MACROSCOPIC LACK OF LOCALIZATION IN QUANTUM RADIATIVE PROCESSES

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ABSTRACT

We use results on beamstrahlung and pair production in bunch collisions to illustrate the macroscopic lack of localization of the corresponding quantum processes. Localization is usually associated with a coherent length. However, for some relatively important contributions to the production rate, the concept of inside and outside a bunch of particles loses its meaning. The effect should be clearly observable with electron-positron colliders in the multi-TeV range.

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1 The quantum aspect of beamstrahlung

In quantum mechanics, the space resolution of one event is directly related to the momentum transfer which it implies and therefore to the precision with which each momentum can be specified. We are used to this “quantum blur” and it is at the basis of many estimates readily achieved when dealing with quantum processes.

The deep nature of quantum mechanics still leads to challenging foods for thoughts when the acknowledged lack of resolution extends over macroscopic distances. It is therefore interesting to study such examples and their underlying physics.

The purpose of this paper is to present one of them, which recently arose in the analysis of radiative phenomena at very high energy and in very intense fields. Whilst experimental information is not available yet, the calculations based on quantum electrodynamics are straightforward on the theoretical side [1],[2].

Over the past few years we have done some extensive calculations in beamstrahlung [1],[3], following questions raised by the operation of linear electron-positron colliders in the TeV and multi-TeV range [4]. We refer the readers to these papers for a detailed derivation of the results and only focus here on the physics issue which they raise.

Beamstrahlung is the coherent radiation of electrons traversing a positron bunch. This is a generalization of synchrotron radiation, as is well known [5]. However, future colliders are worth constructing only if they can achieve a luminosity which increases at least as the square of the collision energy, since bench mark cross-sections, such as that for $e^+e^- \rightarrow \mu^+\mu^-$, decrease as the square of the reaction energy. In practice this calls for very dense bunches with tiny radii and, as a result, to extremely high fields. These high field regions are traversed by very energetic electrons which, accordingly, radiate as they are accelerated. The intense radiation process yields energetic photons which easily turn into electron-positron pairs in the high field of the bunch. Abundant pair production becomes part of the beamstrahlung process at high energy and high field [6]. Such conditions indeed lead one into the so-called “deep quantum” regime. The two basic processes, namely photon radiation and pair formation are shown in Fig. 1.

To be more specific, the standard approach to synchrotron radiation makes much use of a parameter:

$$\Upsilon = \frac{\gamma^2}{m\rho_c} \quad (1)$$

Here $\gamma = \frac{E}{m}$ is the Lorentz contraction factor, m is the electron mass and ρ_c is the radius of curvature.

In the classical approach, Υ is also the ratio between the typical energy radiated in the classical regime, which is defined by the cut-off frequency, and the energy of the incident electron. One has in practice $\Upsilon \ll 1$.

The conditions of very high energy and very high luminosity, and hence very high field, can, however, lead to values of Υ which are much larger than one. This is most likely the case with a collision energy of order of 1 TeV and a luminosity reaching accordingly beyond $10^{33} \text{ cm}^{-2}\text{s}^{-1}$. The classical approach breaks down. It has to be replaced by a full quantum one. One is in the so-called “deep quantum” regime ($\Upsilon \gg 1$).

Our approach to that problem uses a Feynman graph calculation [1]. The lowest-order processes are shown in Fig. 2. The calculation of the corresponding graphs can be considered as a distorted wave Born approximation procedure. In the case of photon radiation, we also

investigated the case of varying bunch densities [3] and also the leading contribution to all orders [3]. The key physics aspects are, however, already present to the full in the lowest-order calculation corresponding to the graphs of Fig. 2 and also when dealing with cylindrical uniform bunches (Fig. 1), of length L_b and radius R .

The use of Feynman graphs readily brings a new feature which does not appear in the usual calculations.

A Feynman amplitude is usually written in terms of momenta. It corresponds to an integral over all space, which is extended to infinity. This ensures momentum conservation in the overall process.

In the present case the integral over space involves a special feature since there is a region where the field is strong ($|z| < \frac{L_b}{2}$ and $|x| < R$ in Fig. 1) and a region, the remaining one, where the field is practically vanishing. Radiation results from the bending occurring inside the high field region but also from the presence of boundaries between the high field and vanishing field regions. There are classical analogies in both cases, namely synchrotron radiation and transition radiation, respectively. The Feynman graph approach brings them together and gives a superposition of spectral shapes for both processes. Each contribution can be measured separately experimentally. Their origin cannot be precisely localized in space.

A key feature of the deep quantum regime is that radiation proceeds through the emission of hard photons. That part of the radiation spectrum proportional to the bunch length L_b is [1]:

$$I(x) \sim \frac{1 + (1 - x)^2}{(1 - x)^{1/3} x^{2/3}} \quad (2)$$

whereas for pair production, which is then also copious, we have

$$I(x) \sim \frac{x^2 + (1 - x)^2}{(x(1 - x))^{1/3}} \quad (3)$$

In the former case x is the ratio between the photon energy and the incident electron energy while in the latter case it is the ratio between the produced electron energy and the incident photon energy [7].

The question which we wish to discuss in some detail is where such violent processes occur.

2 Localization of radiative processes

Inside the bunch, the integral over space is strongly restricted and this allows one to locate rather precisely a reaction zone.

The production amplitude has a phase with rapid oscillations in the high-field region and the production rate receives practically all its contributions from the zone where the phase is stationary. The stationarity conditions in terms of impact parameter and of localization alongside the bunch are defined by the initial and final kinematics, the latter one being eventually integrated upon in the calculation of the global rate. The calculation of the Feynman graph (Fig. 2) readily introduces a coherent length ℓ_c [1].

The introduction of a coherent length is a well-known move in synchrotron radiation. The produced photon is the quantum aspect of a wave emitted by the electron as it is accelerated. Emission occurs mainly within a cone of opening angle $1/\gamma$. The radiation remains coherent

as long as the electron trajectory has not been bent by more than $1/\gamma$. This defines the usual coherent length L_c [5].

In quantum physics, there is another length which naturally introduced itself. It is the radiative length L_e , over which an electron freely emits a photon or a photon freely transforms itself into an electron-positron pair. Quantum fluctuations provide for the change in energy. One can define L_e as

$$L_e = \hbar \frac{\gamma}{mc} \quad (4)$$

In spite of the fact that L_e is a quantum length, with typical machine parameters in the TeV range, it may become an order of magnitude larger than L_b . One is used to think of a quantity proportional to the Planck constant $\hbar$ as microscopic, and that such quantities are smaller than macroscopic quantities that do not involve any factor of $\hbar$. The situation here is characterized by the appearance of the factor γ , which is roughly 10^6 for a 1 TeV electron-positron collider. It is this large factor that makes a "microscopic quantity" L_e larger than a macroscopic quantity L_b .

The coherent length relevant to the deep quantum regime, and which comes naturally from the Feynman graph calculation, combines L_e and L_c . It is [1]

$$\ell_c = (L_e L_c^2)^{1/3} \quad (5)$$

The deep quantum regime corresponds to $L_e \gg L_c$ and hence $\ell_c \gg L_c$. Indeed one finds that [1]

$$\Upsilon = \left(\frac{\ell_c}{L_c} \right)^3 \quad (6)$$

Once ℓ_c is introduced, the contribution to the rate corresponding to the integral over the bunch takes a very simple form being proportional to $\frac{\alpha}{\pi} \frac{L_b}{\ell_c}$, both for photon radiation [1],[8] and for pair production [6]. This limit in localization is quite natural.

In the case of radiation the photon cannot be distinguished from the radiating electron until the electron has collected enough recoil from its interaction with the bunch. This requires a length of order ℓ_c [1]. In the case of pair formation, the pair cannot be acknowledged as such until the electron and the positron have trajectories which have been bent differently by the effect of the bunch. This again requires length of the same order ℓ_c .

The coherent length is, however, no longer simply related to the electron mass [9]. Indeed we operate in a regime where fields are high and typical bending momenta are therefore much higher (by two orders of magnitude) than the electron mass. The definition of ℓ_c (5) actually eliminates the mass which appears in both L_e and L_c when considering their explicit centre-of-mass expressions [1]. The coherent length ℓ_c is proportional to the incident energy to the power 1/3 and inversely proportional to the maximum bending momentum to the power 2/3. The fractional third powers are related to the cubic behaviour of the phase near the stationary point.

We readily understand this lack of resolution in space. One needs a certain distance travelled, and therefore a certain global reaction from the bunch traversed, in order to be able to distinguish an electron-photon state from an electron state, or an electron-positron state from a photon state.

The quantum aspect of radiation, however, shows more violently its nature when considering the contribution to the amplitude corresponding to regions extending outside the bunch.

The stationary phase conditions lead to simple saddle point integrals over the transverse coordinates [1]. We are left with integrals along the bunch length taken as the z axis (Fig. 1) for the reaction amplitudes. When calculating the rate we have in practice to perform a double integral over z and z' , which both extend from $-\infty$ to $+\infty$. The integrands which involve the matrix element squared already integrated over the transverse coordinates are of course different whether we are in the high field region ($|z| < \frac{L_b}{2}$) or outside it. They can be separately calculated in both cases [1],[6].

In order to discuss the new effect, we can consider the front end of the bunch. We have already dealt with the first integral with $-\frac{L_b}{2} < z < \frac{L_b}{2}$, $-\frac{L_b}{2} < z' < \frac{L_b}{2}$. The integral with $z < -\frac{L_b}{2}$ and $z' < -\frac{L_b}{2}$ is easily dealt with. It is of order α and not multiplied by a large factor $\frac{L_b}{\ell_c}$ as the one found for the first integral [1],[2]. The integral with either $z < -\frac{L_b}{2}$ and $z' > -\frac{L_b}{2}$ or $z' < -\frac{L_b}{2}$ and $z > -\frac{L_b}{2}$, comes up, however, with a large factor. It involves L_e through the quantity $\ln \frac{L_e}{\ell_c}$ which at very high energy becomes important [1],[2],[6]. The contribution to the spectrum has a particular shape, namely

$$I(x) \sim \frac{1 + (1 - x)^2}{x} \quad (7)$$

in the case of beamstrahlung and

$$I(x) \sim x^2 + (1 - x)^2 \quad (8)$$

in the case of pair production [6].

They are different from those found for the terms proportional to the bunch length (2) and (3) [7]. Whilst the former ones are proportional to $\frac{\alpha L_b}{\pi \ell_c}$ the latter ones are proportional to $\frac{\alpha}{\pi} \ln \frac{L_e}{\ell_c}$.

Such openly quantum processes with rates proportional to $\ln L_e$ eventually compare in yields with the more classical ones, proportional to the bunch length, when the energy increases. With these particular spectral shapes their contributions can in principle be clearly separated out in the overall rate.

One may of course try to talk about radiation or pair production “before” bunch crossing. The process is then seen as occurring through a quantum fluctuation. The electron, or the members of the pair are “off shell” but they come back on shell through their “later” interaction with the bunch. However, this is somewhat misleading. The event occurs because of the boundary whilst it cannot be located at the boundary. Its presence translates the existence of two regions with different field properties and the corresponding rate corresponds to two integrals over wide and different domains.

One would naively like to see the pair forming in a particular zone in space. The quantum calculation gives a complete lack of localization for the terms proportional to $\frac{\alpha}{\pi} \ln \frac{L_e}{\ell_c}$. Although one may say that the pair becomes observable only after a distance ℓ_c inside the bunch, pair formation results in part from interferences affecting macroscopic distances and even questions such as “inside the bunch” or “outside the bunch” lose any meaning. At high energy, a considerable amount of radiation or pair formation results from the presence of a boundary but yet their origin cannot be in any way located near that boundary.

3 Conclusions

We have shown that with future very high energy electron-positron colliders operating in the multi-TeV range, a considerable part of the radiation process and of the pair formation process, which both correspond to specific spectrum shapes, are associated with an impressive lack of localization in space, since they result from what takes place both inside and outside a bunch.

In quantum physics this is quite normal. We deal with a stationary process with a double integration over space, or time if one wishes to see it that way. A good fraction of the pairs are produced when the photon is both inside and outside. A good fraction of the photons are radiated when the electron is both inside and outside. The relative amount would increase with ribbon bunches which one may use to minimize beamstrahlung [10]. This will be a clearly seen phenomenon, involving macroscopic distances, and which resists any classical description.

References

- [1] M. Jacob and T.T. Wu, *Phys.Lett.* **B197** (1987) 253; *Nucl.Phys* **B303** (1988) 373; *Nucl.Phys.* **B303** (1988) 389.
- [2] M.A. Bell and J.S. Bell, *Part.Accel.* **22** (1988) 301; **24** (1988) 1.
- [3] M. Jacob and T.T. Wu, *Phys.Lett.* **B216** (1989) 442; *Nucl.Phys.* **B314** (1989) 334 and **B318** (1989) 53; CERN Preprint TH. 6050 (1991) and CERN Preprint TH. 6163 (1991).
- [4] B. Richter, *IEEE Trans.Nucl.Sci.* **32** (1985) 3928;
U. Amaldi, *Phys.Lett.* **B61** (1976) 313;
R. Blankenbecler and S.D. Drell, *Phys.Rev.* **D36** (1987) 277.
- [5] A.A. Sokolov and I.M. Turnov, *Synchrotron Radiation*, Pergamon (1968);
T. Himel and J. Siegrist, SLAC-PUB 3572 (1985).
- [6] M. Jacob and T.T. Wu, *Phys.Lett.* **B221** (1989) 203; *Nucl.Phys.* **B327** (1989) 285;
R. Blankenbecler, S.D. Drell and N. Kroll, *Phys.Rev.* **D40** (1989) 2462;
P. Chen, SLAC-PUB 4822 (1988).
- [7] The proportionality factor giving the actual radiated intensity after integration over $x dx$ depends on the bunch properties. For a realistic multi-TeV machine it is typically of order 0.1.
- [8] V.N. Baier, V.M. Katkov and V.M. Stratkovenko, *Zh.Exp.J.Fiz.* **94** (1988) 125.
- [9] The standard approach makes great use of the classical electron radius $\propto \frac{\hbar}{m}$. This parameter loses its importance in a regime where momentum transfers are much higher (two orders of magnitude) than the electron mass.
- [10] R. Blankenbecler and S.D. Drell, *Phys.Rev.Lett.* **61** (1988) 2324.

Figure 1: The two basic processes, beamstrahlung (a) and pair formation (b). They are presented in the bunch frame where the particles experience only an intense radial electric field. In practice $L_b \gg R$. The reaction process inside the bunch occurs over a distance of order ℓ_c , the coherence length and at an impact parameter r_0 related to the global momentum transferred to the bunch.

Figure 2: Lowest order Feynman graphs for beamstrahlung (a) and pair production (b). The crosses stand for the interaction with the bunch treated by solving the wave equation in the external field.