LEPTON SPECTRA AS A MEASURE OF
 b QUARK POLARIZATION AT LEP

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Abstract

At LEP, b quarks are produced in Z decays with a large longitudinal polarization. A possible strategy for detecting some polarization effect through the study of the leptonic energy spectrum in the inclusive semi-leptonic decay of b hadrons is studied in detail. We especially focus on the large theoretical uncertainties due to the ambiguities on the b -fragmentation function. The related uncertainties in the lepton spectrum, which could fake polarization effects, can be controlled, by making use of both LEP data and lower-energy PETRA data on b production. A first comparison with experimental results is shown as well.

In e^+e^- collisions at the Z^0 peak, b quarks are produced with a large longitudinal polarization. The mean value P_b of this quantity, which has little angular dependence, is given by:

$$P_b = \frac{\sigma(b_R) - \sigma(b_L)}{\sigma(b_R) + \sigma(b_L)} = -\frac{2v_b a_b}{v_b^2 + a_b^2} = -0.936, \quad (1)$$

where v_b and a_b are respectively the vector and axial coupling of the b quark to the Z^0 ($v_b = -1 + \frac{4}{3}\sin^2\theta_W$, $a_b = -1$) and $\sin^2\theta_W = 0.23$ is assumed. Just for comparison, the τ longitudinal polarization is $P_\tau = -0.16$ in the same conditions.

Here, we discuss the possibility of detecting the effects of such a large value of P_b in the existing LEP data on the b semileptonic decay (see also ref. [1]).

For a free quark (assuming no fragmentation, no hadronization, and no QCD corrections), a rather straightforward way to observe polarization effects would be through the study of the semileptonic decay $b \rightarrow l\bar{\nu}c$ with $l = e, \mu$. In fact, first the branching ratio relative to this mode is quite high (about 20%). Secondly, the lepton-energy spectrum is rather sensitive to the b -quark polarization P . The normalized decay distribution in the b rest frame [2] is

$$\frac{1}{\Gamma} \frac{d^2\Gamma}{dyd(\cos\alpha)} = \frac{1}{f(\epsilon)} \frac{y^2(1-\epsilon-y)^2}{(1-y)^2} \left\{ 3 - 2y + \epsilon \left(\frac{3-y}{1-y} \right) + P \cos\alpha \left(1 - 2y - \epsilon \left(\frac{1+y}{1-y} \right) \right) \right\} \quad (2)$$

$$f(\epsilon) = 1 - 8\epsilon + 8\epsilon^3 - \epsilon^4 - 12\epsilon^2 \log \epsilon.$$

In eq. (2), $y = 2E_l'/m_b$, where E_l' is the lepton energy in the b rest frame, α is the angle between the lepton and the b spin quantization axis, and $\epsilon = m_c^2/m_b^2$. In the laboratory system, the b quark moves with velocity $\beta = p_b/E_b \leq \sqrt{1 - (2m_b/M_Z)^2}$. If we quantize the b spin along the direction of $\vec{\beta}$, the lepton-energy decay distribution in the laboratory frame is given by

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \int_{y^-}^{y^+} \frac{2}{\beta y} \frac{1}{\Gamma} \frac{d^2\Gamma}{dyd(\cos\alpha)} dy, \quad (3)$$

where

$$x = \frac{E_l}{E_b}, \quad \cos\alpha = \frac{1}{\beta} \left(\frac{2x}{y} - 1 \right),$$

$$y^- = \frac{2x}{1+\beta}, \quad y^+ = \min \left\{ \frac{2x}{1-\beta}, (1-\epsilon) \right\},$$

m_c (GeV)	$P = 0$	$P = P_b$
1.2	0.309	0.344
1.3	0.303	0.337
1.4	0.298	0.329
1.5	0.291	0.322

Table I: Mean value of the l -energy fraction $< x >$ for different choices of P and m_c . Fragmentation effects are not included.

and E_l is the lepton energy in the laboratory system. In the LEP case, where $\beta \simeq 1$, one gets

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \frac{(1-\epsilon-x)}{3f(\epsilon)} \left\{ 5 + 5x - 4x^2 + P(1+x-8x^2) + \frac{\epsilon}{(1-x)^2} [-22 + 5\epsilon + (27-4\epsilon)x + 5\epsilon x^2 - 5x^3 + P(-8-17\epsilon + (9+28\epsilon)x + (18-17\epsilon)x^2 - 19x^3)] \right\} + \frac{6\epsilon^2}{(1-\epsilon-x)} \log \left(\frac{1-x}{\epsilon} \right) [3-\epsilon + P(3+\epsilon)], \quad (4)$$

with $f(\epsilon)$ defined in eq. (2).

In fig. 1, the results for $\frac{1}{\Gamma} \frac{d\Gamma}{dx}$ for a naïve b polarization P_b (solid lines) are compared with the unpolarized case (dashed lines), assuming two different values (1.2 and 1.5 GeV) for the c -quark mass in both cases and $m_b = 5$ GeV. Here, any b -fragmentation effect is neglected and the final lepton-energy fraction in the laboratory frame becomes $x = 2E_l/M_Z$. The harder spectra correspond to $m_c = 1.2$ GeV, the softer ones to $m_c = 1.5$ GeV. One can see that a negative value of the b polarization makes the l -energy spectrum harder.

In Table I, the mean value $< x >$ of the distribution is shown, for both the polarized and the unpolarized cases, for different values of m_c . By going from the unpolarized case to the naïve maximum b polarization P_b , an increase of about 10% in the mean value of the lepton-energy fraction is observed (the same is true for the $\bar{b} \rightarrow l\nu\bar{c}$ case, in the limit of exact CP invariance). However, a heavier m_c softens the x spectrum. As far as $< x >$ is concerned, the polarized case with $m_c = 1.5$ GeV is

not very different from the unpolarized case, with $m_c = 1.2$ GeV (cf. Table I). So, the uncertainty from m_c could partially hide polarization effects, if only $\langle x \rangle$ was considered. As seen from fig. 1, a more detailed study of the spectrum beyond the first moment is required to overcome the limited knowledge of m_c .

QCD corrections to the leptonic spectrum of eq. (2) have not yet been computed, to our knowledge, for the polarized case. This calculation is now in progress [9]. Anyhow, the order- α_s correction for unpolarized b [4,5] differs from a constant value only near the end point of the spectrum, so that it amounts to a nearly constant factor. Indeed the uncorrected shape of the lepton-energy distribution is not significantly altered.

Let us now consider hadronization and hard fragmentation effects. Clearly, all memory of the original b -quark polarization is lost when a pseudoscalar B meson is formed, either directly or as a result of a cascade from a B^* meson. Thus, as far as hadronization effects are concerned, only b baryons are expected to retain some spin information. In particular, heavy-quark effective theories suggest that a Λ_b always keeps the initial b polarization [6]. As a consequence, assuming a B.R. of order 10% for Λ_b production from b quarks, one would expect to observe a polarization effect of the order of $P_b/10 \sim -0.1$ in the inclusive semileptonic decay channels.

Gluon radiation from the b quark before its hadronization and decay or, more in general, b fragmentation is the main source of theoretical uncertainty in the measure of a polarization effect by means of the l -energy spectrum in the semi-leptonic b decay. In fact, although perturbative QCD conserves helicities in the limit of negligible quark masses, and therefore does not much influence the final b polarization, gluon radiation considerably changes the final energy fraction of the b quark. As a consequence, the decay l spectrum in the laboratory frame is modified, and a good sensitivity to polarization effects requires an accurate knowledge of the b -fragmentation process. On the other hand, a complete theoretical control of the b -fragmentation function $D_b(x_b, Q)$, which describes the final b -energy spectrum at the energy scale Q , is difficult to achieve. The fragmentation function $D_b(x_b, Q)$ is defined as

$$D_b(x_b, Q) = \frac{1}{\sigma_b^{\text{tot}}} \sum_{H_b} \frac{d\sigma_{H_b}(x_b)}{dx_b}, \quad (5)$$

where the sum is over all the bottom-flavoured hadrons. $D_b(x_b, Q)$ can be thought

of as given by the convolution of a perturbative part, which includes QCD evolution-equation effects and is dependent on Q , with a non-perturbative function that takes into account the observed moderate departures from the perturbative evolution at scales near m_b . To the accuracy required for polarization studies, m_b is not really large enough to completely neglect the non-perturbative component in $D_b(x_b, Q)$. Correspondingly, the moments of $D_b(x_b, Q)$

$$\langle x_b^N \rangle (Q) = \int x_b^N D_b(x_b, Q) dx_b, \quad N = 1, 2, 3, \dots, \quad (6)$$

will be given by products of the form

$$\langle x_b^N \rangle (Q) = \langle x_b^N \rangle_{\text{Pert}} (Q) < x_b^N \rangle_{NP}. \quad (7)$$

Here, $\langle x_b^N \rangle_{\text{Pert}} (Q)$ is calculable to any order in perturbative QCD. At any finite order in α_s , it depends on the particular choice of some parameters [7]: the matching scale of the initial condition for the perturbative evolution equation, μ_0 , that should be of the order of m_b for avoiding large $\log(\mu_0/m_b)$ terms in the fixed-order expression, the value of Λ_{QCD} and the value of m_b (that is however less crucial in this discussion). On the other hand, the non-perturbative component $\langle x_b^N \rangle_{NP}$ in eq. (7) cannot be computed with adequate precision. By the way, its precise magnitude will be influenced by the particular choice of the perturbative parameters. The estimate $\langle x_b^1 \rangle_{NP} \sim 0.9$ can be obtained [7] by subtracting the theoretical perturbative prediction from LEP data on the b -fragmentation function [8].

The perturbative component of the moments in eq. (7) can be determined to the next-to-leading order in α_s by using the results of ref. [7]. In Table II, we show the average value of x_b , $\langle x_b^1 \rangle_{\text{Pert}} (Q)$, for different choices of μ_0 and Λ_s , when $m_b = 5$ GeV and $Q = M_Z$. Here, Λ_s is $\Lambda_{\overline{MS}}$, as defined by the two-loop expression of α_s with five light flavours. The values range from 0.68 to 0.81 for $m_b/2 < \mu_0 < 2m_b$ and $150 \text{ MeV} < \Lambda_s < 350 \text{ MeV}$.

In order to control the non-perturbative effects and also a part of the uncertainties in $\langle x_b^N \rangle_{\text{Pert}} (Q)$, we suggest the following strategy. Consider the ratios of the N -th moments of $D_b(x_b, Q)$ at different values of Q . In particular, take $Q = 91 \text{ GeV}$ (Z^0 mass) over $Q = 32 \text{ GeV}$ (that can be thought of as the mean $\sqrt{s}$ at PETRA, as far

Λ_5 (MeV)	$\mu_0 = m_b/2$	$\mu_0 = m_b$	$\mu_0 = 2m_b$
150	0.757	0.788	0.813
250	0.717	0.759	0.791
350	0.681	0.734	0.773

Table II: $\langle x_b^1 \rangle_{P_{\text{ert}}}$ (Q) for different choices of Λ_5 and μ_0 at $Q = M_Z$.

Λ_5 (MeV)	R_b^1	R_b^2	R_b^3
150	0.912	0.865	0.833
250	0.903	0.852	0.817
350	0.896	0.841	0.804

Table III: The ratio R_b^N , defined by eq. (8), for different choices of Λ_5 and $N = 1, 2, 3$.

as b data are concerned [9]. We call this ratio R_b^N

$$R_b^N = \frac{\langle x_b^N \rangle_{P_{\text{ert}}} (91 \text{ GeV})}{\langle x_b^N \rangle_{P_{\text{ert}}} (32 \text{ GeV})} = \frac{\langle x_b^N \rangle_{P_{\text{ert}}} (91 \text{ GeV})}{\langle x_b^N \rangle_{P_{\text{ert}}} (32 \text{ GeV})}, \quad (8)$$

where we have made use of eq. (7). One important point is that, at PETRA energies, b quarks are produced, to a very good approximation, through γ exchange alone, that is with zero longitudinal polarization. Thus the low-energy data offer a way to calibrate on the unpolarized b quarks. In addition, in R_b^N , low-energy non-perturbative effects drop out. Therefore, R_b^N can be evaluated by using the next-to-leading order calculation of ref. [7]. Note that in R_b^N the dependence on the matching scale μ_0 introduced in the finite-order calculation [7] turns out to also drop out. In Table III, the value of the ratio R_b^N for the first three moments is reported as a function of Λ_5 . Confronting Tables II and III, one can see that the ratio of moments tends to be more stable also with respect to Λ_5 variations.

At this point, we note that, in the collinear approximation, the observed leptonic spectrum in the laboratory is the convolution of the polarization-dependent “naive”

leptonic spectrum of eq. (3) with $D_b(x_b, Q)$:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx_l} = \int dx_b dx D_b(x_b, Q) \frac{1}{\Gamma} \frac{d\Gamma}{dx} \delta(x_l - x_b x) \quad (9)$$

where $x_l = 2E_l/\sqrt{s}$ and $\sqrt{s}$ is the c.m. e^+e^- collision energy.

As a result, the ratios R_l^N of the N -th moments of the leptonic spectrum at $\sqrt{s} = M_Z$ over the ones at PETRA energies can be predicted as functions of the “effective” b longitudinal polarization $\hat{P}$, defined as the fraction of R_b that survives hadronization processes at LEP. In fact, R_l^N is given by R_b^N times the polarization-dependent ratio of the moments of the “naive” lepton spectra at LEP and PETRA

$$\begin{aligned} R_l^N(\hat{P}) &= \frac{\langle x_l^N \rangle_{LEP}(\hat{P})}{\langle x_l^N \rangle_{PETRA}} = \frac{\langle x_b^N \rangle_{P_{\text{ert}}}(91 \text{ GeV})}{\langle x_b^N \rangle_{P_{\text{ert}}}(32 \text{ GeV})} \frac{\langle x^N \rangle_{LEP}(\hat{P})}{\langle x^N \rangle_{PETRA}} \\ &= R_b^N \frac{\langle x^N \rangle_{LEP}(\hat{P})}{\langle x^N \rangle_{PETRA}}. \end{aligned} \quad (10)$$

By the way, factorizing moments in eq. (9) implies an error of the order of m_b^2/E_s^2 (comparable with that deriving from the collinear approximation) due to the dependence on β of the naive leptonic distribution. In the actual calculation of $\langle x^N \rangle_{LEP}$ and $\langle x^N \rangle_{PETRA}$, a fixed β , corresponding to $\langle x_b \rangle_{LEP} = 0.7$ and $\langle x_b \rangle_{PETRA} = 0.8$ respectively [9], was assumed.

In figs. 2, 3 and 4 the values predicted for R_l^N with $N = 1$ (mean-value ratio), 2 and 3, respectively, are shown as a function of the “effective” b polarization $\hat{P}$. As in fig. 1, two different values of m_c are assumed. The band corresponds to Λ_5 varying between 150 GeV (upper curves) and 350 GeV (lower curves) in the prediction of R_b^N . A variation $\Delta\hat{P} = \pm 1$ in the polarization from the unpolarized case ($\hat{P} = 0$) corresponds to a $\mp 10\%$ change in R_l^1 . The study of higher order moments is especially useful for consistency checks on the measured value of $\hat{P}$. Note that, for higher N , R_l^N gets more sensitive to b polarization, because the main difference between the polarized and unpolarized lepton-energy spectra is at large values of x . Moreover, the sensitivity to the exact value of m_c is considerably reduced in the ratio of moments.

In fig. 2, a comparison with presently available experimental data [9] is shown. The dashed area corresponds to the measured value $R_b^1(\text{exp}) = 0.894 \pm 0.015$, that has been determined assuming $P = 0$.

With this assumption, one gets $\langle x^1 \rangle_{LEP} \simeq \langle x^1 \rangle_{PETRA}$ [neglecting β effects in eq. (9)] and, as a consequence, $R_1^1(xp) \simeq R_1^1(xp)$. The present experimental data are already accurate enough to exclude large polarization effects and point to a value of the “effective” b polarization centred around 0 but compatible with the theoretical prediction of about -0.1 .

In the near future, higher statistics on b events at LEP will allow the study of more exclusive channels. In refs. [10] and [11] some studies of decays of polarized Λ_b are presented. A method to measure the polarization of a high-momentum parton from the analysis of the structure of its jet is presented in ref. [12]. We stress, however, that the limitations from the uncertainties in the knowledge of the b -quark fragmentation clearly also apply to polarization measurements based on semileptonic b -baryon decays. The same problem affects the determination of any change in the leptonic spectrum, that may arise, for example, from a possible new right-handed current that mediates b decays [13,14]. Moreover, a relevant question in view of our previous discussion is whether, with a reasonable statistics on semileptonic Λ_b decays (some data are already available [15]), LEP data will not longer need to be normalized with lower-energy data. One could imagine an improvement of our procedure through the study of the ratios $\langle x_l^N \rangle_{\Lambda_b}(\hat{P}) / \langle x_l^N \rangle_B$ of the leptonic-spectrum moments for the Λ_b sample over those for the B -meson sample, both measured at LEP¹. In fact, in this case we have

$$\begin{aligned} R_l^N(\hat{P})_{\Lambda_b} &= \frac{\langle x_l^N \rangle_{\Lambda_b}(\hat{P})}{\langle x_l^N \rangle_B} = \frac{\langle x_l^N \rangle_{Peri}(91 GeV) < x^N >_{LEP}(\hat{P})}{\langle x_l^N \rangle_{Peri}(91 GeV) < x^N >_{LEP}(0)} \\ &= \frac{\langle x^N >_{LEP}(\hat{P})}{\langle x^N >_{LEP}(0)}. \end{aligned} \quad (11)$$

Hence, b -fragmentation effects are mostly cancelled. In addition, in the exclusive Λ_b sample one expects the “effective” polarization to be an order of magnitude larger than in the previous inclusive analysis. In fig. 5, we report the ratios $R_l^N(\hat{P})_{\Lambda_b}$ defined in eq. (11) for the first three moments as functions of the “effective” polarization $\hat{P}$. If hadronization effects do not substantially spoil the effect, a value $\hat{P} \sim P_b$ should be measured in this case. However, the main problem with this method is that we cannot calibrate on the spectrum from an unpolarized Λ_b . Without such a normalization, a possible observed difference between the Λ_b and the inclusive spectra could be blamed

¹We thank P. Checchia for discussions on this possibility.

on the model dependent features of the exclusive decay. In order to get a feeling of the size of model dependent effects, one can vary the effective b and c masses according to $(m_b, m_c) = (m_{\Lambda_b}, m_{\Lambda_c})$ or (m_B, m_D) to mock possible variations in the lepton spectra in going from $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$ to $B \rightarrow D l \bar{\nu}_l$ or $B \rightarrow D^* l \bar{\nu}_l$ respectively. We can see in fig. 6 that such differences are indeed of the order of the polarization effects.

In conclusion, we find that there is indeed the possibility of obtaining important experimental information on the left-over b polarization after hadronization from the lepton spectrum. For a correct measurement of the effective polarization a good control of fragmentation effects is crucial. This can be achieved by a combination of theoretical and experimental inputs. For example, with moderate luminosities, one can calibrate on lower-energy data. With larger integrated luminosities, one can use the Λ_b sample of semileptonic events and compare it with the B -meson sample in order to control fragmentation effects. In the last case, a reliable model for Λ_b semileptonic decay would be needed before polarization effects can really be established. Such a model could possibly be developed starting from the heavy-quark effective theory supplemented by the experimental information that will be collected with time on Λ_b decays.

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Figure captions

- Fig. 1: Comparison between the lepton-energy spectrum in the laboratory frame for a naive b polarization P_b (solid lines) and the unpolarized case $P = 0$ (dashed lines), assuming two different values for m_c (1.2 GeV and 1.5 GeV) and $m_b = 5$ GeV. The b -fragmentation effects are not included and $x = 2E_l/M_Z$.
- Fig. 2: Prediction for R_l^N with $N = 1$ (mean-value ratio) versus the "effective" b polarization $\hat{P}$. Two different values of m_c [1.2 GeV (solid lines) and 1.5 GeV (dashed lines)] are assumed. The band corresponds to Λ_s varying between 150 GeV (upper curves) and 350 GeV (lower curves) in the prediction of R_l^N . The dashed area derives from the measured value of R_l^b $R_l^b(exp) = 0.894 \pm 0.015$.
- Fig. 3: Same as in fig. 2 for $N = 2$.
- Fig. 4: Same as in fig. 2 for $N = 3$.
- Fig. 5: Prediction for the ratios $R_l^N(\hat{P})/\Lambda_s$, as defined in eq. (11), for $N = 1, 2$ and 3, versus the "effective" polarization $\hat{P}$. Two different values for m_c [1.2 GeV (solid lines) and 1.5 GeV (dashed lines)] are assumed. A fixed b energy $E_b = < x_b >_{LEP} \cdot M_Z/2$, with $< x_b >_{LEP} = 0.7$, is assumed in the laboratory boost of the leptonic spectrum (see text).
- Fig. 6: Comparison between the lepton-energy spectrum in the laboratory frame for a naive b polarization P_b (solid lines) and the unpolarized case $P = 0$ (dashed lines), assuming four different sets of values for (m_b, m_c) :
 1) $(m_b, m_c) = (m_{\Lambda_s}, m_{\Lambda_c}) = (5.64 \text{ GeV}, 2.28 \text{ GeV})$;
 2) $(m_b, m_c) = (m_B, m_{D^*}) = (5.28 \text{ GeV}, 2.01 \text{ GeV})$;
 3) $(m_b, m_c) = (m_B, m_D) = (5.28 \text{ GeV}, 1.86 \text{ GeV})$;
 4) $(m_b, m_c) = (5 \text{ GeV}, 1.5 \text{ GeV})$ (as in fig. 1)
 The b -fragmentation effects are not included and $x = 2E_l/M_Z$.

Fig. 1

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