown by the thin solid line in Figure 2a. The result of the inversion matches the original sound-speed difference closely for $0.2 R < r < 0.95 R$, with some smoothing due to the finite resolution of the inversion. (Closer to the center or surface the asymptotic approximation becomes much less accurate.)

Figure 3 shows the results of applying the asymptotic inversion to the solar data of Libbrecht et al. (1990), using Models 1 - 4 as reference models. The uncertainties in the inversions, based on the quoted observational errors (shown in Fig. 3 for Model 2 only), are small compared with the inferred differences. The inversion relative to the non-diffusive model is dominated by a substantial positive $\delta c/e$ close to the base of the convection zone. This is suppressed when diffusion is taken into account. On the other hand, the diffusive models have too large a sound speed in somewhat deeper regions, more so in the models with turbulence. Nonetheless, including diffusion clearly improves the agreement between the models and the Sun.

As mentioned in § 2.3, the frequency separations $\delta f$ test the models in the solar core. Elsworth et al. (1990) obtained $\delta f = 9.00 \pm 0.06 \mu$Hz and $\delta f = 9.36 \pm 0.06 \mu$Hz, from an extensive set of observations. For the diffusive models, $\delta f$ is in excellent agreement with the observed value (see Table 1), and $\delta f$ is within two standard deviations. The values for the non-diffusive model agree less well with the results of Elsworth et al. It should be noted, however, that Toutain & Fröhlich (1992) obtained a somewhat higher value for $\delta f$ from an independent set of observations.
REFERENCES

Danish National Science Research Council.


Inversion when Model 2 is dominant.

The inversion when Model 2 is dominant is based on the observed zonal variation, as shown in Figure 1a. In this case, the inversion is defined as the depth of the transition zone where the average of the zonal gradient reaches a maximum. This depth is determined by the observed zonal gradient and the observed zonal variation.

Figure captions:

Model 1: Model 2

Table 1: Properties of the Models