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We present a comprehensive study of finite volume and lattice spacing effects on observables calculated in the static approximation. Ground state projection using smearing techniques is studied at $\beta = 6.26$. We give high statistics results for the pseudoscalar decay constant after extrapolating to the chiral and continuum limit. In addition results for mass splittings and the distance of string breaking in the $Q\bar{Q}$ potential are presented.

1. Introduction

For the B system it seems very natural to apply the infinite mass effective theory (IMET) on the lattice [1] to calculate interesting quantities like the pseudoscalar decay constant f which has not been measured experimentally so far. A related question which can be studied on the lattice is beyond which mass IMET can be considered a reasonable approximation. Since IMET has important phenomenological implications studying the static limit on the lattice has attracted a lot of attention right after it was proposed. However in the last few years it has become clear that before lattice methods could offer a concrete answer on f and on the validity of the static approximation various systematic errors should first be understood. We aspire to provide answers on some of these issues in this report. Specifically we will be addressing ground state projection using smearing techniques, volume and finite a effects and extrapolation to the continuum limit.

2. Smearing techniques

The first issue concerning the pseudoscalar correlator is whether the ground state can be unambiguously obtained using smearing techniques [2, 3]. A systematic dependence of f and the pseudoscalar mass $\tilde{M}_P$ on the wave function size was reported in ref. [4] where the effect was found to be more pronounced at larger β 's. Here we use $\tilde{M}_P$ to denote the pseudoscalar mass obtained

in the static approximation since this is not the real mass but the mass with the heavy quark mass subtracted. To investigate this issue we performed a careful analysis at $\beta = 6.26$ on a $18^3 \times 48$ lattice with 43 configurations using six wave functions of r.m.s. radius varying from $3.9a$ to $6.5a$. Three of these (denoted by $G1, G2, G3$) are constructed as linear combinations of a local and a gaussian interpolating function, whereas the remaining three are purely gaussian characterised by the two parameters n and α . These wave functions are constructed in a gauge invariant manner and require a very small investment in CPU time. The light quark mass was fixed at about twice the strange quark mass. In order to obtain f we need a good signal for both the "smeared - smeared" (quark fields smeared both at source and sink), and the "local - smeared" (quarks smeared at either source or sink) correlators. Ground state projection can be detected as a plateau in the 'local mass' $\mu(t) = \log(C(t)/C(t-a))$ of the correlator $C(t)$. For the "smeared-smeared" correlator, $C^{SS}(t)$, the plateaus are relatively easy to identify, with the best wave function resulting in a plateau as early as $t = 2a$. The masses obtained by fitting $C^{SS}(t)$ in the plateau yield consistent values for $a\tilde{M}_P = 0.565(8)$. On the other hand identifying the plateaus for the "local-smeared" correlator $C^{LS}(t)$ is much trickier as it can be seen from fig 1. The wave function $G1$ with the smallest r.m.s. radius reaches the plateau at a large value of t where noise dominates. As we increase

the radius the plateaus set in at smaller t until we reach an optimum radius giving rise to the best plateau. Increasing the radius even further results again in reaching the asymptotic value at larger t e.g. wave function with $n = 160$, $\alpha = 5$.

Table 1

Lattices used in this work. N gives the number of independent configurations. The inverse lattice spacing a^{-1} is obtained using the ρ meson mass ($a_{M_\rho}^{-1}$) and, in square brackets, the string tension $\sqrt{\sigma}$ [5]. The spatial extension L of the lattice was estimated using a_{M_ρ} .

β	L/a	L_t/a	N	a^{-1}/GeV	L/fm
5.74	4	24	404	1.45(19) [1.12]	0.55
	6	24	131		0.82
	8	24	270		1.10
	10	24	213		1.37
	12	24	140		1.65
5.82	6	28	100	1.72[3]	0.70
6.00	6	36	227	2.25(10) [1.88]	0.53
	8	36	100		0.70
	12	36	304		1.05
	18	36	27		1.58
	12	48	103		0.64
6.26	18	48	76		0.96

Therefore without prior knowledge of the height of the plateau one is liable to misidentify the position of the plateau and thus arrive at wave function dependent results. For this purpose, we have inserted into fig. 1 the plateau as obtained from a fit to $C^{SS}(t)$ using the best wave function with $\alpha = 4$, $n = 100$. As can be seen the t -dependent masses finally end up in the same plateau. The procedure of first fitting the “smeared-smeared” correlator to obtain the mass $\tilde{M}_P$ and then using this value of $\tilde{M}_P$ in a constrained fit of the “local-smeared” correlator in the range of the plateau yields the same result for f (within error bars) for *all* wave functions.

Having thus established the wave function independence of the static results, for the rest of the lattices we performed the calculation using the best wave function.

Figure 1. Local mass for “local - smeared” correlator for six different wave functions at $\beta = 6.26$. The error band arises from fitting $C^{SS}(t)$ for the best gaussian wave function $n = 100$, $\alpha = 4$.

3. The pseudoscalar decay constant

Our systematic study of volume and finite a effects was performed using the lattices listed in table 1. The quantity

$$\hat{F} = f\sqrt{M_P} \left(\frac{\alpha_s(e^{-2/3}M_P)}{\alpha_s(e^{-2/3}M_B)} \right)^{6/25} \quad (1)$$

is considered since it has a finite limit as $M_P \rightarrow \infty$. In order to check scaling we form the dimensionless ratio $\hat{F}/\sigma^{3/4}$ where the string tension σ is obtained from ref. [5]. We show our results at different β values as a function of $L\sqrt{\sigma}$ in fig. 2 with the light quark mass fixed so that the corresponding pseudoscalar mass $M_P(l, l)$ satisfies $M_P^2(l, l)/\sigma = 4$. If finite a effects are negligible then all data should follow a universal curve. Since all data are statistically independent the relatively reasonable fit of the data (dashed line in fig. 2) to a function depending solely on L is tantamount to a effects smaller than the 10% level of the general error bars. For our most accurate data with errors smaller than 10% finite a effects are of course visible. From fig. 2 we also learn that finite volume effects are negligible beyond lengths of $L\sqrt{\sigma} \sim 3$ for the light quark mass considered

here. Using our accurate results at $\beta = 5.74$ for the lattices with $L/a = 12$ and $L/a = 8$ we do not see any volume dependence also for the lighter quark masses that we used in this study. We thus can use our data at $L\sqrt{\sigma} \sim 3$ for which we have most statistics for a more detailed analysis.

Figure 2. $\hat{F}/\sigma^{3/4}$ vs $L\sqrt{\sigma}$ for the lattices of table 1. The light quark mass is fixed to about twice the strange quark mass.

We extrapolate to $\kappa_{critical}$ by fitting linearly our results for M_π^2 . To fix the physical scale we use both the ρ mass M_ρ and the string tension σ . From our data we seem to find that using f_π is equivalent to using M_ρ since f_π/M_ρ is within our accuracy independent of the light quark masses and consistent with the experimental value. The κ corresponding to the strange quark mass is found by matching to the kaon mass. In fig. 3 we show the results at $\beta = 5.74, 6$ and 6.26 for the decay constant at the chiral limit using both σ and M_ρ to fix the scale. Equivalent plots can be made for the strange quark mass. Setting the scale using $\sqrt{\sigma}$ makes finite a effects rather small and a linear extrapolation to $a = 0$ gives results within error bars of the value obtained at $\beta = 6.26$. On the other hand when M_ρ is used the errors are much larger. The final (extrapolated) results are, however, in agreement for the

two scales. We obtain in $GeV^{3/2}$

$$\hat{F}_s = 0.61(4)(7), \quad \hat{F}_u = 0.53(5)(6) \quad (2)$$

with $\sqrt{\sigma}$ and

$$\hat{F}_s = 0.67(18)(7), \quad \hat{F}_u = 0.59(18)(6) \quad (3)$$

from the M_ρ scale. The first errors are statistical and the second result from the uncertainty in $Z_{stat} = 0.71(8)$ which was determined by using renormalised perturbation theory [6]. At the B meson mass we obtain in MeV

$$f_{bs} = 266(22)(26), \quad f_{bu} = 230(22)(26) \quad (4)$$

and at the D meson mass the extremely high value of $f_{cs} = 405(27)(46)$ MeV making $1/M_P$ corrections necessary.

4. Mass splittings

The mass difference between the scalar and the pseudoscalar, Δ_S , and between the Λ baryon and the pseudoscalar, Δ_Λ , were determined directly by looking at the ratio of the respective correlators. The signal for Δ_S is reasonable but not as good as for the pseudoscalar. Therefore our results for Δ_S are not accurate enough to allow a definite statement on the light quark mass dependence of this quantity. The linear extrapolation to the continuum using $\sqrt{\sigma}$ gives 344(37) MeV at quark mass, m_{2s} , twice that of the strange quark. For Δ_Λ we can only quote a rough estimate of 600 MeV since our signals were not good enough for any extrapolation.

5. String breaking

Here we estimate the distance R_b where the full QCD potential flattens off using only quantities calculated in the quenched approximation ref. [7, 8]. For distances up to about 0.7 fm the QCD potential is approximated rather well by the quenched $Q\bar{Q}$ potential. Also at very large distances we expect the potential to be well represented by the quenched approximation. The asymptotic behaviour of the potential is given by the mass of two mesons in the static approximation. Thus we can obtain an upper bound for R_b

Figure 3. $F = \hat{F}/Z_{stat}$ is shown vs a , in (a) in units of $\sqrt{\sigma}$ and in (b) in units of M_ρ .

by setting $V(R_b) = 2\tilde{M}_P$, with $V(R)$ being the quenched potential [5] including the self-energy term which cancels in the given relation. In fig. 4 we show the a -dependence of R_b at the chiral limit measured in units of $\sqrt{\sigma}$. The corresponding figures for quark masses up to m_2 , show very similar results. Our extrapolation to $a = 0$ gives for all quark masses up to m_2 ,

$$R_b = 1.9(2)(2) \text{ fm.} \quad (5)$$

Figure 4. $R_b/\sqrt{\sigma}$ vs $a\sqrt{\sigma}$ at the chiral limit

Because of the weak dependence of this quantity on the light quark mass, we expect the screening of the potential in full QCD to appear at distances that are rather independent of the dynamical quark mass and hence can be studied with relatively large quark masses.

REFERENCES

- 1 E. Eichten, in *Field Theory on the Lattice*, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 147.
- 2 S. Güsken, in *Lattice 89*, Nucl. Phys. B (Proc. Suppl.) 17 (1990) 361.
- 3 C. Alexandrou, S. Güsken, F. Jegerlehner, K. Schilling and R. Sommer, Phys. Lett. B 256 (1991) 60.
- 4 S. Hashimoto and Y. Saeki, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 381.
- 5 G. S. Bali and K. Schilling, Phys. Rev. D46 (1992) 2636, and K. D. Born *et al*, Nucl. Phys. B (Proc. Suppl.) 20 (1991) 394.
- 6 G. P. Lepage and P. B. Mackenzie, Nucl. Phys. B (Proc. Suppl.) 20 (1991) 173.
- 7 R. Sommer, C. Alexandrou, S. Güsken, F. Jegerlehner and K. Schilling, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 493.
- 8 C. Alexandrou, S. Güsken, F. Jegerlehner, K. Schilling and R. Sommer, PSI preprint, PSI-PR-92-27.