EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN LIBRARIES, GENEVA

CM-P00063433

CERN AT / 92-28 (MA)
LHC Note 206

Optimization of the Coils of the LHC Main Dipole

S. Russenschuck, T. Tortschanoff

Abstract

Mathematical optimization techniques are applied in order to find the most appropriate design for the superconducting bending magnets for the CERN Large Hadron Collider. By this means a coil block arrangement for a 56 mm aperture is derived, having a considerably reduced multipole content compared to that obtained in previous design studies.

Presented at the 1992 Conference on Electromagnetic Field Computation
(CEFC), Claremont, Ca, 3-5 August 1992

Geneva, Switzerland
November 1992

Optimization of the Coil of the LHC main Dipole

S. Russenschuck, T. Tortschanoff
CERN, 1211 Geneva 23, Switzerland

Abstract - Mathematical optimization techniques are applied in order to find the most appropriate design for the superconducting bending magnets for the CERN Large Hadron Collider. By this means a coil block arrangement for a 56 mm aperture is derived, having a considerably reduced multipole content compared to that obtained in previous design studies.

I. INTRODUCTION

The Large Hadron Collider (LHC) project is proposed as the future extension of the CERN accelerator complex. The LHC requires twin aperture superconducting dipoles of highest possible field to guide the proton beams in the existing LEP tunnel of 26.7 km circumference and to make possible colliding experiments of up to 2×8 TeV [1]. Following a recent review suggesting an increased aperture and further reduction of the higher multipoles, especially of the 14 pole and 18 pole component, it became evident that a new coil geometry with new cable distribution had to be found. Mathematical optimization techniques have been applied in order to find a coil block arrangement with low multipole errors, high dipole field and a working point not higher than 95 % along the load line.

II. SIMULATION OF THE LHC DIPOLE COILS

The cables of the main dipoles are Rutherford type trapezoidal cables made of NbTi/Cu composite strands, and are arranged in blocks in two layers. The position of each cable is evaluated with respect to its height, width and keystone angle. This allows the investigation of different cable dimensions and the taking into account of the influence of the insulation material. The fact that the keystoneing of the cables is not sufficient to allow their edges to be positioned on the curvature of a circle, is fully respected. This effect increases with the inclination of the coil blocks versus the radial direction. The characteristic data of the cable is given in table 1. In this table $J_c(10.8T, 2K)$ is the minimum guaranteed current density with the cable immersed in a field of 10.8 T and cooled to 2 K.

Each cable is then approximated by line currents carrying the same current located at equally spaced radial and azimuthal positions inside the cable. Thus the grading of the current density due to the keystoneing is accounted for. The influence of the iron yoke is considered by imaging the line currents, assuming that the yoke iron has an infinite permeability. Saturation effects and the quadrupole and sextupole errors due to the two-in-one construction are independent of the coil design. By applying Biot-Savart's law for each line current, the field is calculated at a given radius in the aperture (10 mm) and in the coil itself.

	Cable for inner layer	Cable for outer layer
Inner width (mm) (inc. 0.1 mm insulation)	2.04	1.58
Outer width (mm) (inc. 0.1 mm insulation)	2.36	1.85
Height (mm) (inc. 0.19 mm insulation)	15.78	15.68
Diameter of strands (mm)	1.14	0.84
Number of strands	26	36
Cu/Sc	1.6	1.8
$J_c(10.8T, 2K) (A/mm^2)$	1087.	1032.
$dJ_c/dB (A/mm^2T)$	505.	590.
$dJ_c/dT (A/mm^2K)$	500.	500.

Table 1: Characteristic data of the cable

As the magnetic length of the dipoles is much larger than their aperture, the two-dimensional field calculation is appropriate. By this approach we are able to calculate the multipole content and the peak field in the coils with a higher accuracy than with finite-element calculations (this is useful, as the multipoles to be minimized create fields of the order of $10^{-7}T$ at the nominal radius).

Convergence tests have been carried out in order to guarantee accuracy with a minimum of computation time. It has been found that a discretization of $N_1 = 3$ (azimuthal direction) by $N_2 = 30$ (radial direction) line currents per cable is sufficient. For the harmonic analysis the field is evaluated at 80 points, equally spaced on the radius of the multipole analysis. The field is then harmonically analyzed at this radius by means of the program TRICOF from the CERN Library. For the estimation of the peak field in the cable it is sufficient to divide into $N_1 = 1$ by $N_2 = 7$

elements.

The current density in the superconductor is given by

$$J_{sc} = \frac{I_{cable}}{A_{sc}} = \frac{I_{cable} \cdot (1 + \frac{Cu}{Sc})}{n \cdot \frac{d^2 \pi}{4}} \quad (1)$$

where I_{cable} is the total current in the cable, n is the number of strands in the cable and d is the strand diameter (cf. table 1). From the critical current density J_c and dJ_c/dB the percentage of the critical current density in the working point can be evaluated. The short sample $B - J$ characteristic curves are approximated by

$$J_{sc} = -\frac{dJ_c}{dB} \cdot B + J_c \quad (2)$$

at 1.8 Kelvin (cf. table 1). The load line characteristic for a dipole with different cable parameters in the inner and outer layer is plotted in fig. 1.

Figure 1: Load line characteristic

III. THE OPTIMIZATION PROBLEM

An optimization problem can be expressed as

$$\min f(\vec{X}) \quad f: R^n \Rightarrow R \quad (3)$$

with the design variable vector $\vec{X} = (x_1, x_2, \dots, x_n)$ and the

general nonlinear objective function f , subject to the following nonlinear constraints

$$g_i(\vec{X}) \leq 0 \quad i = 1, \dots, l \quad (4)$$

$$h_j(\vec{X}) = 0 \quad j = 1, \dots, m \quad (5)$$

and

$$x_{ll} \leq x_l \leq x_{lu} \quad l = 1, \dots, n \quad (6)$$

with x_{ll} and x_{lu} the lower and upper bounds of the design variables. The design variables in the vector $\vec{X}$ describe the geometry. The feasible domain is given by

$$R = \{ \vec{X} \in R^n \mid g_i(\vec{X}) \leq 0; h_j(\vec{X}) = 0; x_{ll} \leq x_l \leq x_{lu} \forall i = 1, \dots, l; j = 1, \dots, m; l = 1, \dots, n \} \quad (7)$$

For the LHC main dipole the design variables may be the angles φ_i and the inclination angle α_i of the blocks 2,4,5,6. The numbering of the blocks can be seen in fig. 2 for the initial design together with the design variables. The number of turns was as from previous design studies $2 \cdot (9 + 15)$ in the outer layer and $2 \cdot (4 + 4 + 3 + 2)$ in the inner layer.

Figure 2: Coil block arrangement (initial design) and design variables for the optimization problem

The optimization problem for the dipole is given by

$$\min \sum_{i=3,5,7,9} p_i \cdot b_i(\vec{X})^2 \quad (8)$$

with

$$\vec{X} = (\varphi_{i=2,4,5,6}, \alpha_{i=2,4,5,6}, I) \quad (9)$$

subject to nonlinear constraints

$$b_1(\vec{X}) > 10 \quad (10)$$

$$r_{JB}(\vec{X}) < 95 \quad (11)$$

and in addition lower and upper bounds for the design variables (6) arranged in the design variable vector $\vec{X}$. The $b_i(\vec{X})^2$ are the calculated normal multipoles and p_i are weighting factors to compensate for the different sensitivity of the multipoles. Because of the symmetry there appear no skew components. $b_1(\vec{X})$ is the dipole field in Tesla and $r_{JB}(\vec{X})$ the relative performance (in %) along the load line. The constraints are that the dipole field is higher than 10 T, or at least as high as possible, and that the working point is not higher than 95 % of the load line. If the cable dimensions are not chosen carefully, one may not obtain a 10 T field without violating the load line constraint. The current in the cables I has then to be decreased. Therefore the current I is considered as an additional design variable of the mathematical optimization problem.

IV. MATHEMATICAL TREATMENT OF THE OPTIMIZATION PROBLEM

One way of treating nonlinear constrained optimization problems is by transformation into a set of unconstrained problems by means of a penalty function method and the application of an optimization algorithm for the solution of unconstrained problems.

The main idea of the penalty function method [2] is to add a penalty term to the objective function which depends on the degree to which the constraints are violated and vanishes if the constraints are satisfied. The optimization problem (3)-(5) transformed into the penalty function reads:

$$P(\vec{X}, p) = f(\vec{X}) + \sum_{i=1}^l p_i \cdot \max^2(0, g_i(\vec{X})) + \sum_{j=1}^m p_j \cdot h_j^2(\vec{X}) \quad (12)$$

As the design variables of the optimization problem may only be changed between upper and lower bounds (6), a modified objective function is applied for the $k+1$ th. function evaluation.

$$Q(\vec{X}^{k+1}) = \begin{cases} P(\vec{X}^{k+1}, p) & \text{no bound violated} \\ 10^5 \cdot P(\vec{X}^k, p) + r(\vec{X}) & \text{bound violated} \end{cases} \quad (13)$$

with

$$r(\vec{X}) = \sum_l r_l \begin{cases} (x_l - x_{lu} + 1)^2 & \text{if } x_l > x_{lu} \\ (x_{lu} - x_l + 1)^2 & \text{if } x_l < x_{lu} \\ 0 & \text{else} \end{cases} \quad (14)$$

and a sufficiently high penalty parameter r_l . The advantage of this procedure is that the bounds are checked before a function evaluation is carried out and in case of a violation the process is immediately returned to the optimization algorithm.

In order to achieve a feasible result large penalty factors p_i have to be chosen. Large penalty factors, however, lead to illconditioned function topologies, and the optimization algorithms need many function evaluations to find the minimum. It was therefore proposed to solve a sequence of unconstrained minimization problems with increasing penalty factors (SUMT) [2]. However, as the objective function is discontinuous from step to step, one has to restart the optimization algorithm with updated determination criteria. In practical applications it seems more reliable to adjust the penalty factor once and keep these values for the next runs. It should be pointed out that from experience the efficiency of the whole optimization procedure mainly depends on the treatment of the constraints, i.e. the choice of the penalty parameters, whereas for a given unconstrained minimization problem the performance of the algorithms differs only slightly.

The resulting unconstrained minimization problem reads:

$$\min \{ \sum_{i=3,5,7,9} p_i \cdot b_i(\vec{X})^2 + p_b \cdot \min^2(0, b_1(\vec{X}) - 10) + p_r \cdot \max^2(0, r_{JB}(\vec{X}) - 95) \} \quad (15)$$

with the penalty factors $p_3 = 6$, $p_5 = 100$, $p_7 = 600$, $p_9 = 1000$, $p_b = 10$ and $p_r = 1000$ which are appropriate for good convergence of the method and a good representation of the requirements. They have been found in an iterative decision making process [3] and are dependent on the user's preference and on the sensitivity of the different multipoles (cf. table 3). A method to guide the choice of the penalty factor other than experience is the Lagrange-multiplier estimation which has been described in [4].

A different approach is the application of the exact penalty method where the absolute value is used in (12) and (15) instead of the squares. The residual is then greater near the optimum point and the penalty factors may be reduced. This leads to non-differentiable objective functions in the optimum where the treatment with search or gradient algorithms converges poorly. For these applications stochastic algorithms are usually applied [5].

As in our case the optimization problem is convex in at least an area around the optimum, we stick to the search algorithms which show a much faster convergence behavior. The uniqueness of the solution (convexity of the objective function) has been checked by starting from a number of different starting points and achieving the same optimum.

The applied optimization algorithm is the search routine EXTREM by Jacob [6] which proved to be very efficient in a sense of different criteria such as convergence behavior, robustness (the success in obtaining a solution without problem dependence or user supplied parameters) and accuracy.

V. RESULTS

The optimization procedure allows the investigation of many different coil block arrangements with different numbers of turns in the blocks, and to compare them in their individually optimized form. It became evident that in block 2 of the outer layer, turns had to be added. In the inner layer the coil design with 5 turns in block 4 is appropriate, whereas block 5 contains only 2 turns. The cross section of the optimal design for the specific cable and an aperture of 56 mm is given in fig. 3. The characteristic data of the dipole are given for the optimized design in table 2. The design meets perfectly the requirements of $b_7 < 2 \cdot 10^{-6}$ and $b_9 < 5 \cdot 10^{-7}$.

Table 3 shows the sensitivity of the multipoles on the design variables. The values are a measure for errors due to perturbations in the construction process.

	Inner layer	Outer layer
Number of turns	$2 \cdot (4 + 5 + 2 + 2)$	$2 \cdot (10 + 17)$
Peak field (T)	9.99	9.05
r_{JB} (%)	94.7	95.0
Current (A)	13349	
Dipolefield (T)	9.758	
Quenchfield (T)	10.246	
$b_3/b_1 \cdot 10^{-4}$	0.0052	
$b_5/b_1 \cdot 10^{-4}$	0.0000	
$b_7/b_1 \cdot 10^{-4}$	0.0000	
$b_9/b_1 \cdot 10^{-4}$	-0.0006	
φ_2 (deg.)	24.99	
α_2 (deg.)	29.93	
φ_4 (deg.)	20.27	
α_4 (deg.)	20.31	
φ_5 (deg.)	47.47	
α_5 (deg.)	49.76	
φ_6 (deg.)	65.06	
α_6 (deg.)	59.03	

Table 2: Characteristic data for the optimized design

	$\partial b_1/$	$\partial b_3/$	$\partial b_5/$	$\partial b_7/$	$\partial b_9/$
$\partial \varphi_2$	-0.0467	-4.6122	0.1504	0.0094	-0.0001
$\partial \alpha_2$	-0.0074	-0.7021	0.0158	0.0014	0.0000
$\partial \varphi_4$	-0.0123	-5.4675	-0.3782	0.0471	0.0097
$\partial \alpha_4$	-0.0026	-1.0706	-0.0841	0.0034	0.0012
$\partial \varphi_5$	-0.0077	-0.8932	0.3329	-0.0037	-0.0053
$\partial \alpha_5$	-0.0019	-0.2165	0.0585	0.0002	-0.0007
$\partial \varphi_6$	-0.0091	0.9312	0.1040	-0.0381	0.0054
$\partial \alpha_6$	-0.0022	0.1448	0.0275	-0.0063	0.0006

Table 3: Sensitivity analysis for the optimized design

Figure 3: Cross section of the optimized design

VI. REFERENCES

- [1] Perin, R.: The Superconducting Magnet System for the LHC, *IEEE-Transactions on Magnetics*, 1991
- [2] Himmelblau, D.M.: Applied nonlinear Programming, *McGraw-Hill*, 1972
- [3] Stadler, W.: Preference optimality and application of Pareto-optimality in Marzollo, Leitmann: Multicriterion Decision Making, CISM Courses and Lectures, *Springer*, 1975
- [4] Russenschuck S.: Application of Lagrange-multiplier estimation to the design optimization of permanent magnet synchronous machines, *IEEE Transactions on Magnetics*, Vol-Mag 28, 1992
- [5] Gottvald, A.: Comparative analysis of optimization methods for magnetostatics, *IEEE Transactions on Magnetics*, 1988
- [6] Jacob, H.G.: Rechnergestützte Optimierung statischer und dynamischer Systeme, *Springer*, 1982.