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from a Given Multipole Content**

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From the measurements of the multipole content of a superconducting dipole model magnet an inverse field problem is formulated. Solving this problem by means of mathematical optimization methods leads to the coil block displacements due to manufacturing tolerances.

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Estimation of the Errors of Conductor Positioning in the LHC Main Magnets from a given Multipole Content

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I. INTRODUCTION

CERN is preparing for the construction of a new high energy collider, the Large Hadron Collider (LHC). This new facility is based on a double ring of high field superconducting magnets installed in the 26.7 km long LEP tunnel, above the LEP machine components. Several model magnets have been built in common development programs with industry [1]. Their measured field distribution exhibits multipole components which are due to perturbations of the coil block arrangements in the manufacturing process. In order to reduce these random errors it is necessary to know where they come from, thus allowing to specify appropriate tolerances for the construction.

The paper discusses the inverse field calculation problem associated with this question. It differs considerably from design optimization for the following reasons.

- When optimizing a design a relatively small number of design variables is used, as the designer has already a "knowledge base" and the magnets are considered to be symmetric [2].
- For the inverse problem a large number of design variables have to be considered as there may occur a variety of displacements and conductor errors not respecting the symmetries of the magnet.
- In design optimization of magnetostatic problems the objective function is in most cases at least partly convex around a carefully chosen starting point resulting in a unique solution. The multipole content however can be due to different effects thus leading to a function topology with multiple optima (non convex).
- The efficiency of optimization procedures is very much influenced by the nonlinear constraints and their treatment, whereas in the inverse problem described

here only upper and lower bounds for the design variables are given.

II. THE SUPERCONDUCTING DIPOLE MAGNET

The cross section of the superconducting prototype magnet in its cryostat for LHC is shown in fig. 1. It shows the two beam channels in a common yoke, characteristic of the two-in-one design. The inside diameter of each coil is 50 mm and the axes of the two coils are separated by 180 mm. Each coil is formed of two shells of cables arranged in blocks, surrounded by aluminium alloy collars. The structure is surrounded by an iron yoke which provides a return path for the flux.

Figure 1: Superconducting dipole in its cryostat

The cables are Rutherford type trapezoidal cables made of NbTi/Cu composite strands. The coil block arrangement is supposed to be symmetric with respect to the poles (cf. fig. 1).

The magnet is operated at a 0.6 Tesla field at injection up to a maximum field of about 10 Tesla.

III. THE INVERSE FIELD PROBLEM

Table 1. shows the measured relative multipole content (normal components b_i and skew components a_i at 10 mm radius in units of 10^{-4}) for one of our model magnets built in industry. The multipoles given are those measured at 4.8 T dipole field, where no iron saturation occurs in the yoke and the influence of persistent currents can be neglected. The second column shows the "intrinsic" terms as expected from the coil block arrangement and the two-in-one design with its influence on the normal quadrupole b_2 and the normal sextupole b_3 .

	Measured		Intrinsic	
	Normal	Skew	Normal	Skew
b_2	-1.078		-0.815	
a_2		-1.347		0.000
b_3	2.832		-2.310	
a_3		-0.476		0.000
b_4	-0.142		0.000	
a_4		-0.032		0.000
b_5	0.043		0.097	
a_5		-0.016		0.000
b_6	0.031		0.000	
a_6		-0.009		0.000
b_7	0.066		0.212	
a_7		-0.157		0.000

Table 1: Multipole content of the model magnet measured at 4.8 T, and intrinsic values as predicted for a symmetric design.

The objective function is given by

$$\min z(\vec{X}) = \min \sum_{i=1}^7 p_i \cdot (\|f_i(\vec{X})\|_n + \|g_i(\vec{X})\|_n) \quad (1)$$

with the residues

$$f_i(\vec{X}) = b_i^*(\vec{X}) - b_i \quad (2)$$

$$g_i(\vec{X}) = a_i^*(\vec{X}) - a_i \quad (3)$$

subject to upper and lower bounds for the design variables

$$x_{ll} \leq x_l \leq x_{lu} \quad l = 1, \dots, n \quad (4)$$

where $b_i^*(\vec{X}), a_i^*(\vec{X})$ are the calculated and b_i, a_i are the measured multipoles normalized to the main dipole field in units of 10^{-4} . $\vec{X}$ is the vector of the design variables for the inverse problem, i.e. the possible perturbations of the coil blocks. The disadvantage of least squares objective functions with norm $\|\cdot\|_2$ is the low sensitivity for residue smaller than one. Therefore sufficiently high weighting factors p_i have to be introduced. If the absolute value norm is applied the disadvantage is the nondifferentiable objective function in the optimum.

IV. OPTIMIZATION ALGORITHMS

The optimization procedure consists only of an algorithm for finding the minimum value of an unconstrained objective function as there are no nonlinear constraints to be considered. The treatment of upper and lower bounds for the perturbations (design variables) is identical to the method used in [2]. Both search algorithms without using derivatives of the objective function (ROSENbrock [3] and EXTREM [4] by Jacob) and gradient methods (Davidon-Fletcher-Powell [3] and LEVENberg-Marquard [5]) have been applied. These algorithms have been proved to be efficient in design optimization work as performed in [2].

Because of the appearance of multiple optima the stochastic algorithm (EVOLution strategy by Schwefel [6]) has been tested as well, it showed however only poor performance with a population of 10 "parents" and 60 "children". It cannot be regarded as being user friendly as it is not easy to find the appropriate number of parents and children for the specific problem.

The optimization algorithm EXTREM consists of one-dimensional minimizations by means of Powell extrapolations [3] in a main search direction (user supplied) and a orthogonal direction evaluated by Gram-Schmidt orthogonalization. The main search direction is updated after the one-dimensional minimizations by a vector pointing from the initial outline to the minimum of the previous step. The user has to supply an initial step-size which we have taken to $(x_{lu} - x_{ll})/10$.

The Rosenbrock algorithm carries out search steps with step-size λ in all the n directions of the coordinates. Afterwards there is an extrapolation step carried out over the minimum point found in the search steps. If this extrapolation is a success, λ is increased, otherwise it is decreased. After such a cycle the search directions are updated using the direction towards the success and the remaining $n-1$ directions evaluated from Gram-Schmidt orthogonalization. The user has to supply an expansion factor and a contraction factor (set to 2 and 0.5 respectively).

The Davidon-Fletcher-Powell algorithm uses a quadratic approximation of the objective function where the step-size and direction is given by

$$\Delta \vec{X} = -\mathbf{H}^{-1} \nabla z(\vec{X}) \quad (5)$$

The Hesse Matrix $\mathbf{H}$ does not have to be calculated in each point but is updated iteratively beginning with the unity Matrix $\mathbf{I}$ [3].

The Levenberg-Marquard algorithm was originally developed for nonlinear regression problems using least squares objective functions. We can apply it therefore to our inverse field problem. Assuming the objective function

given by

$$\min z(\vec{X}) = \min \vec{F}(\vec{X})^T \vec{F}(\vec{X}) \quad (6)$$

with the residues f_i arranged in the vector

$$\vec{F}(\vec{X}) = (f_1(\vec{X}), f_2(\vec{X}), \dots, f_k(\vec{X})) \quad (7)$$

and the Jacobi Matrix $\mathbf{J}(\vec{X})$ of $z(\vec{X})$ it yields:

$$\nabla z(\vec{X}) = 2 \cdot \mathbf{J}(\vec{X})^T \vec{F}(\vec{X}) \quad (8)$$

$$\nabla^2 z(\vec{X}) = 2 \cdot \mathbf{J}(\vec{X})^T \mathbf{J}(\vec{X}) + 2 \frac{\partial \mathbf{J}(\vec{X})}{\partial \vec{X}} \vec{F}(\vec{X}) \quad (9)$$

Using again the quadratic approximation of $z(\vec{X})$ and neglecting the second term in eq. 9 the stepsize and direction is given by:

$$\Delta \vec{X} = -\frac{1}{2} [\mathbf{J}(\vec{X})^T \mathbf{J}(\vec{X}) + \lambda \mathbf{I}]^{-1} \cdot \mathbf{J}(\vec{X})^T \vec{F}(\vec{X}) \quad (10)$$

$\lambda \mathbf{I}$ can be regarded as an approximation for the neglected term. With a high λ the algorithm starts in a Gauss-Newton direction and λ is decreased in the optimization procedure because the neglected term gets less and less important with smaller and smaller residue.

To test the convergence of the optimization algorithms we calculated the possible perturbations which lead to a multipole content as given in table 1. As variables of the inverse problem we took 20 positioning angles and 24 radial positions of the blocks as design variables for the optimization problem (cf. fig. 2). The positioning angles of the blocks 1-2, 5-6, 13-14 and 17-18 are taken identical as the gap between these blocks is well defined by a kapton layer of 0.1 mm thickness, thus azimuthal deformations are assumed to be the same for these pairs of blocks.

In table 2 some data for the convergence behavior of the optimization algorithms are given for two different objective functions $z(\vec{X})$ with norm $\| \cdot \|_2$ and penalty factors $p_i = 1000$. resp. with the absolute value norm $\| \cdot \|$. The number of function evaluations (nfunc) and the obtained values for $z(\vec{X})$ are given. In the first case the absolute value norm in the obtained solution is given in addition in order to compare the results with the minimum values in the second case.

In the beginning, there is a better convergence when the absolute value norm is taken but the achieved accuracy is less because of the non-differentiable function topology. The algorithms determined earlier because no further minimization of the objective function value could be achieved.

$z(\vec{X})$	$\sum 1000 \cdot \ \cdot \ _2$			$\sum \ \cdot \ $	
	nfunc	$z(\vec{X})$	$\sum \ \cdot \ $	nfunc	$z(\vec{X})$
ROSEN	1795	200.9	1.333	711	6.96
EXTREM	975	150.4	1.172	737	5.46
LEVEN	1272	41.5	0.299	-	-
DFP	-	-	-	380	6.65

Table 2: Performance of optimization algorithm

The results are more accurate with a least squares objective function and a penalty factor set to 1000. The DFP algorithm, however, showed only poor performance in this ill-conditioned function topology. The most appropriate algorithm for this kind of minimization problems is the Levenberg-Marquard algorithm which is specially adopted to least squares minimization. It aborted in the case of the absolute value error function.

V. RESULTS

Figure 2 and 3 show the displacements of the coil blocks found by the Levenberg-Marquard algorithm after about 1200 function evaluations applying least squares objective function. Because there are far more unknowns than residues we cannot expect unique solutions of the problem. The algorithm has therefore been started with different initial guesses. Two different results have been displayed in figure 2 and 3 in which all displacements are represented by linearly scaled arrows.

Figure 2: Coil block displacements

- [6] Schwefel H.P.: *Numerische Optimierung von Computer-Modellen mittels der Evolutionsstrategie*, Birkhauser Verlag, 1977

Figure 3: Coil block displacements

The arrow in fig. 3 indicating the azimuthal displacement in block 23 represents 0.46 deg. The arrow in fig. 2 pointing in radial direction represents 0.3 mm in block 23. Although the solutions differ slightly, it can be seen that the spacers between block 5 and 7, 6 and 8, 18 and 20 and 17 and 19 in the inner layer are smaller than designed. This result is in accordance to geometric measurements made of the coils before the collaring. The prestress has been chosen so that at high excitation the Lorentz forces on the coils are compensated for. At 4.8 T however the coils are still slightly elliptic as can be seen from the radial displacements. It is interesting that this deformation is not symmetric with respect to the poles. That comes from the two in one design, where the forces in the cooldown do not act symmetrically.

VI. REFERENCES

- [1] Perin, R.: The Superconducting Magnet System for the LHC, *IEEE Transactions on Magnetics*, 1991
- [2] Russenschuck S., Tortschanoff T.: Optimization of the Coil of the LHC main Dipole, *companion paper*
- [3] Himmelblau, D.M.: *Applied nonlinear Programming*, McGraw-Hill, New-York 1972
- [4] Jacob, H.G.: *Rechnergestützte Optimierung statischer und dynamischer Systeme*, Springer, 1982.
- [5] Kuester, J. Mize J.H.: *Optimization Techniques with Fortran*, McGraw-Hill, 1973.